AI Science and High School Mathematics

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Version 1.3

Artificial Intelligence & Information Analysis Lab

AI Science and Mathematics

- **What is AI?**
- Data and Vectors
- Clustering
- Abstraction
- Classification
- Neural Networks
- Computer Vision
- Natural Language Processing
- Knowledge
- AI and Society
- AI and the Environment

What is AI?

- ***AI Science and Engineering*** (AISE) is the interdisciplinary, scientific study and engineering of ***Artificial Systems*** that mimic and/or surpass ***human intelligence*** in information analysis and ***human interaction*** with the world.

- Core AISE disciplines are:
  - ***Machine Learning*** (ML),
  - Classical (Symbolic) ***Artificial Intelligence*** (AI)

What is AI?

- Closely related AISE disciplines:
  - ***Robotics,***
  - Autonomous Systems,
  - Digital Signal/Image Processing and Analysis,
  - Data Science and Data Analytics
  - ***Network Theory***.

- Very useful in defining:
  - Data analysis modes, applications.

What is AI?

- Complementary AISE-related disciplines:
  - Cognitive Science,
  - Neuroscience,
  - Psychology,
  - ***Philosophy, Ethics***
  - Linguistics
  - Sociology.

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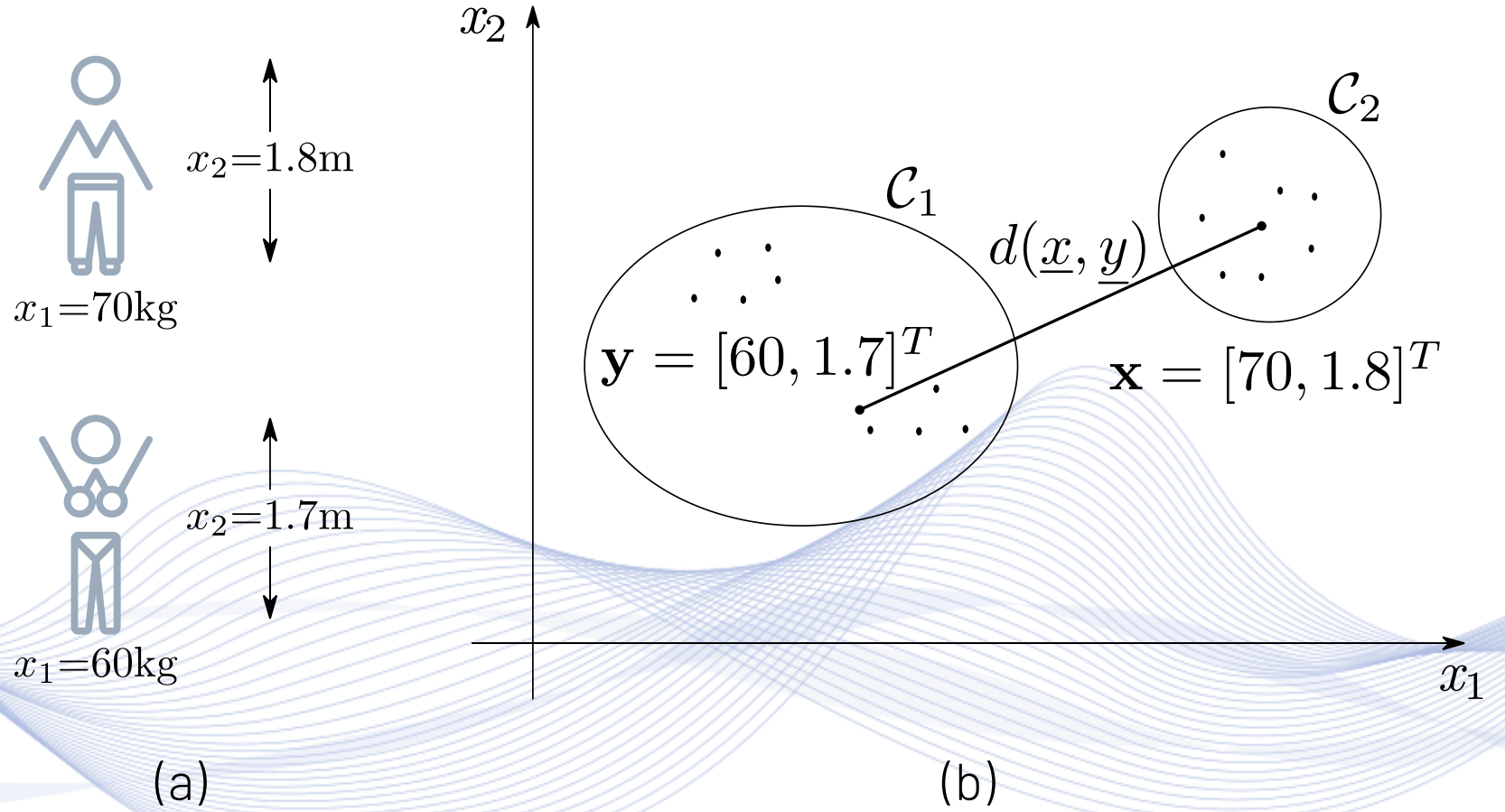
Data and Vectors

- ***Data***: measured quantities related to nature and/or human activities.

- ***Data are primarily numbers*** representing object characteristics (***features***).
- ***Measured in bits.***

- ***Data can be organized in vectors.***

Data and Vectors



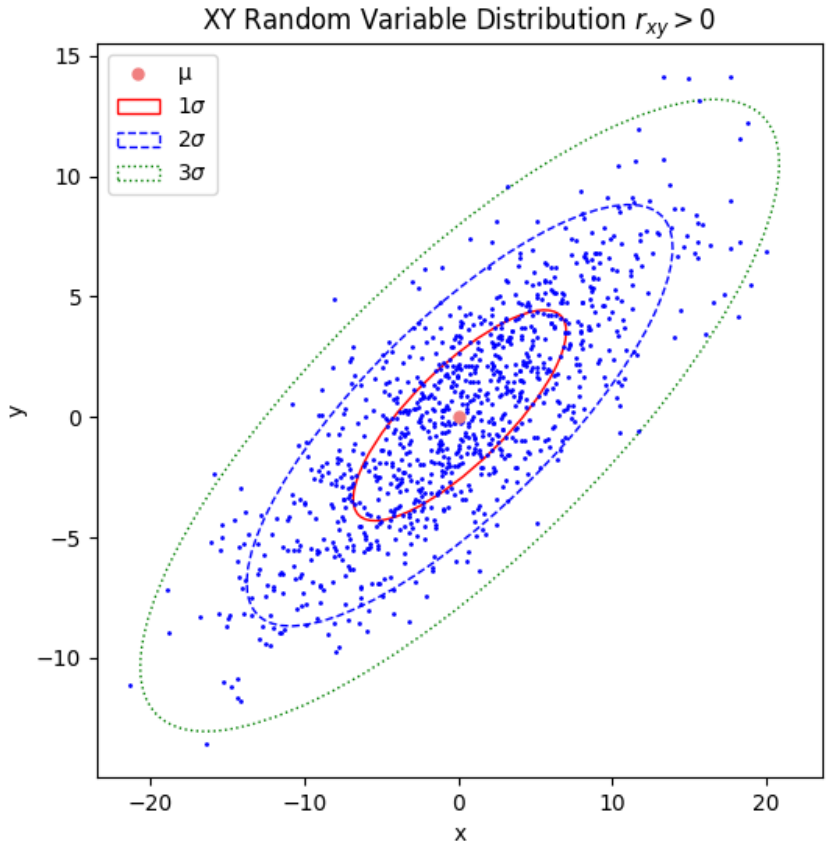
Measuring humans and producing their weight and height vectors.

Data and Vectors

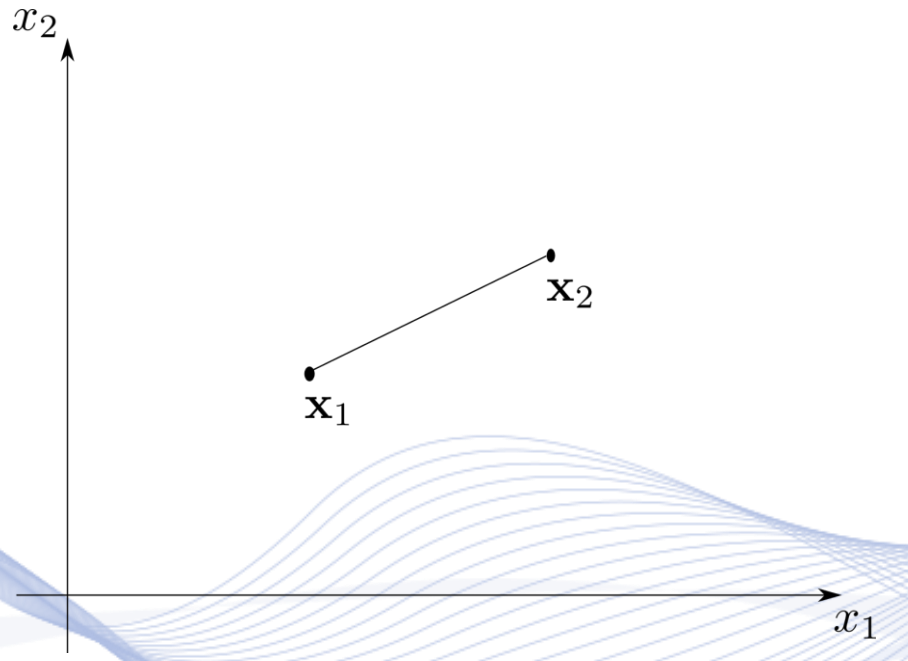
Measurements and data may:

- contain noise and/or
- have natural variability.
- Their features may be correlated.

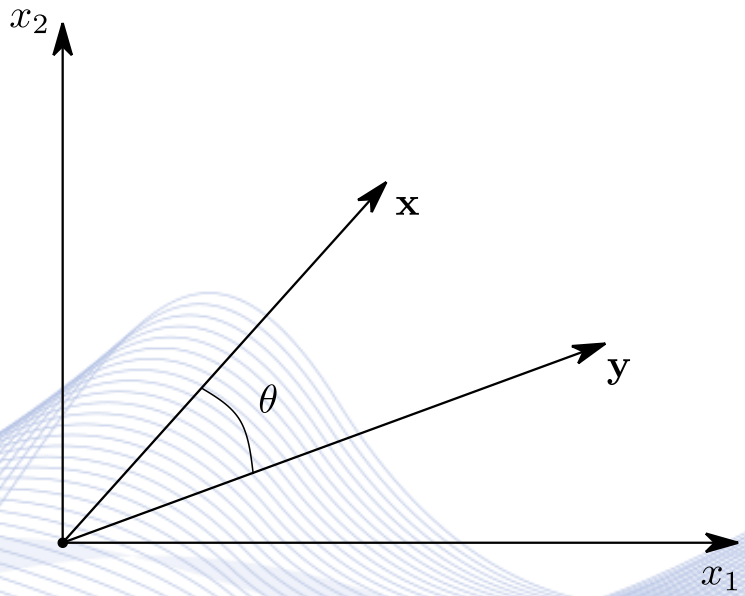
They can be treated using Probability Theory and/or Statistics.



Data and Vectors



Distance between two points (e.g., two humans).



Angle between two vectors.

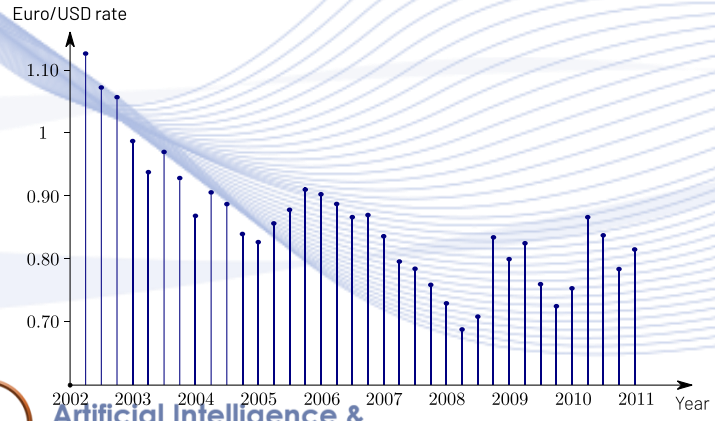
Data and Vectors

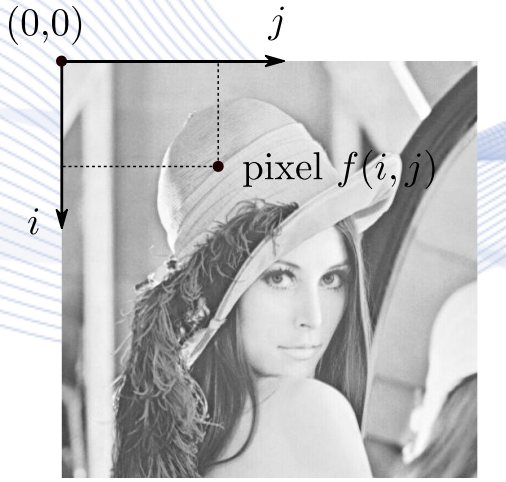
- Signals and object features can be represented by ***vectors***:

$$\mathbf{x}^T = [x_1, x_2, \ldots, x_n].$$

Data vectors can also represent:

- Music, financial time series etc.
- images and videos.



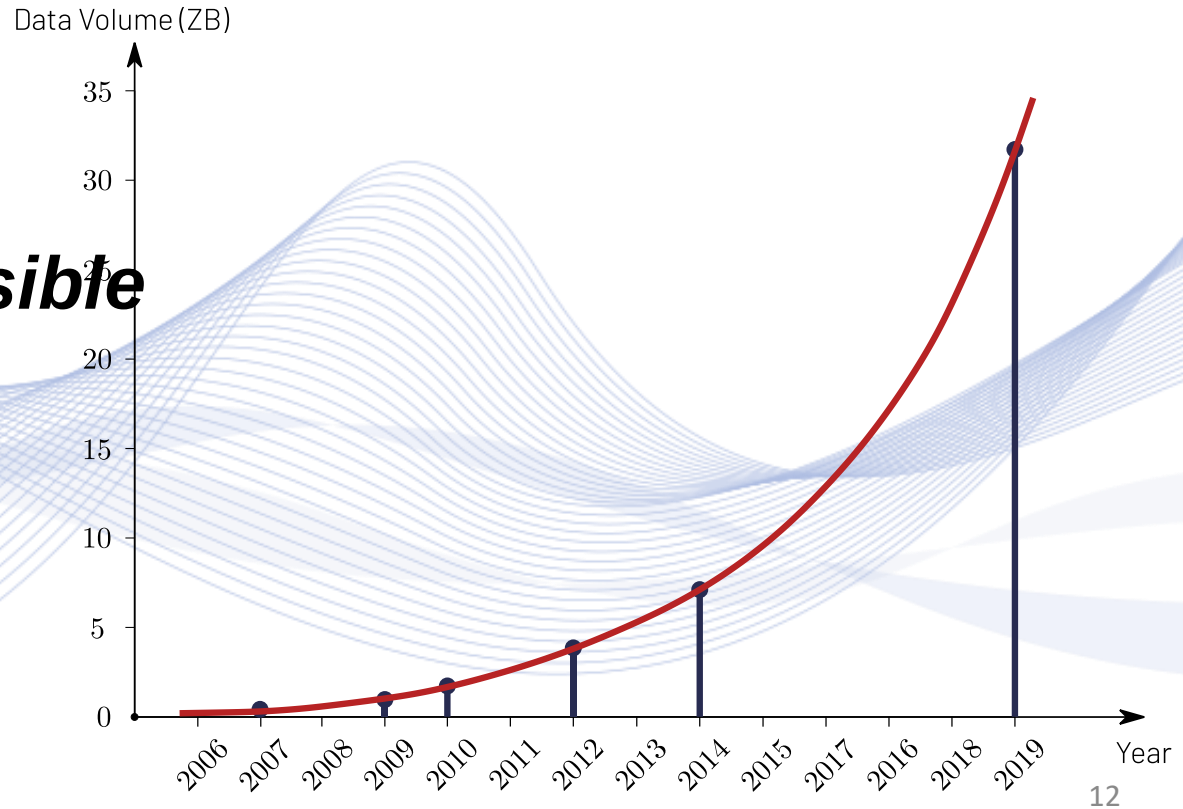


Data and Vectors

Exponential data increase:

- Proliferation of sensors
- Detailed recording of nature and humans
- Sensing automation.

Big data analytics is only possible through Machine Learning.



Data volume increase in past decade.

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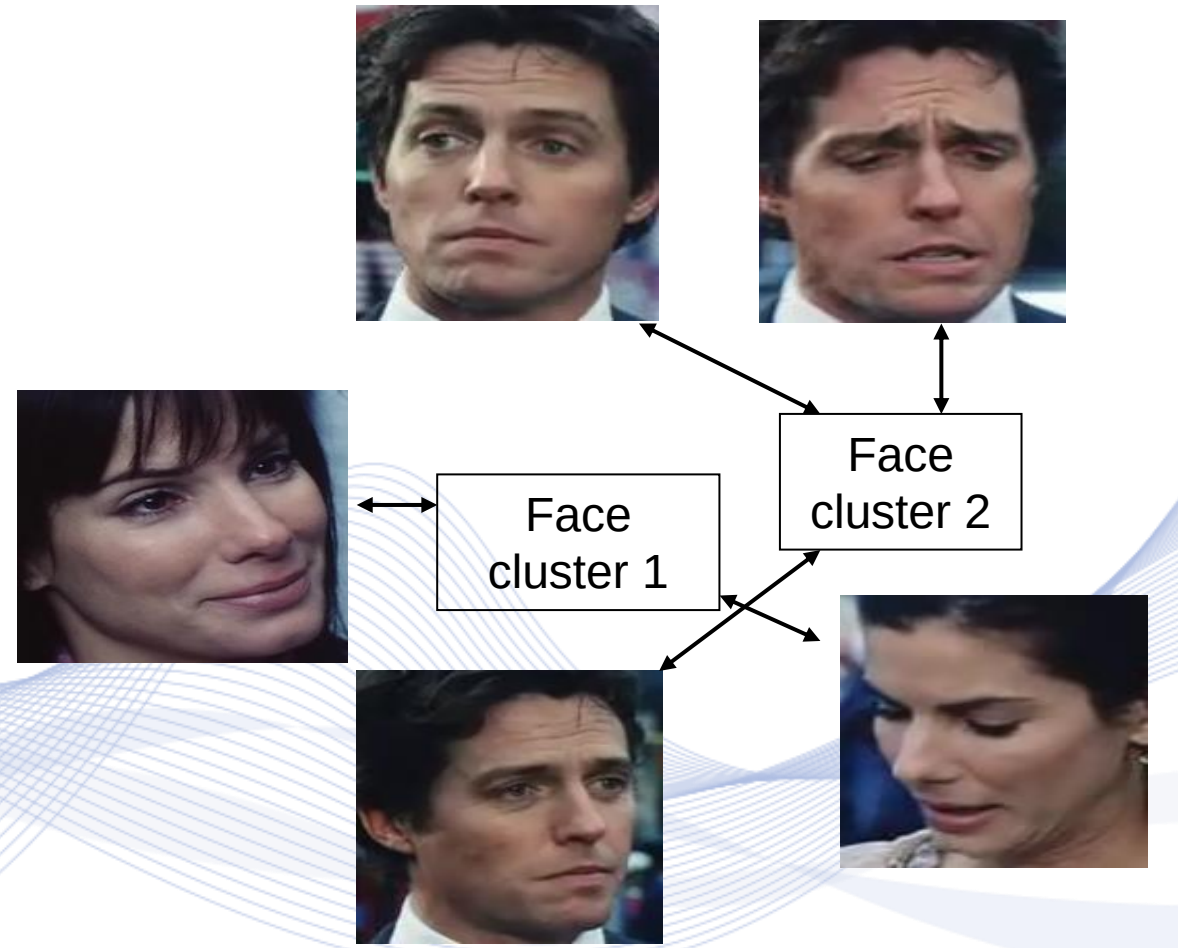
Data clustering

Face clustering:

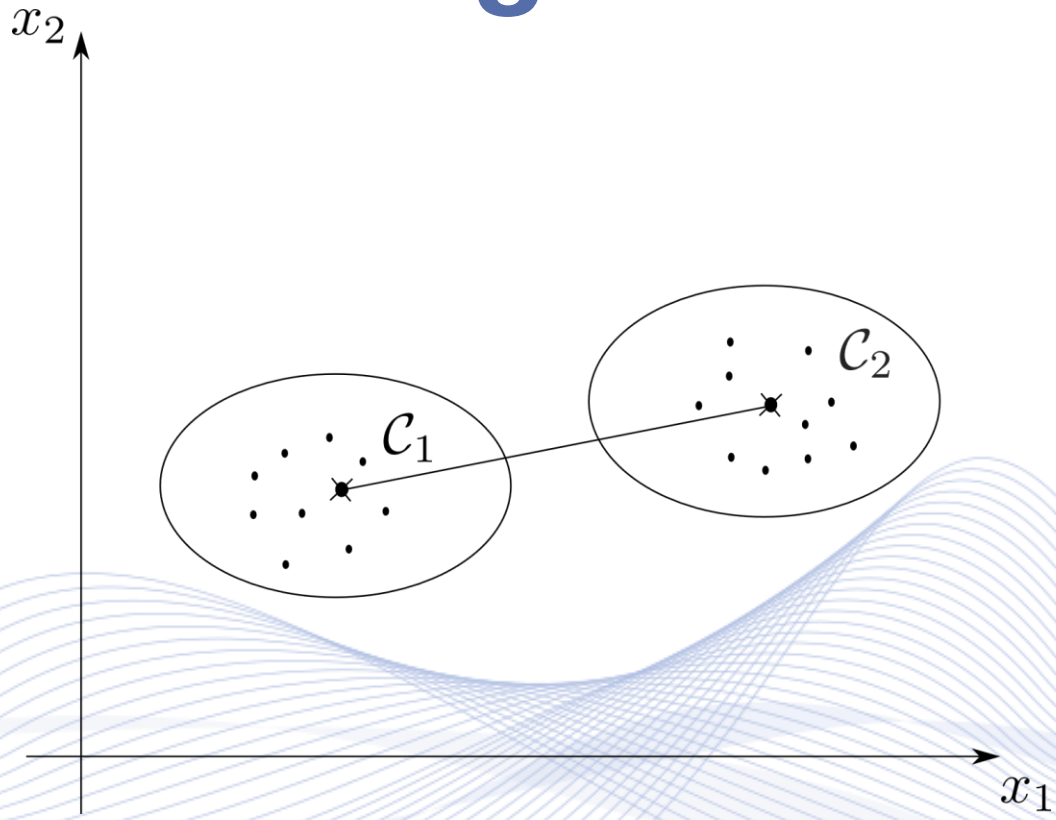
- ***Input***: many facial ROIs
- ***Output***: facial image clusters.

- ***Unsupervised learning.***

- Applications:
  - Biometrics
  - Surveillance applications
  - Video analytics.



Data clustering



Set partitioning. Data clusters should: a) be homogeneous; b) distant from each other.

Data clustering

Data clustering offers:

- Description of data geometry.
- Data visualization.

- ***Abstraction.***
- ***Data compression.***

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Abstraction

Concepts and ideas (‘ιδέες’ in Greek).

- Concepts are specific mental constructs residing in our mind (brain?) that refine and abstract ideas.

- ***Concept instances.***

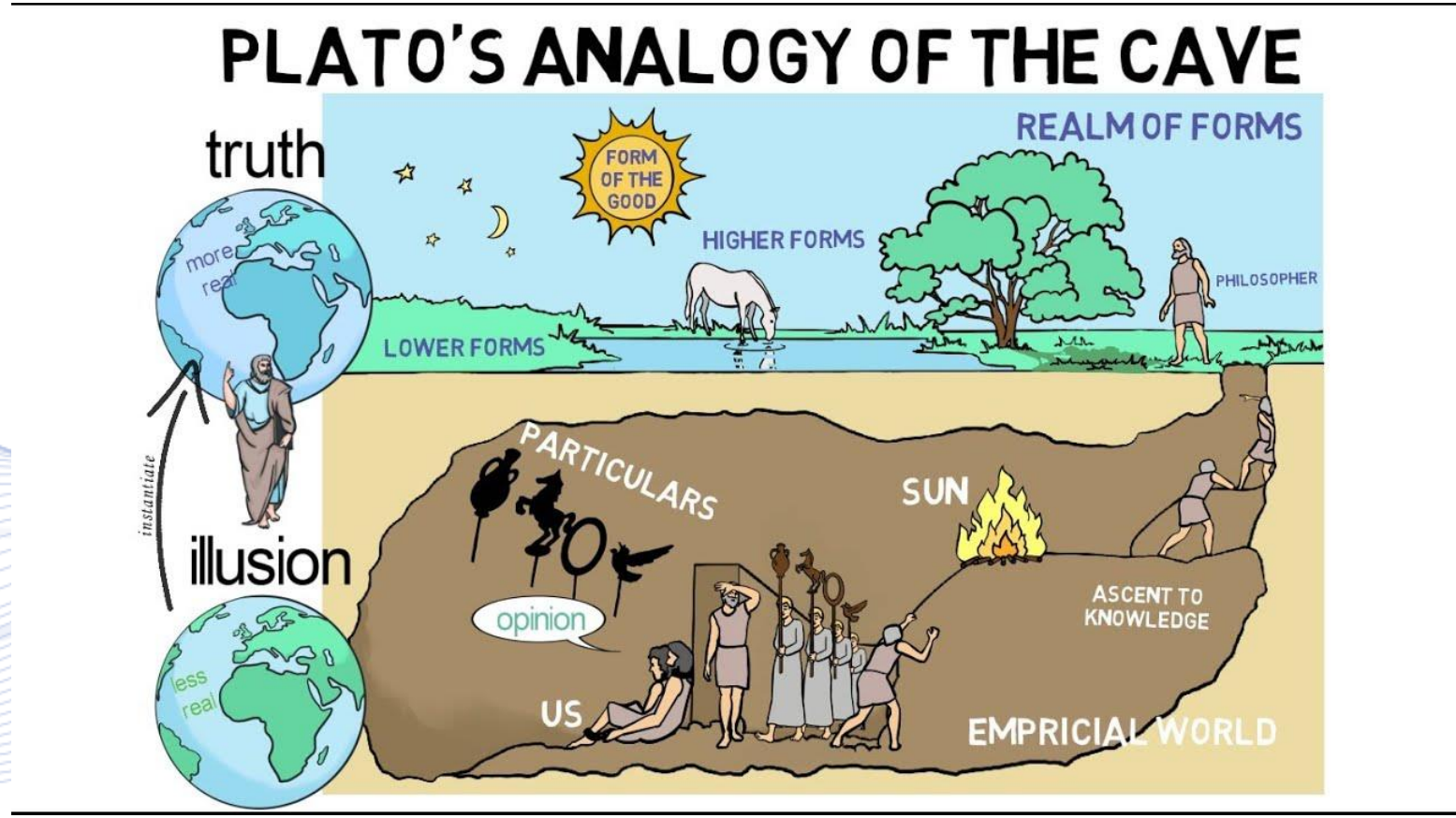
Instances of a triangle.

- ***Abstraction and generalization***:
  - Simplification and data compression.
  - Important procedures in education.

Abstraction

Ideas in Philosophy.

- Plato's cave.
- ***Idealism***: reality is a reflection of ideas.
- ***Materialism***: ideas are shadows of matter on itself (brain).



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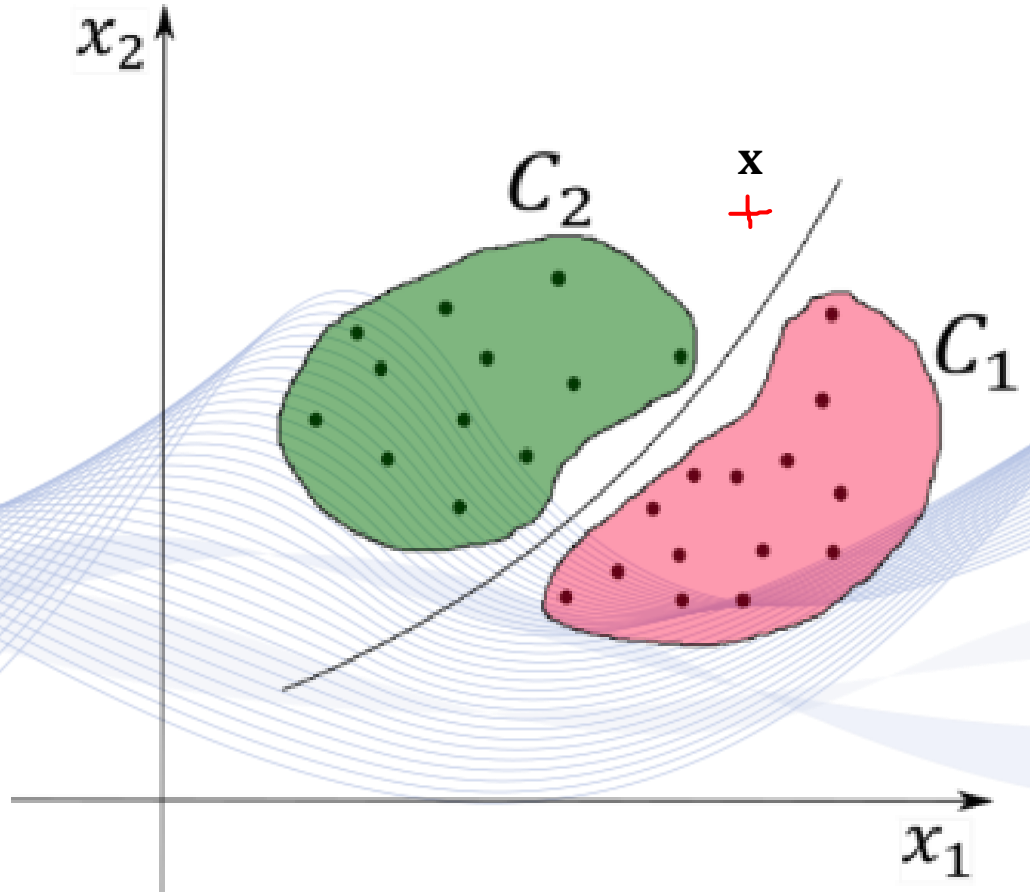
Classification

Supervised Machine Learning

Decision-making theory:

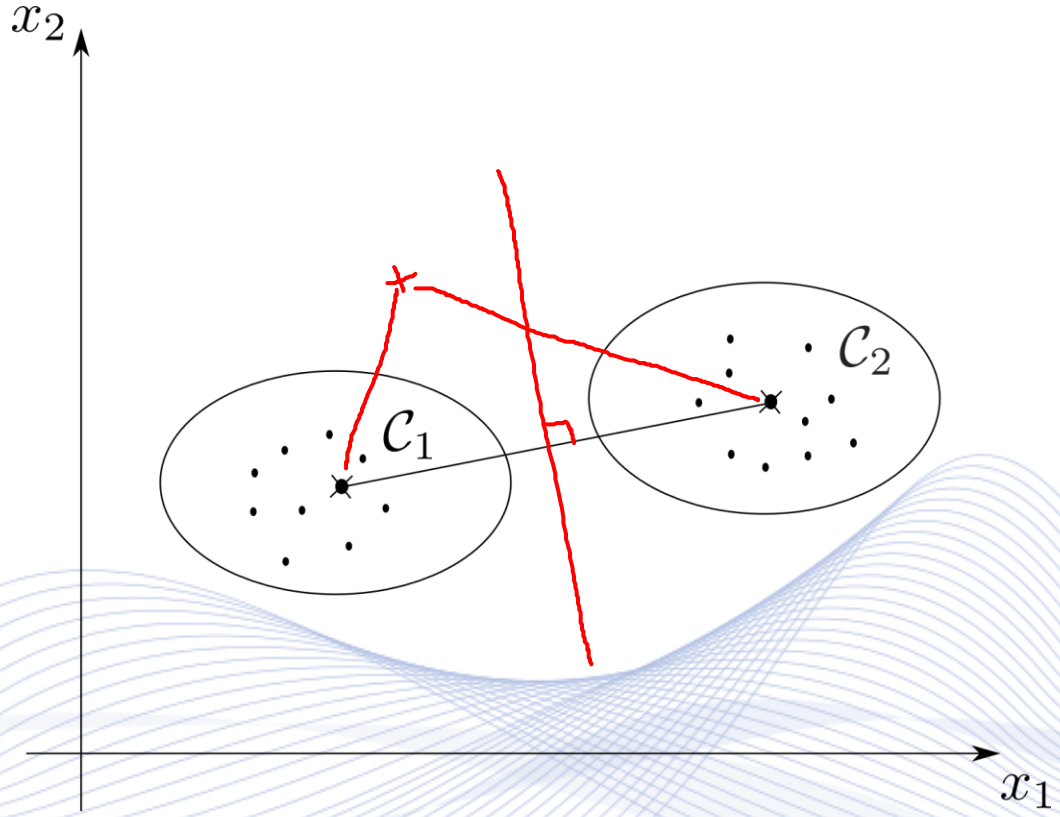
- Does object $\mathbf{x}$ belong to class $\mathcal{C}_1$ or class $\mathcal{C}_2$?
- E.g., ‘car’ or ‘pedestrian’?



Examples:

- Autonomous systems.
- Medical diagnosis.

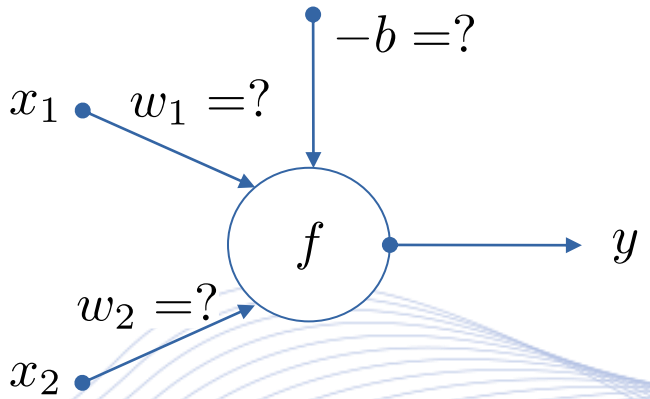
Classification

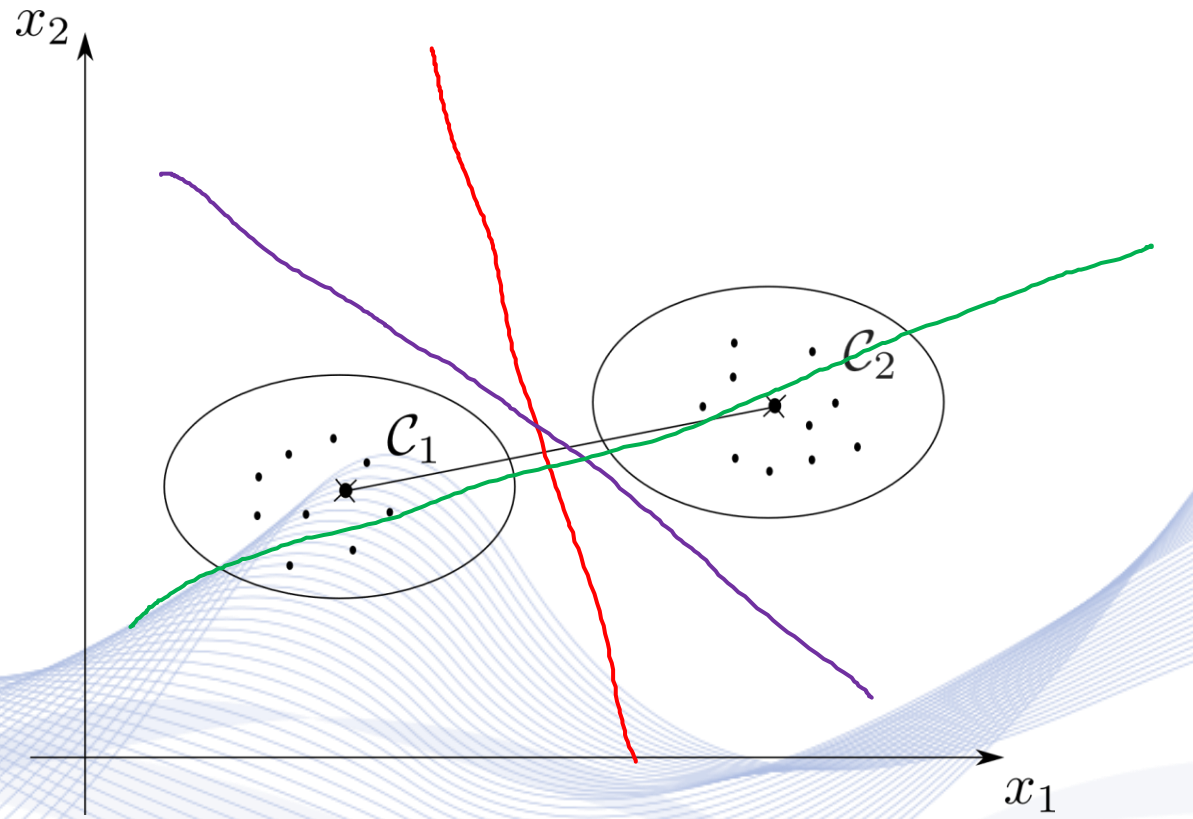


Distance-based classification.

Classification

2D perceptron.





Separating line: $w_1 x_1 + w_2 x_2 + b > 0.$

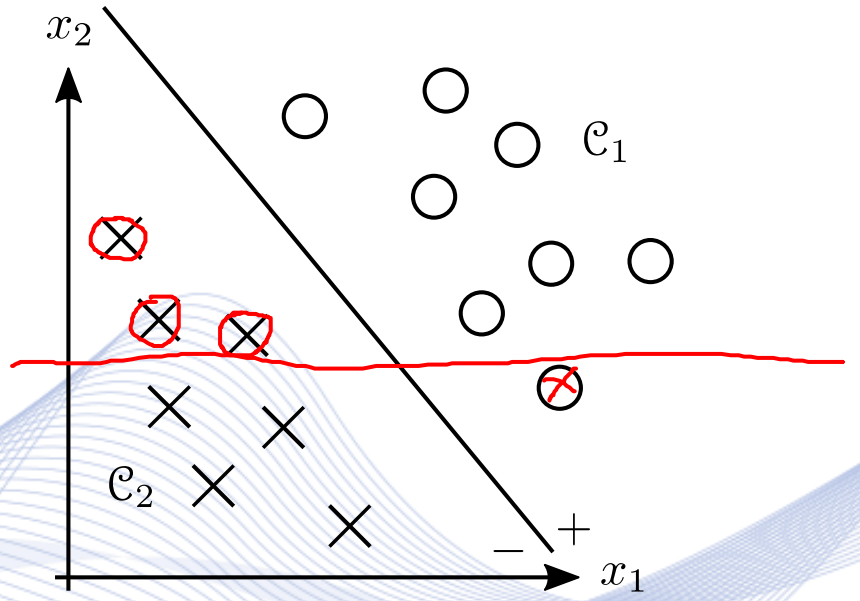
Classification

2D perceptron.

Decision line: $w_1 x_1 + w_2 x_2 + b > 0$.

Classification error minimization:

$J(w_1, w_2, b) = 4$.



Optimization problem.

- Use gradients to find the minimum!

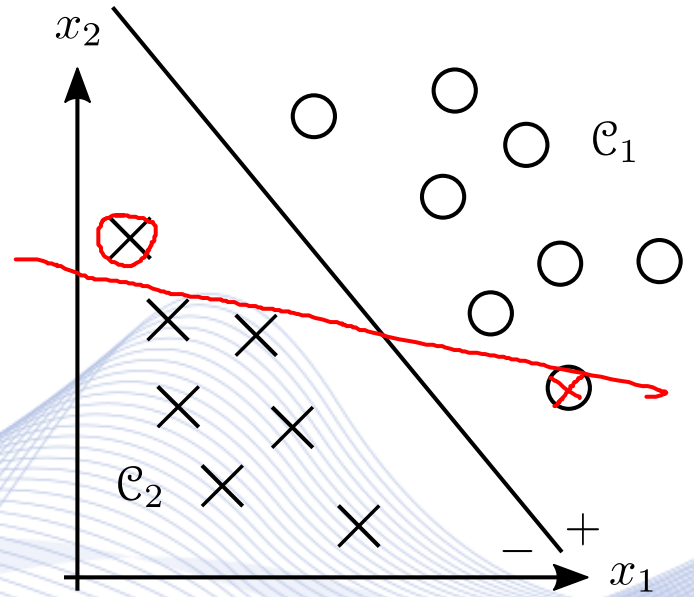
Classification

2D perceptron.

Decision line: $w_1 x_1 + w_2 x_2 + b > 0$.

Classification error minimization:

$J(w_1, w_2, b) = 2$.



Optimization problem.

- Use gradients to find the minimum!

Classification

2D perceptron.

Decision line: $w_1 x_1 + w_2 x_2 + b > 0.$

Classification error minimization:

$J(w_1, w_2, b) = 0.$

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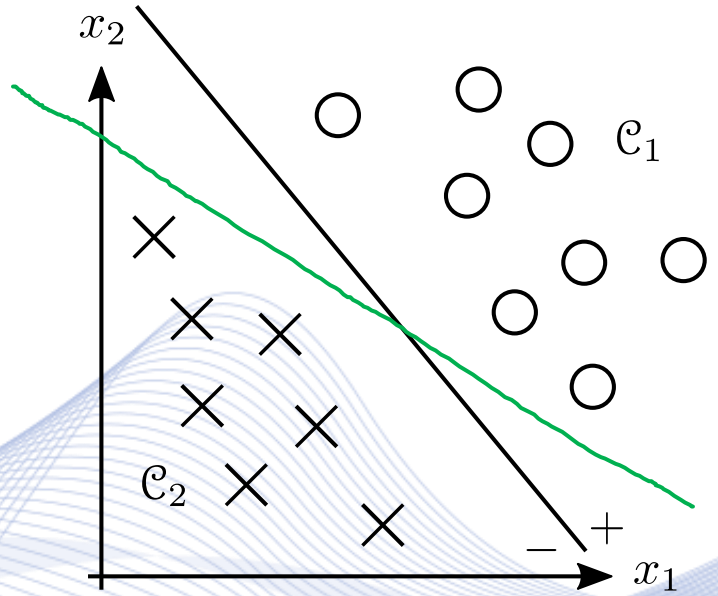
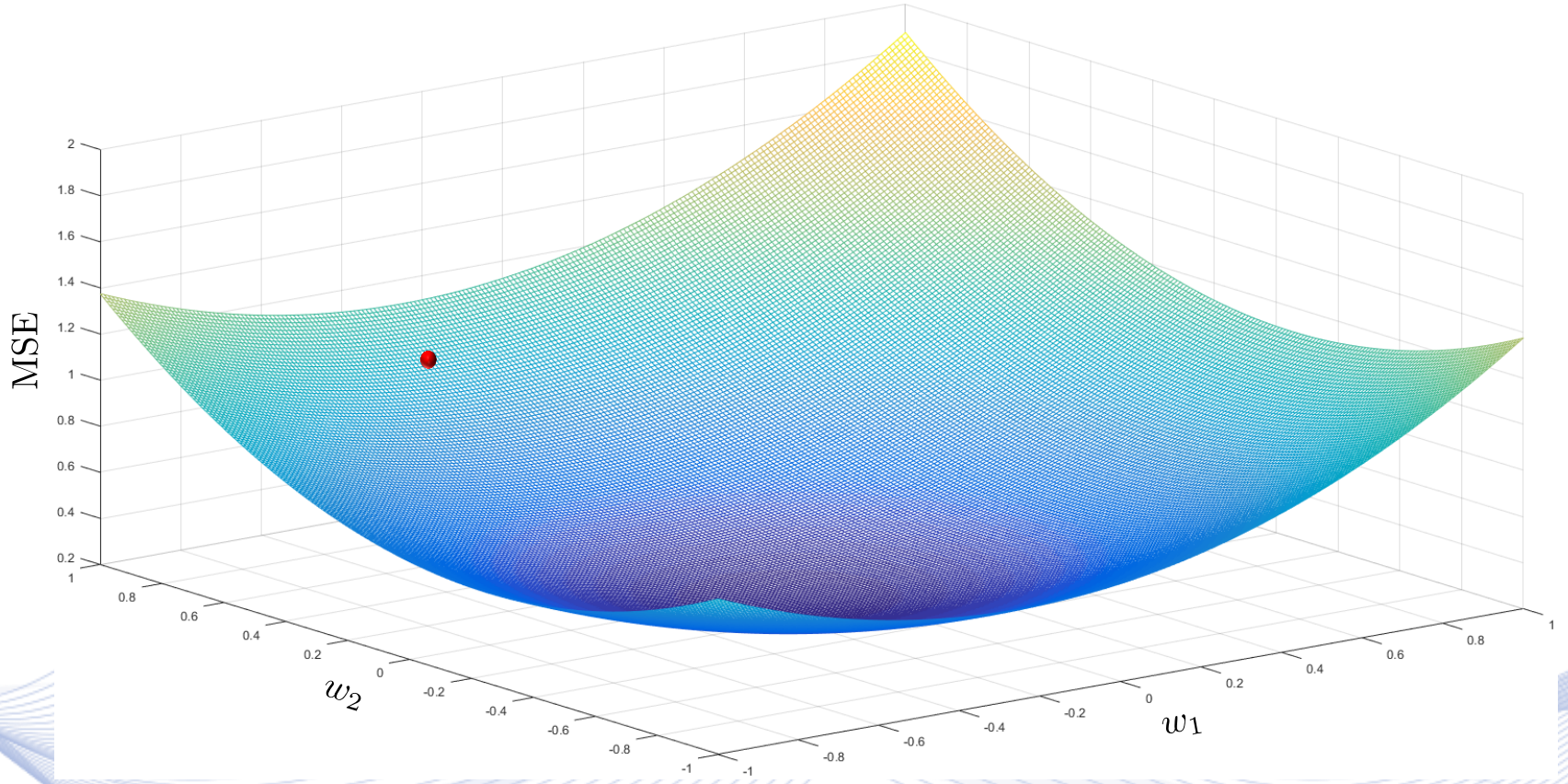


Optimization problem.

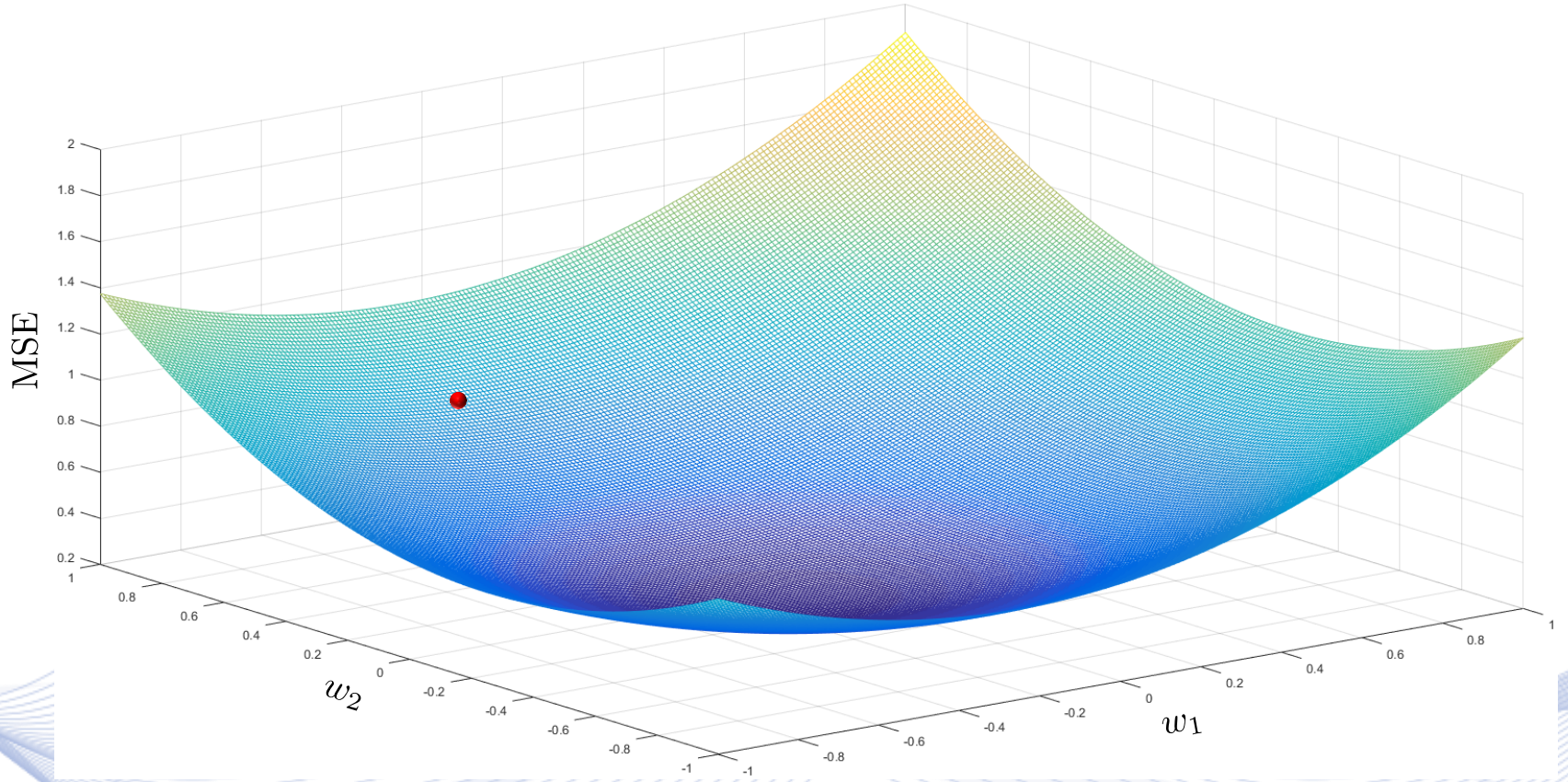
- ***Use gradients (derivatives) to find the minimum!***

Classification



Perceptron training through classification error $J(w_1, w_2)$ minimization.

Classification



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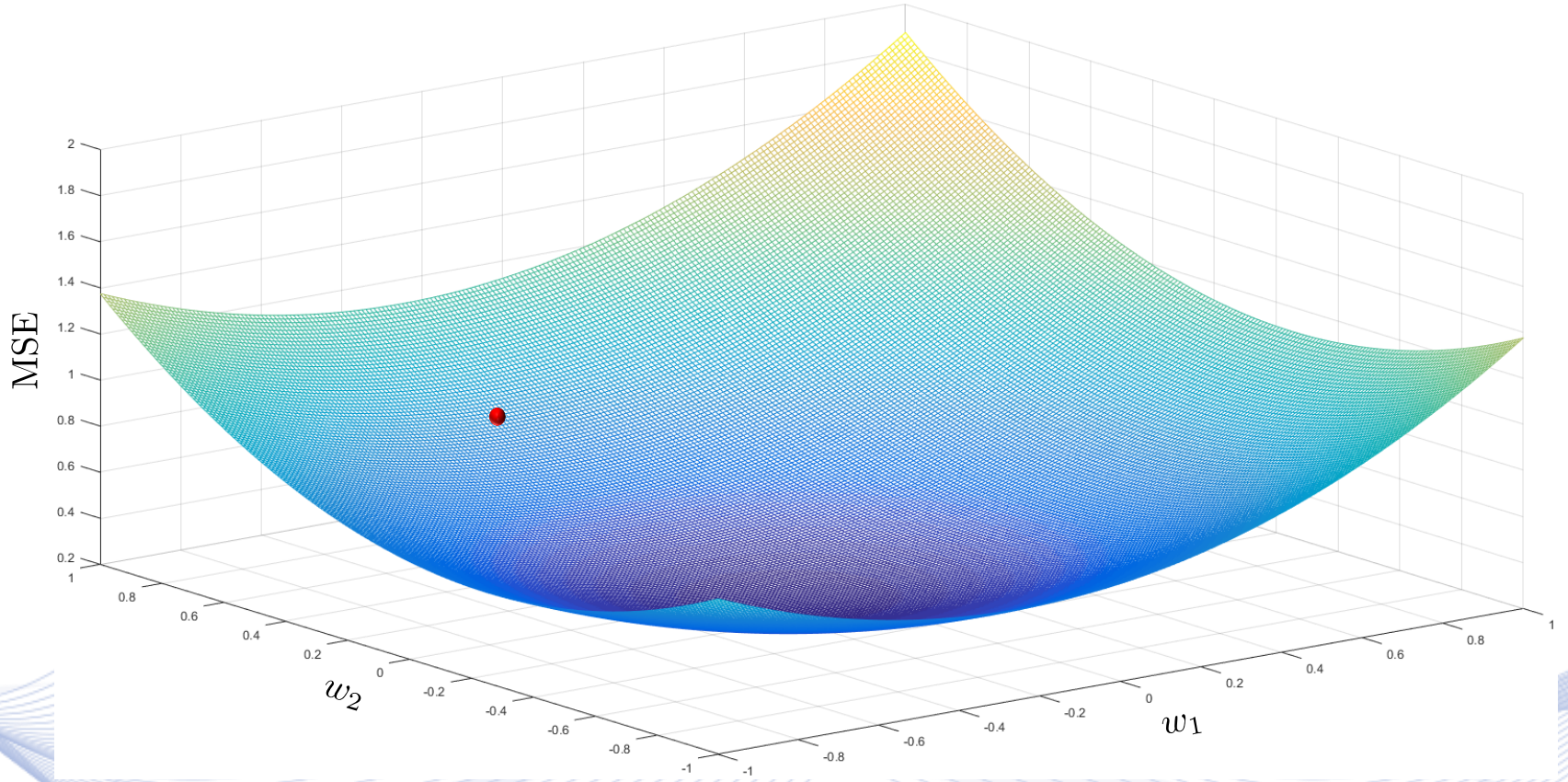

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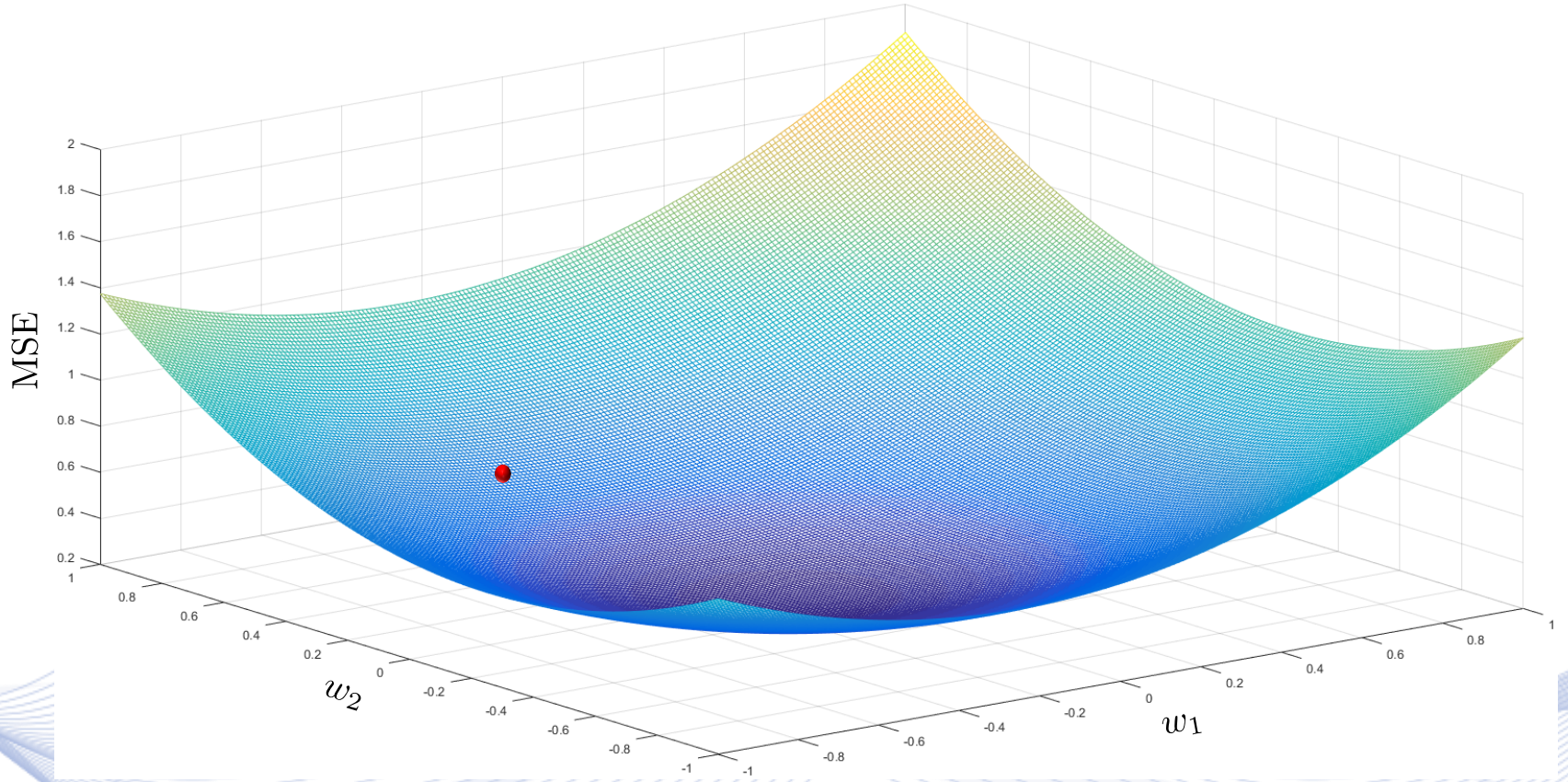

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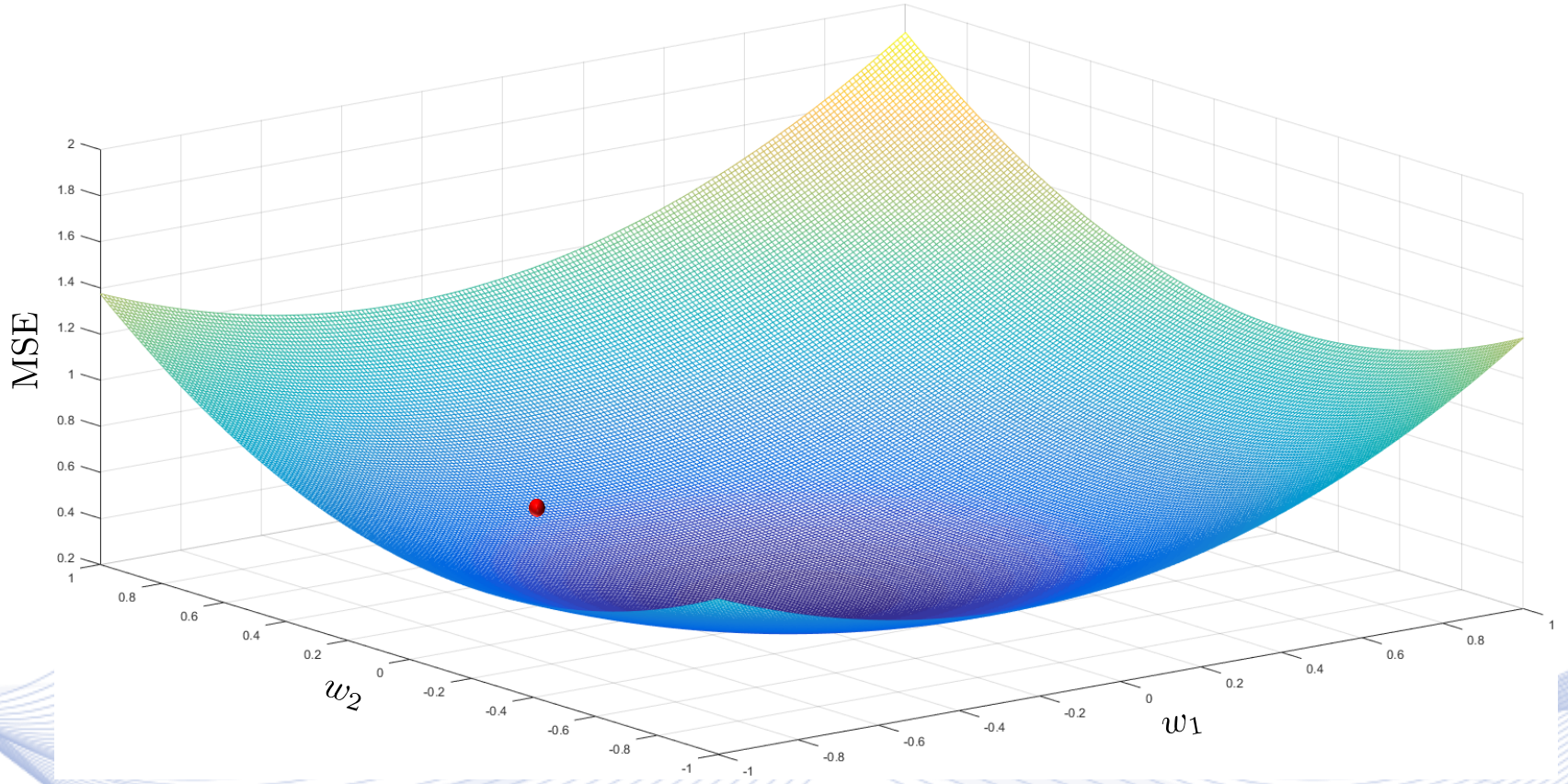

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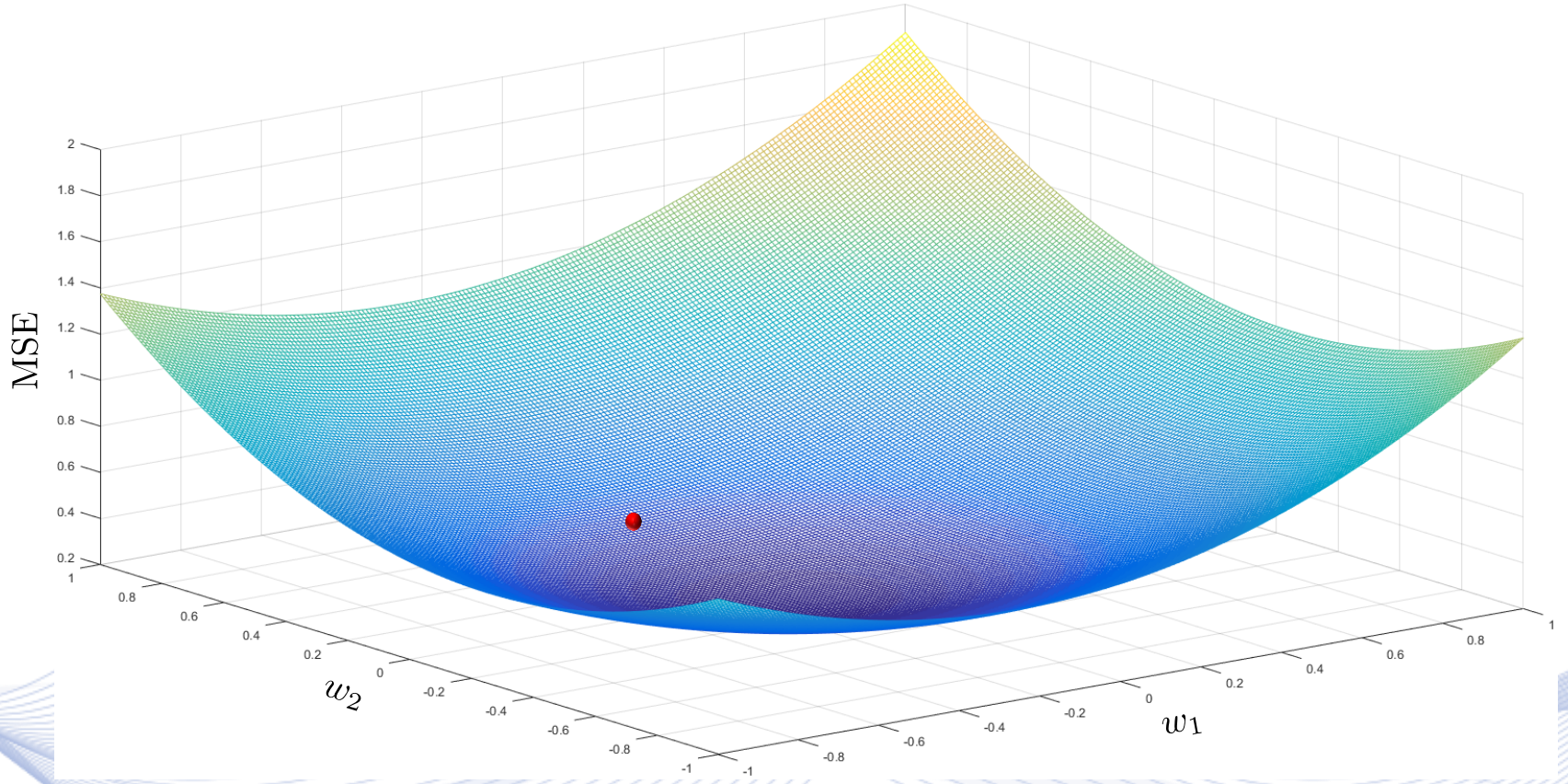

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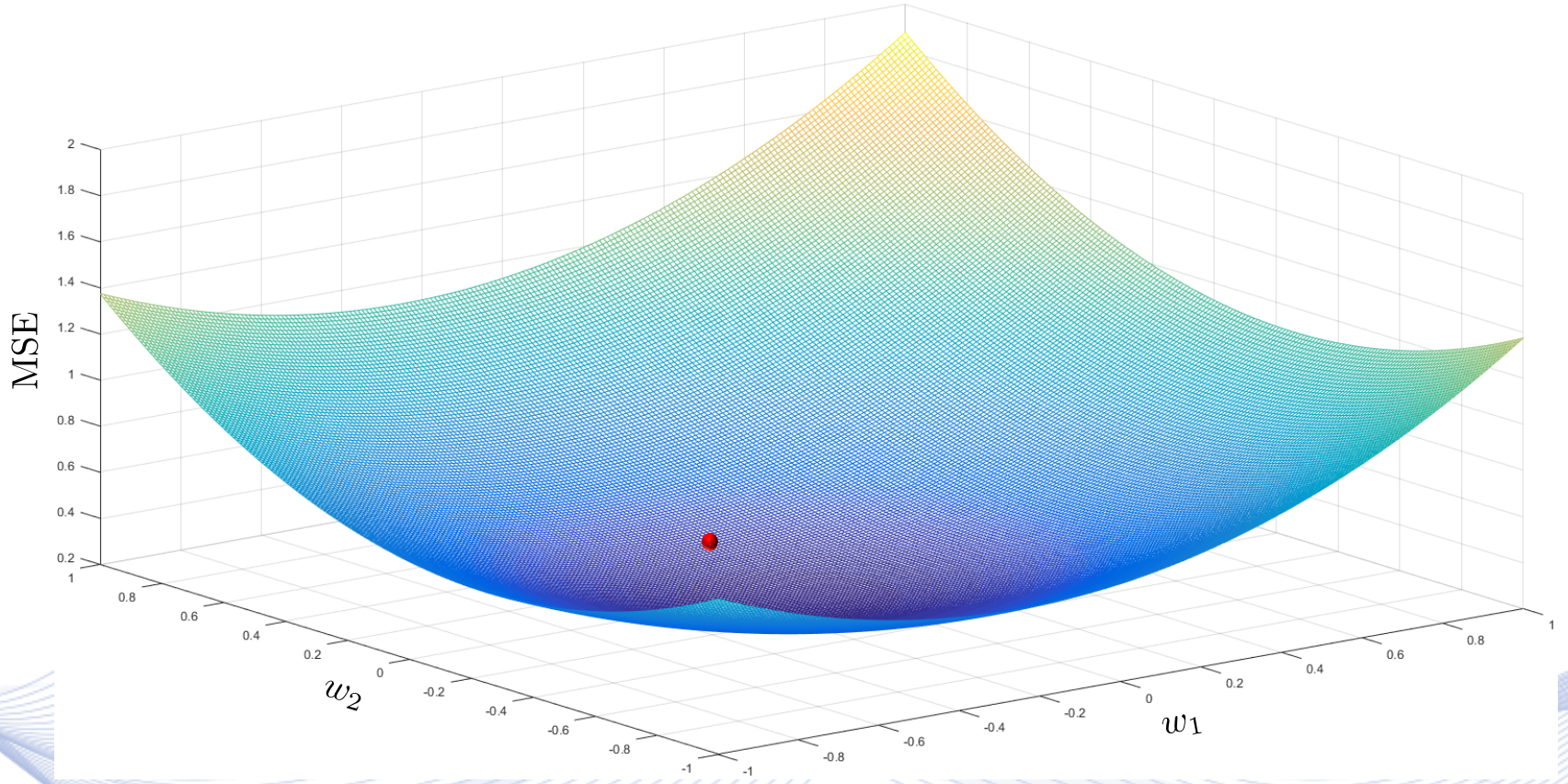

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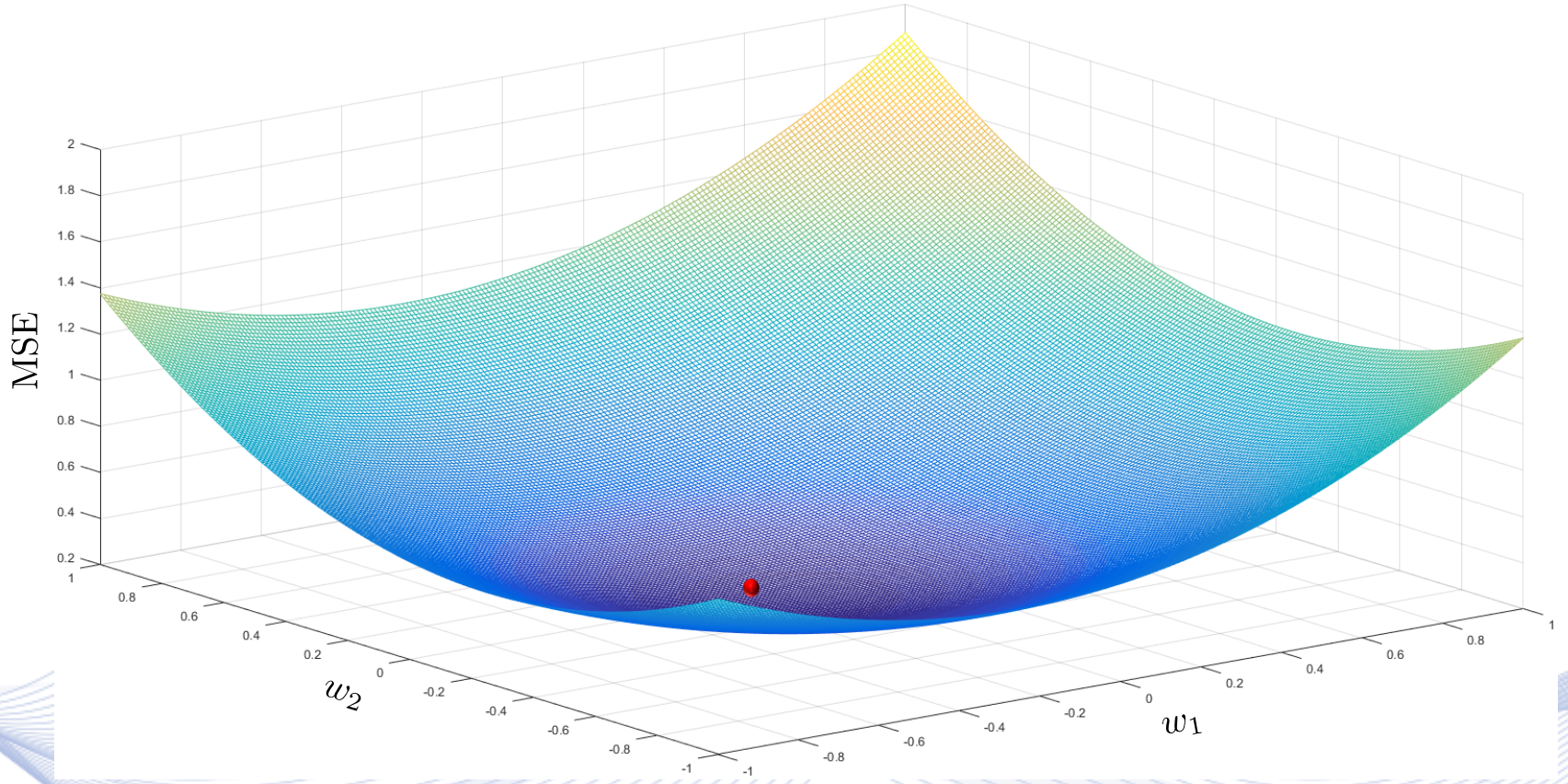
Classification



Perceptron training through classification error $J(w_1, w_2)$ minimization.

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- **Neural Networks**
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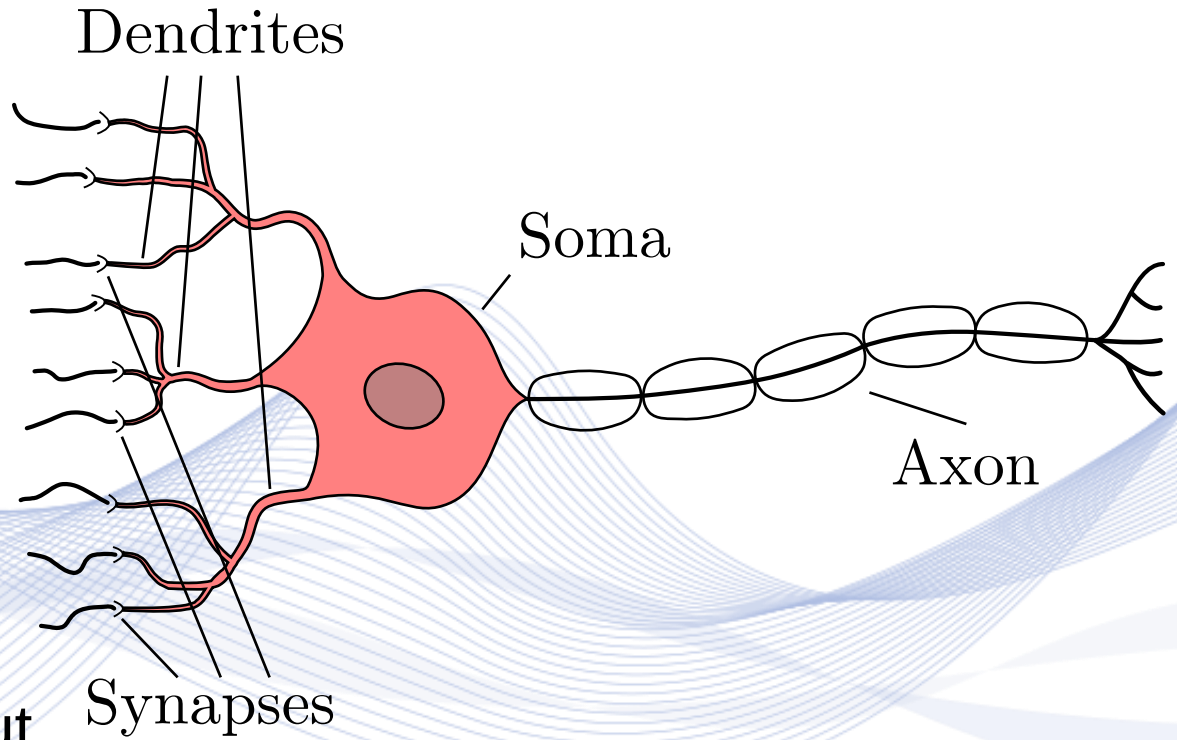
Neural Networks

- Basic computational unit of the brain.
- Main parts:
  - ***Dendrites***
    - They act as inputs.
  - ***Soma***
    - Main body of neuron.
  - ***Axon***
    - It acts as neuron output.



Neural Networks

Artificial neurons are mathematical models loosely inspired by their biological counterparts.

- Previous dendrites fetch the input vector:

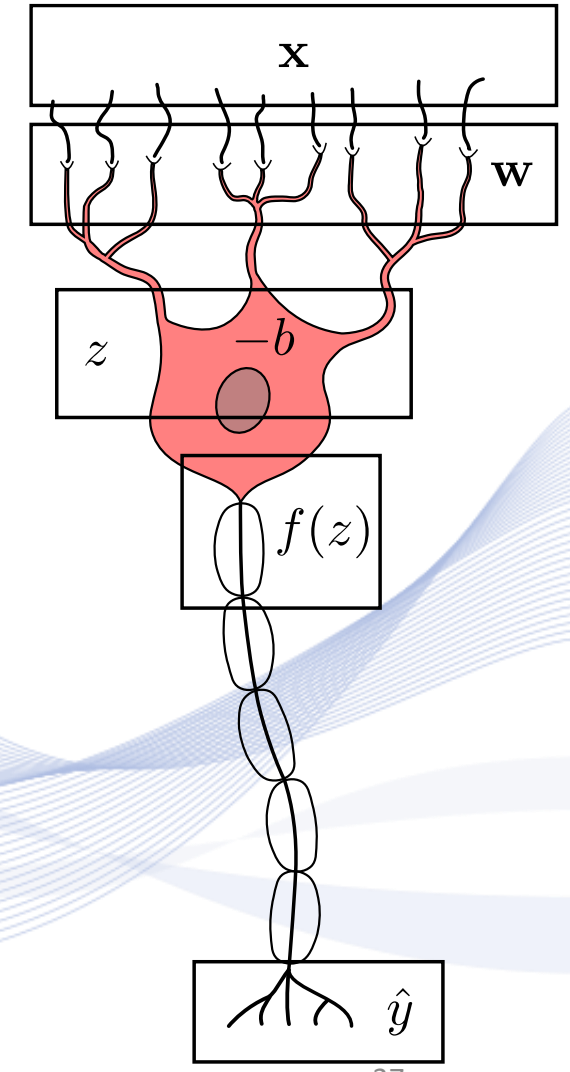
$$\mathbf{x} = [x_1, x_2, \dots, x_n]^T, \qquad x_i \in \mathbb{R}.$$

- The synaptic weights are grouped in a weight vector:

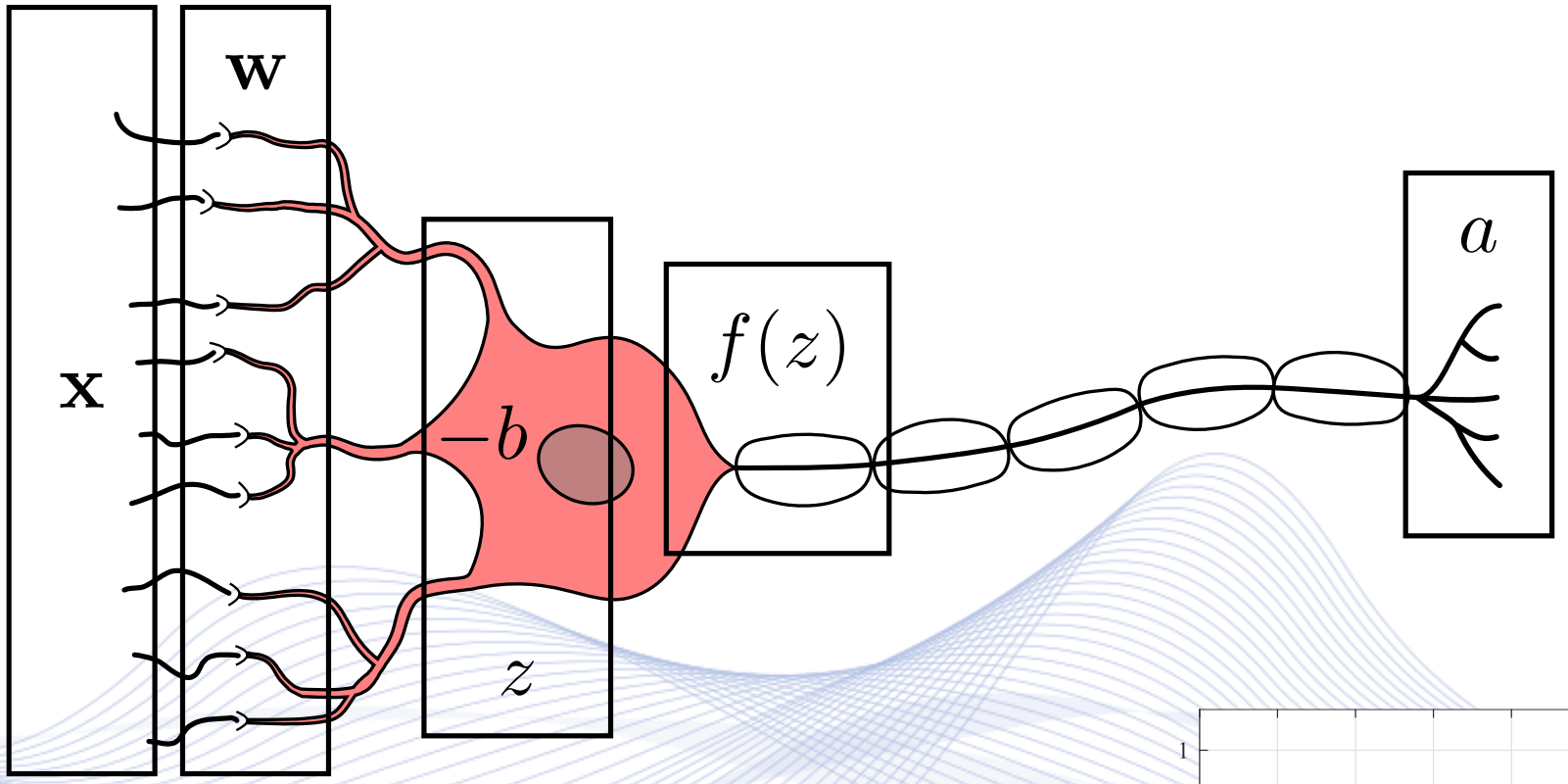
$$\mathbf{w} = [w_1, w_2, \dots, w_n]^T, \quad w_i \in \mathbb{R}.$$

- ***Synaptic integration***:

$$z = w_1 x_1 + w_2 x_2 + \cdots + w_n x_n > b.$$

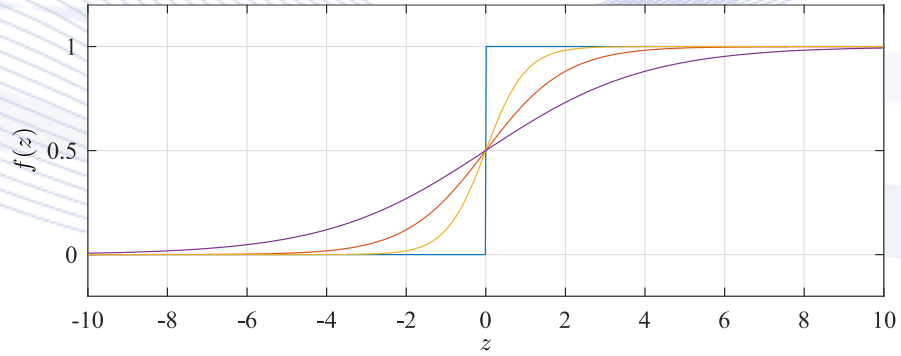


Neural Networks



Perceptron:

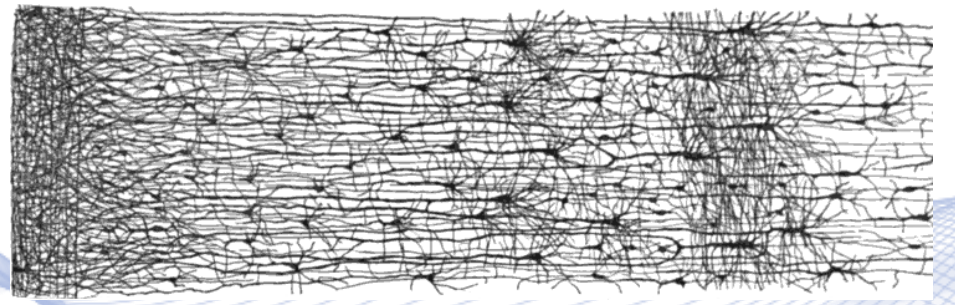
$$a = f(z) = f(w_1 x_1 + w_2 x_2 + \cdots + w_n x_n).$$



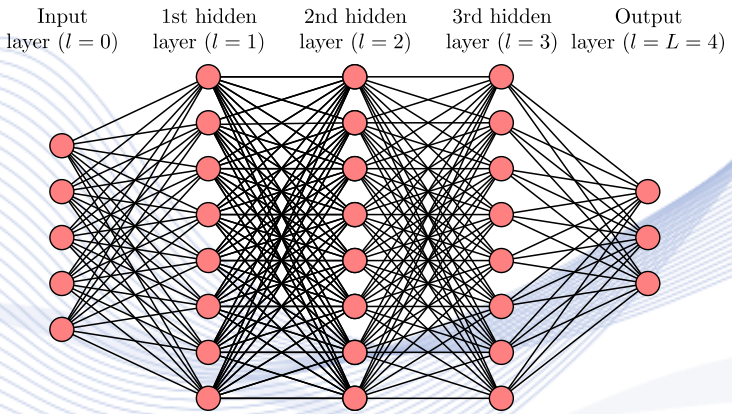
Neural Networks

Artificial and Biological neural networks

- Is ***network complexity*** the basis of both the biological and artificial intelligence?



Biological NN (https://en.wikipedia.org/wiki/Cerebral_cortex).



Multilayer perceptron.

Neural Networks

Classification is a binary function ***prediction*** (estimation):

$$\mathbf{y} = \boldsymbol{f}(\mathbf{x},\mathbf{w}).$$

- ***Input***: $\mathbf{x} = [x_1, x_2, \dots, x_n]^T$, e.g., facial 100×80 pixel image.
- ***Trainable parameters*** (NN weights): $\mathbf{w} = [w_1, w_2, \dots, w_n]^T$.
- ***Output***: $\mathbf{y} = [0, 1, 0, \dots, 0]^T$.
  - Only the correct facial (person) class label is 1.

Neural Networks

Classification.

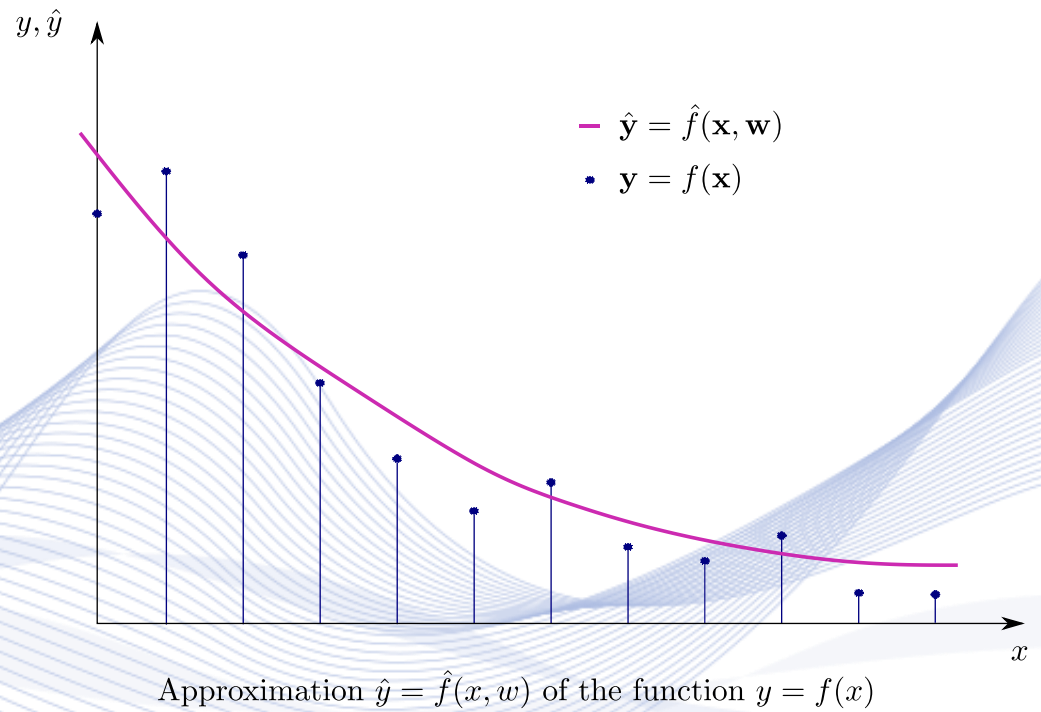
- ***Training***: Use training data $\{(\mathbf{y}_i, \mathbf{x}_i)\}$ to find the optimal parameters $\mathbf{w}$, minimizing the classification error $J(\mathbf{y}_i, \mathbf{x}_i, \mathbf{w})$.
- ***Inference***: Feed the trained NN with data $\mathbf{x}$ to produce the classification label: $\mathbf{y} = \boldsymbol{f}(\mathbf{x}, \mathbf{w})$.

- Classification is a special type of ***regression*** (function approximation).

Neural Networks

Regression is an approximation $\hat{\mathbf{y}} = \hat{f}(\mathbf{x}, \mathbf{w})$ of a real-valued function $\mathbf{y} = f(\mathbf{x})$.

- ***Input***: $\mathbf{x}$ (values in the function domain).
- ***Trainable parameters*** $\mathbf{w}$.
- ***Output vector*** $\hat{\mathbf{y}}$: approximated function values.
- Training and inference.



Approximation $\hat{y} = \hat{f}(x, w)$ of the function $y = f(x)$

Neural Networks

Regression example: object detection.

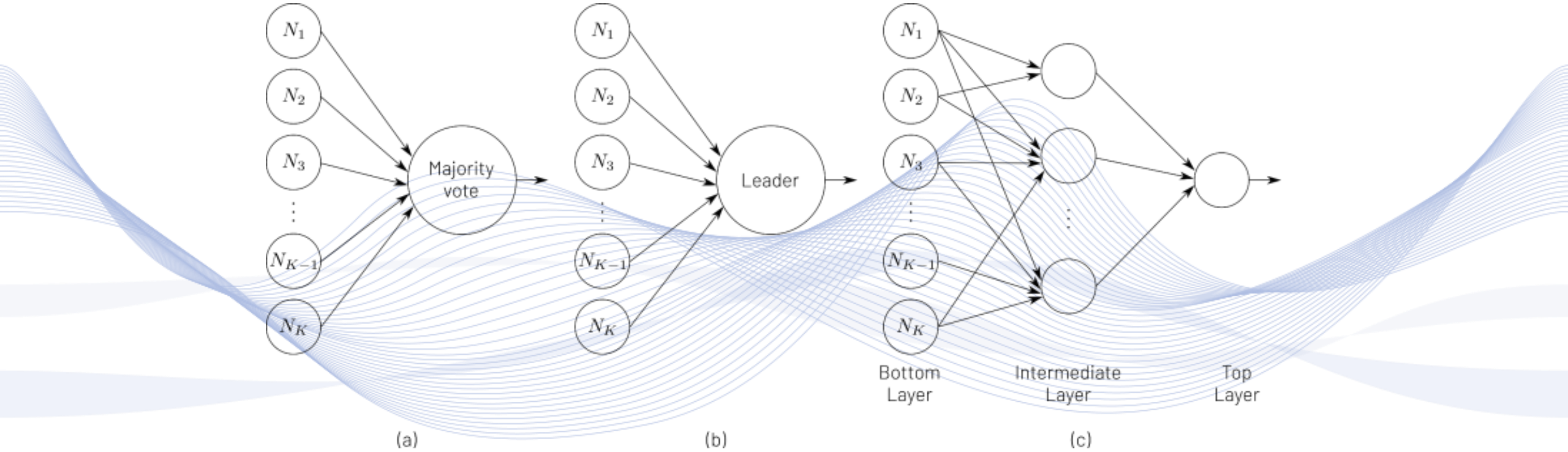
- ***Input***: image $\mathbf{x}$.
- ***Trainable parameters*** $\mathbf{w}$.
- ***Output vector*** $\mathbf{y} = [x_c, y_c, h, w]^T$.
- It describes the ***bounding box*** of an object (center coordinates, height, width).

Athlete detection.

Neural Networks

Political and societal networks.

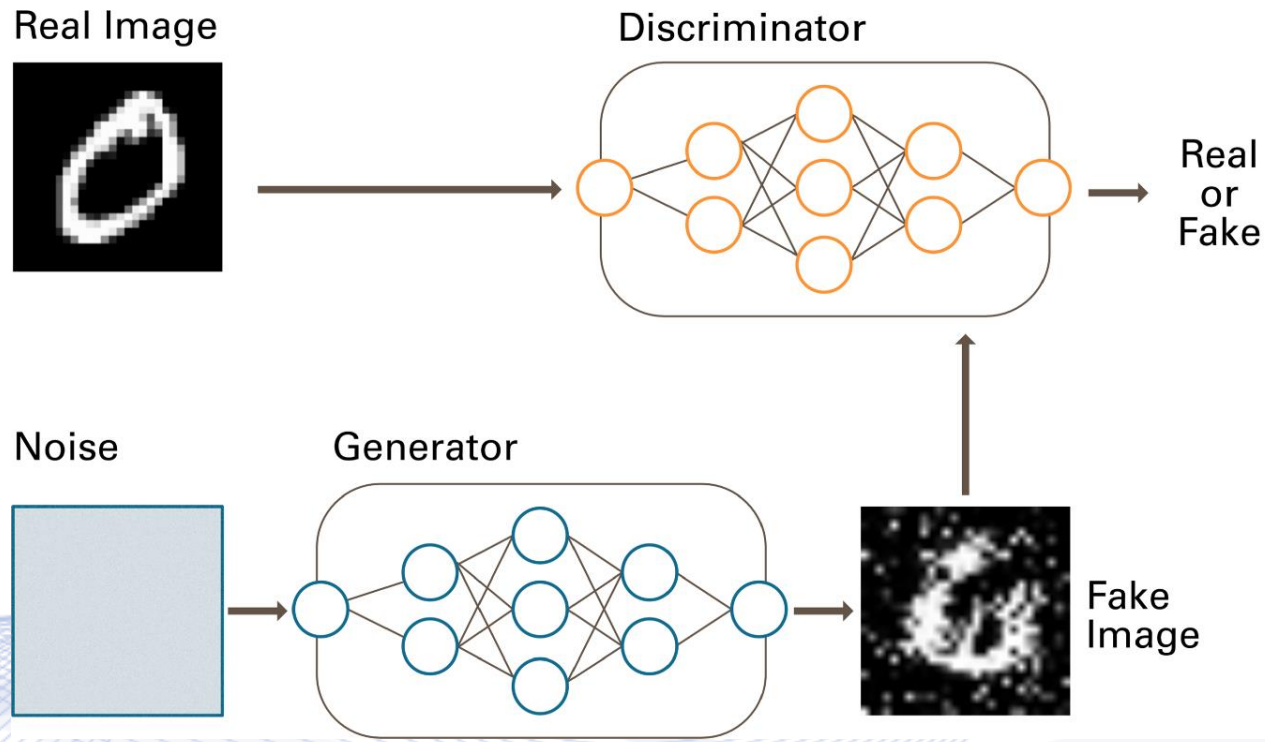
- ***Revising democratic decision-making?***



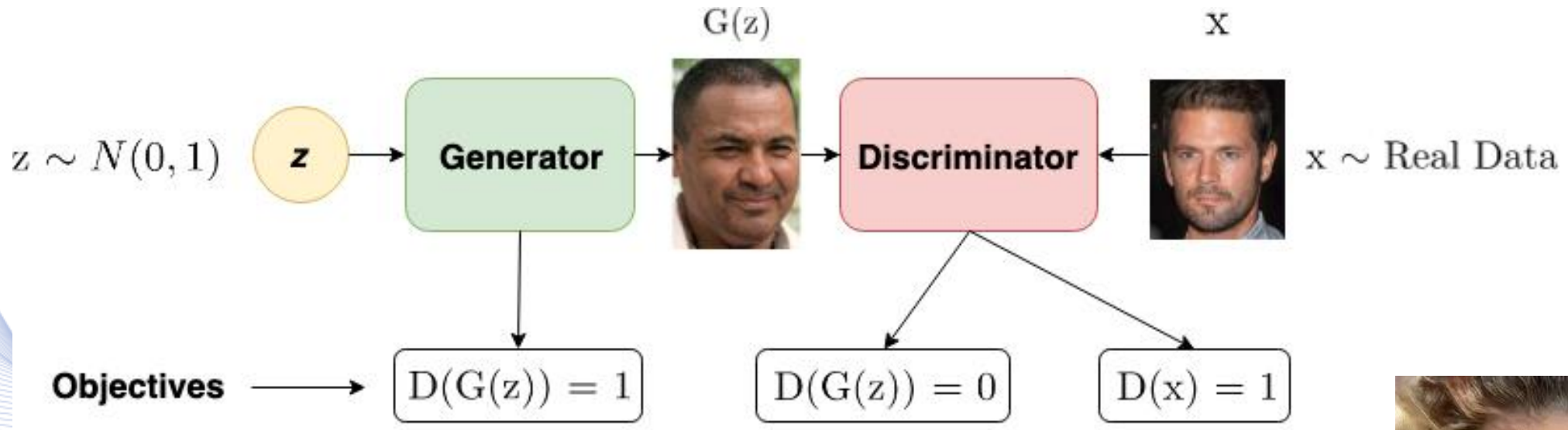
Neural Networks

Generative adversarial networks:

- The ***generator*** NN generates an image.
- The ***discriminator*** NN decides:
  - Real or fake?

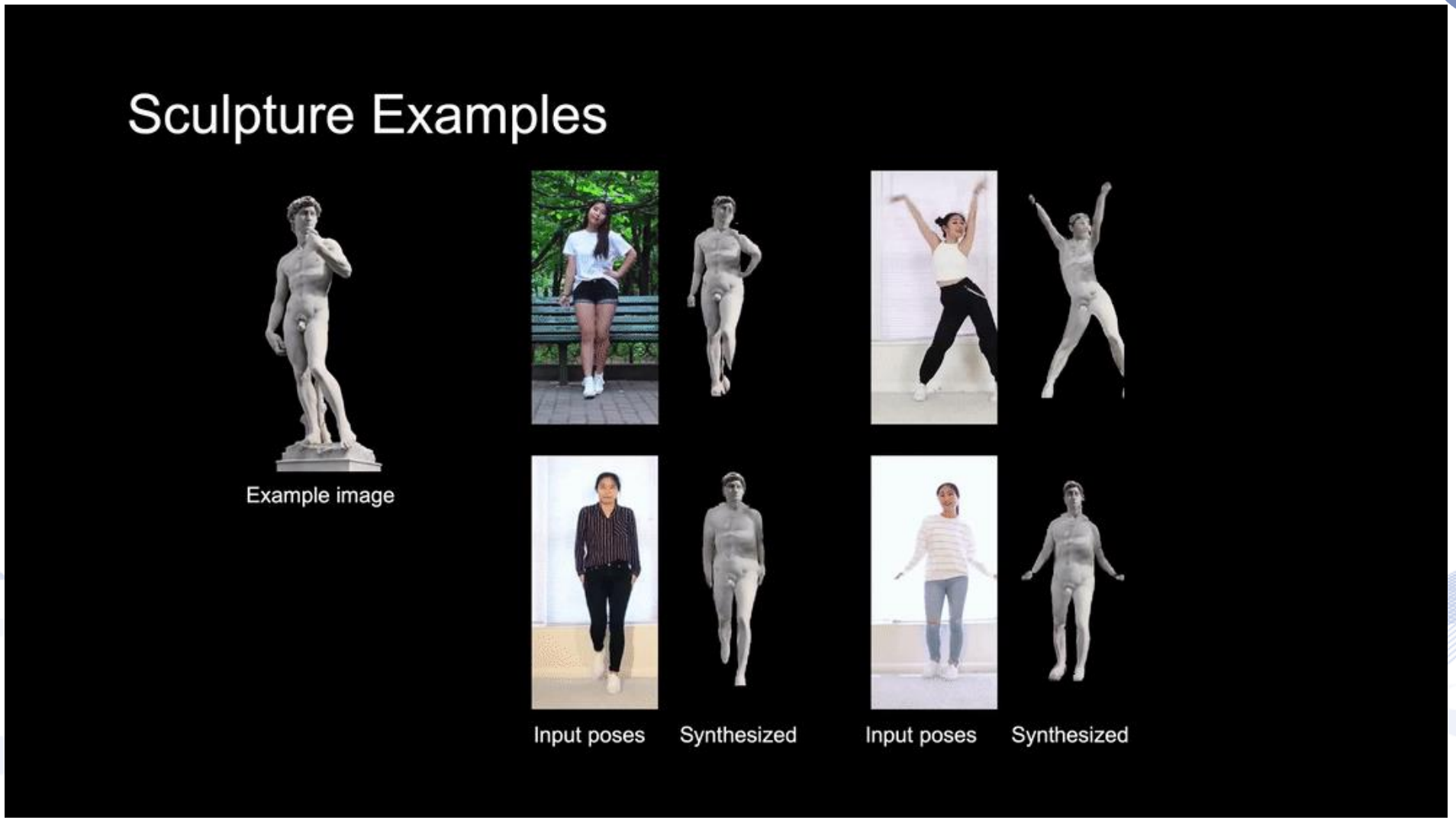


Neural Networks



GANs in image synthesis.

Neural Networks



GANs in video synthesis.

Neural Networks

Advantages

- Very good decision accuracy
  - (frequently above human performance).
- Wide range of applications.
- New generative (creative) arts.

Pitfalls

- Too many data/energy needed for their training.
- Poor explainability.
- Possible decision bias.
- Creation of fake data/news.

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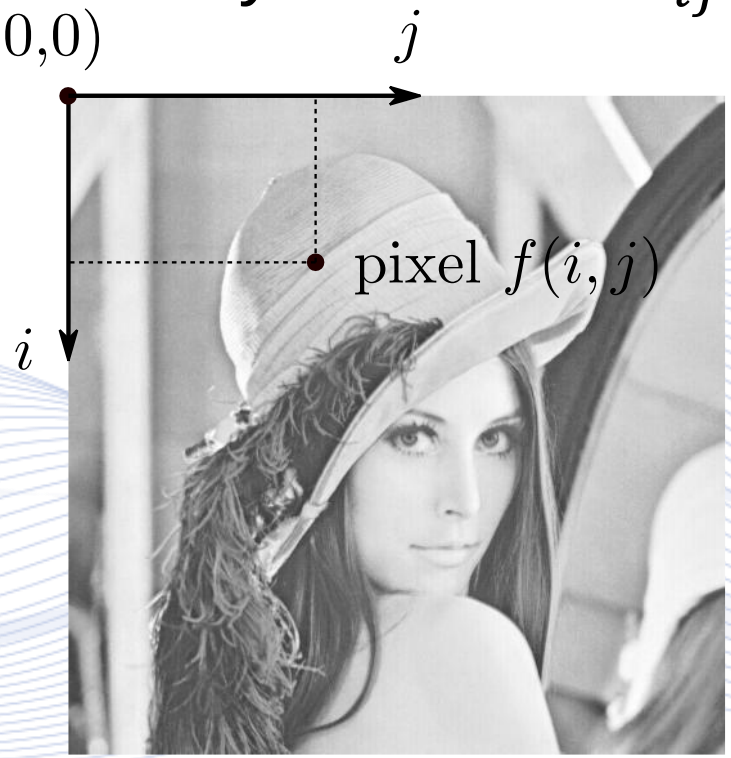
Computer Vision

Digital images consist of ***pixels***.

- They can be represented by ***matrices*** A_{ij} (***Linear Algebra***).



Example: 256×256 pixel image.

Computer Vision

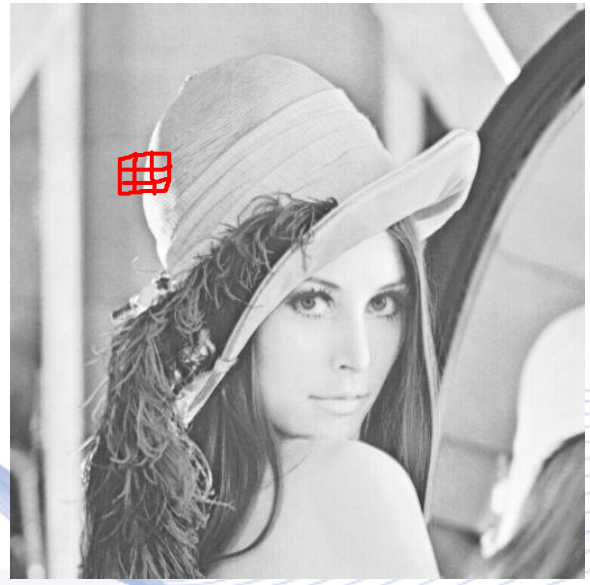
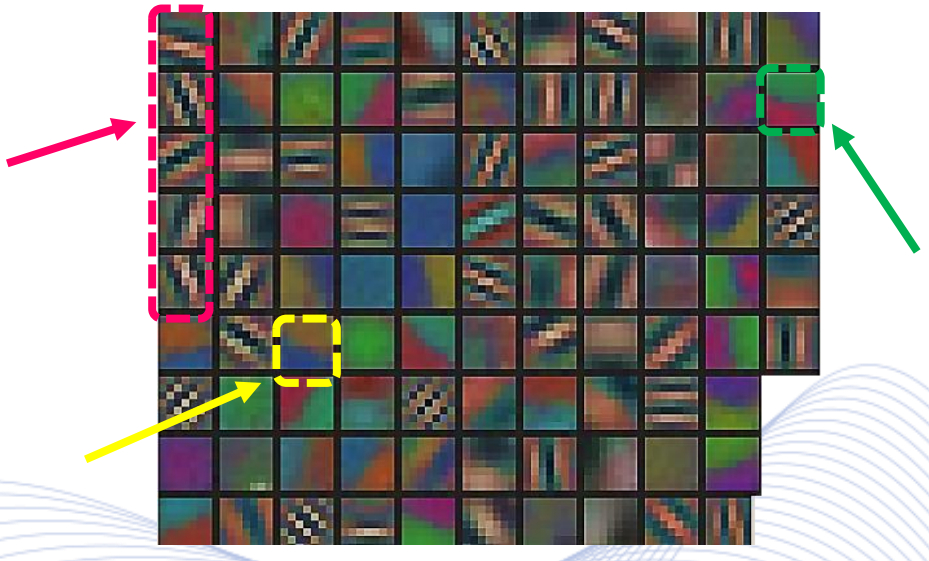


Image is convolved using mask $\mathbf{W}$.

$$\mathbf{W} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}.$$

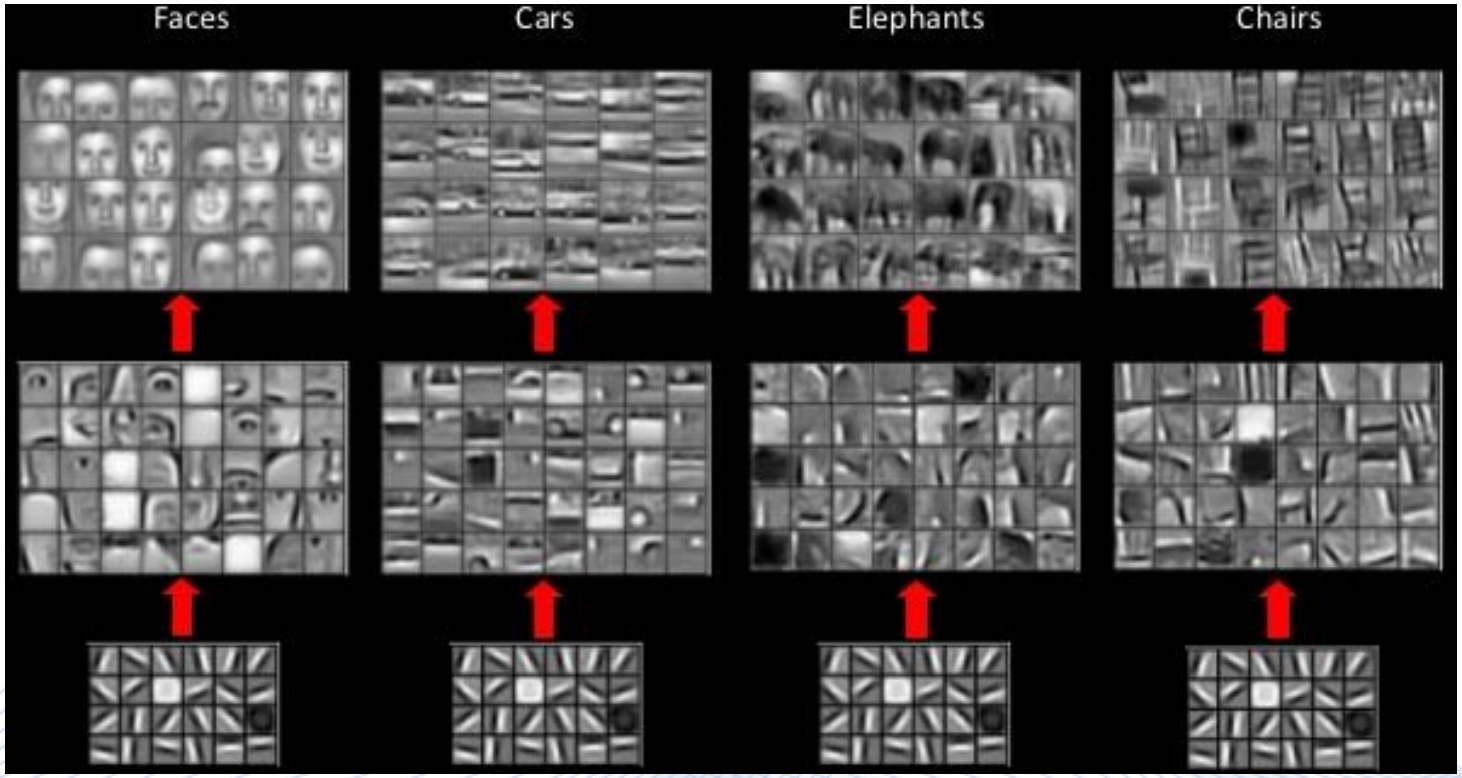
Convolution output: Vertical image edges.

Computer Vision



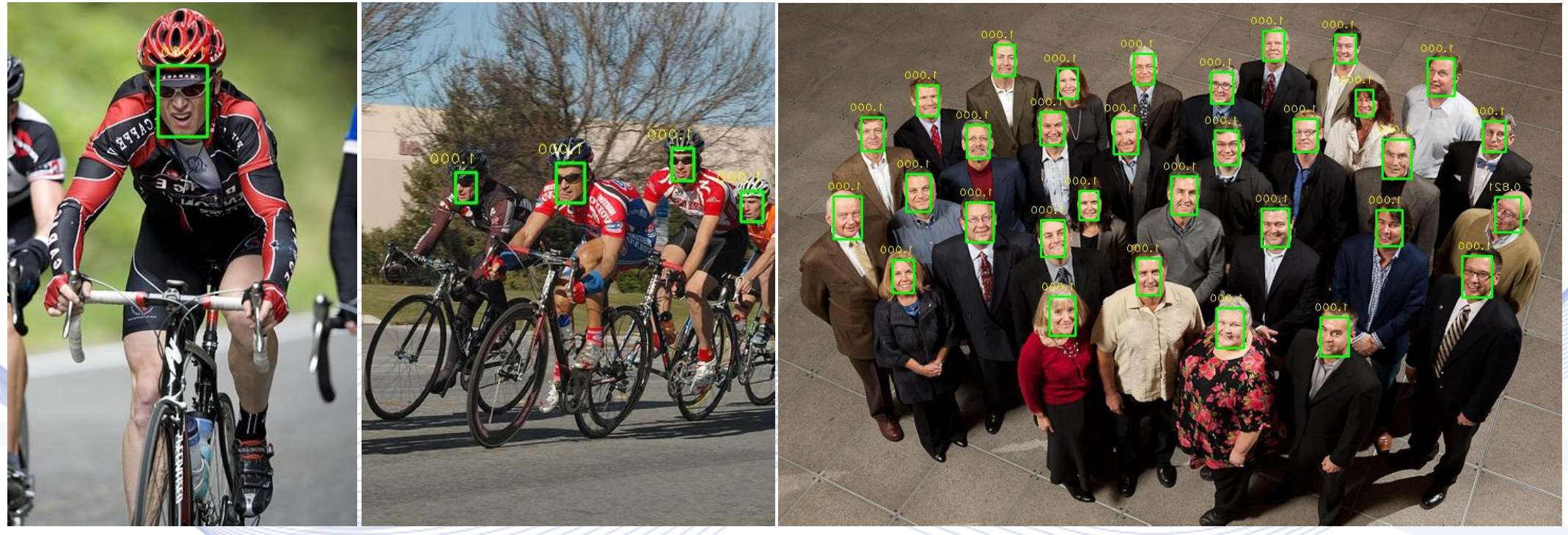
Neural Image Features.

Computer Vision



Convolutional Neural Networks: using neural image features for ML tasks.

Computer Vision



Face detection examples.

Computer Vision

Cyclist detection and tracking.

Computer Vision

Region segmentation.

AI Science and Mathematics

- What is AI?
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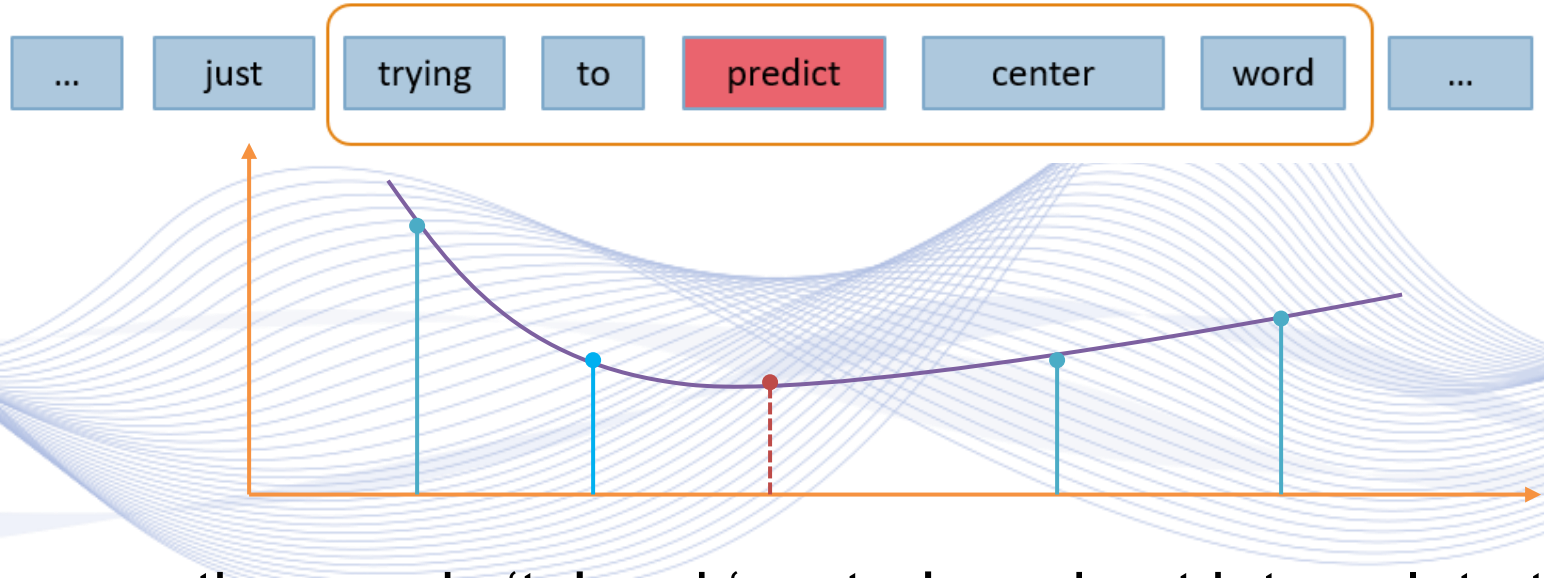
Natural Language Processing

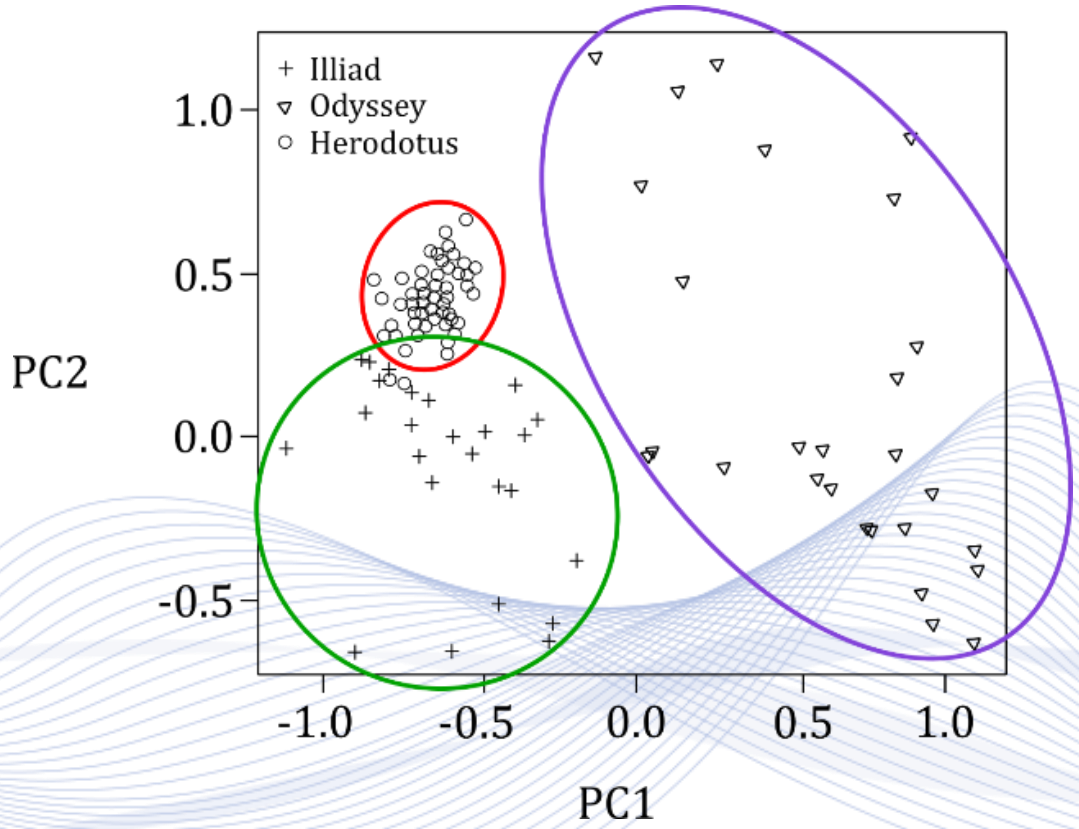
Word embeddings

- Transforming words in vectors.
- Predicting word order.



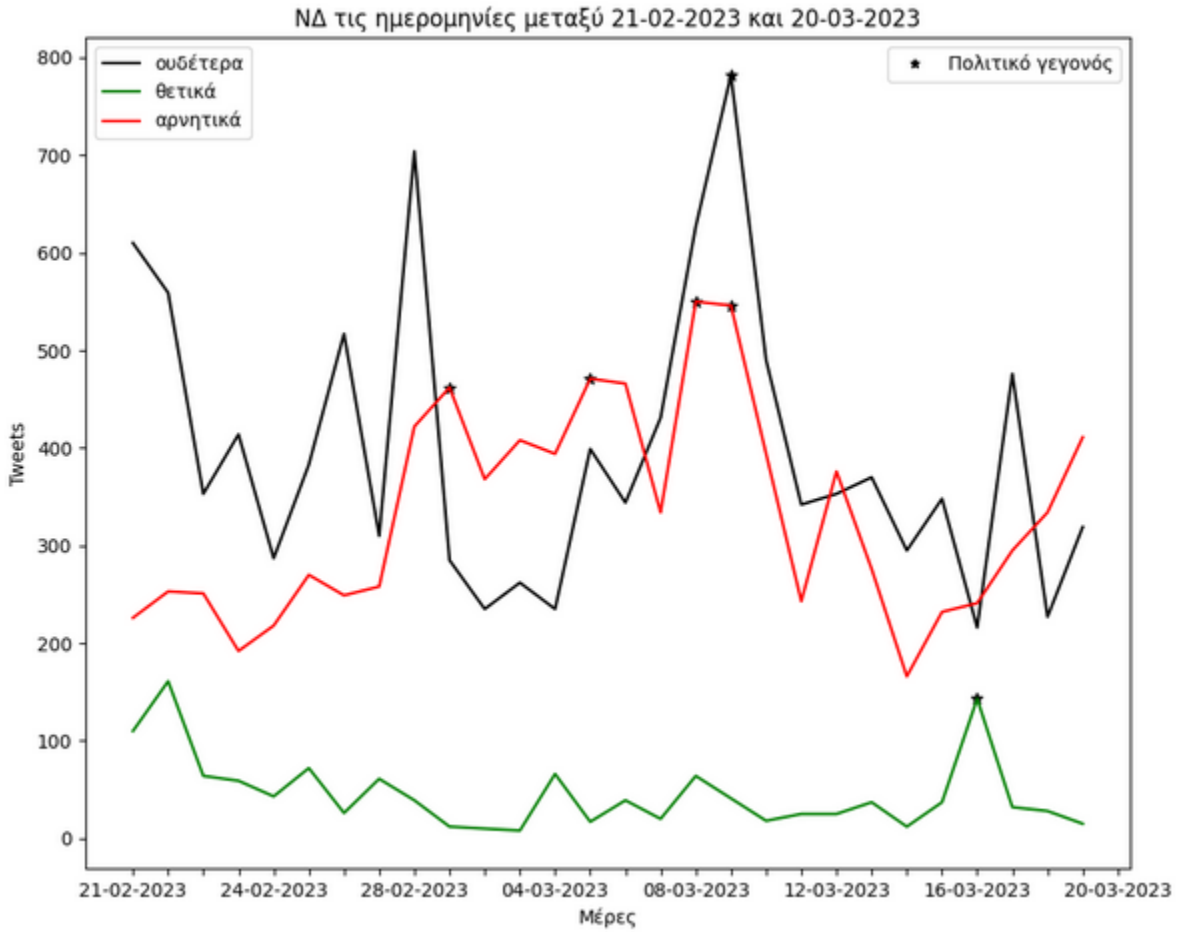
Vectors representing words ‘to’ and ‘center’ can best interpolate the ‘predict’ vector.

Natural Language Processing



Representing texts by vectors:
Principal component analysis of Homer's Iliad and Odyssey.

Natural Language Processing



Artificial Intelligence & Information Analysis Lab

Sentiment Analysis of Greek Political Tweets.

Natural Language Processing

Large Language Models

- ChatGPT, GPT-4
- ***Mathematical Language Modeling*** (word embedding).
- Smooth text production.
- Not intended to offer inference capabilities.
- Code programming.
- Certain mathematical skills.

- Big question: ***what is its best use in education?***

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Knowledge

Information

- Notoriously vague definitions.
- My definition: ***Information is the result of the manual or automatic Data Analysis***.

Taxonomy: Data → Information → Knowledge.

Machine Learning/inference produces ***information*** (including metadata).

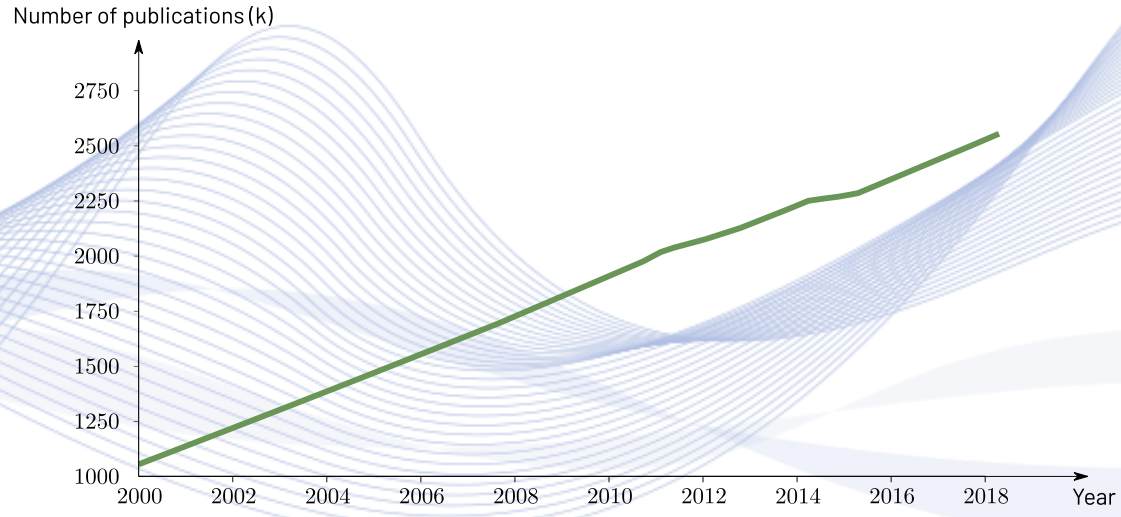
- ***Information theory/entropy: bits (once more)!***

Knowledge

Knowledge is primarily a product of reasoning.

- Is knowledge finite?
- ***Can we measure knowledge?***
- Knowledge increase is linear.

- ***Encyclopedias***
- ***Research publications.***



Global research output (publication) growth.

Knowledge

Current AI revolution:

- ***AI means ML, which means Deep Neural Networks***
- Stagnation of symbolic AI
- Resurrection of a dead term: AI

Major breakthrough needed:

- Advancement of symbolic AI
- ***Fusion of Machine Learning and symbolic AI.***

Knowledge

Data/Information society:

- Exponential data growth.
- Data acquisition automation.
- ***Information extraction automation through ML.***

Sustainability?

- More sensors, more processors, Moore's law.
- ***Energy-intensive data and information extraction.***

Knowledge

Knowledge society:

- Exponential knowledge growth.
- Not there yet: ***knowledge production and communication is still manual***.

- ***Real danger: inability of humanity to grow and uptake knowledge.***
- Past devastating setbacks in knowledge uptaking:
  - Dark ages (beginning of the Medieval times).

Knowledge

Sustainability of knowledge growth:

- Limitations in brain capacity.
- Solution: ***social swarm intelligence***
- Example: collective memory.
- Knowledge communication through ***education*** is way suboptimal:
  - New education mode needed, stressing ***critical thinking*** and ***abstraction***.
  - ***Morphosis***: formation of knowledgeable citizens.
  - ***Global education***: diminishing social and regional barriers to education.
- ***Unified machine and human learning theories?***

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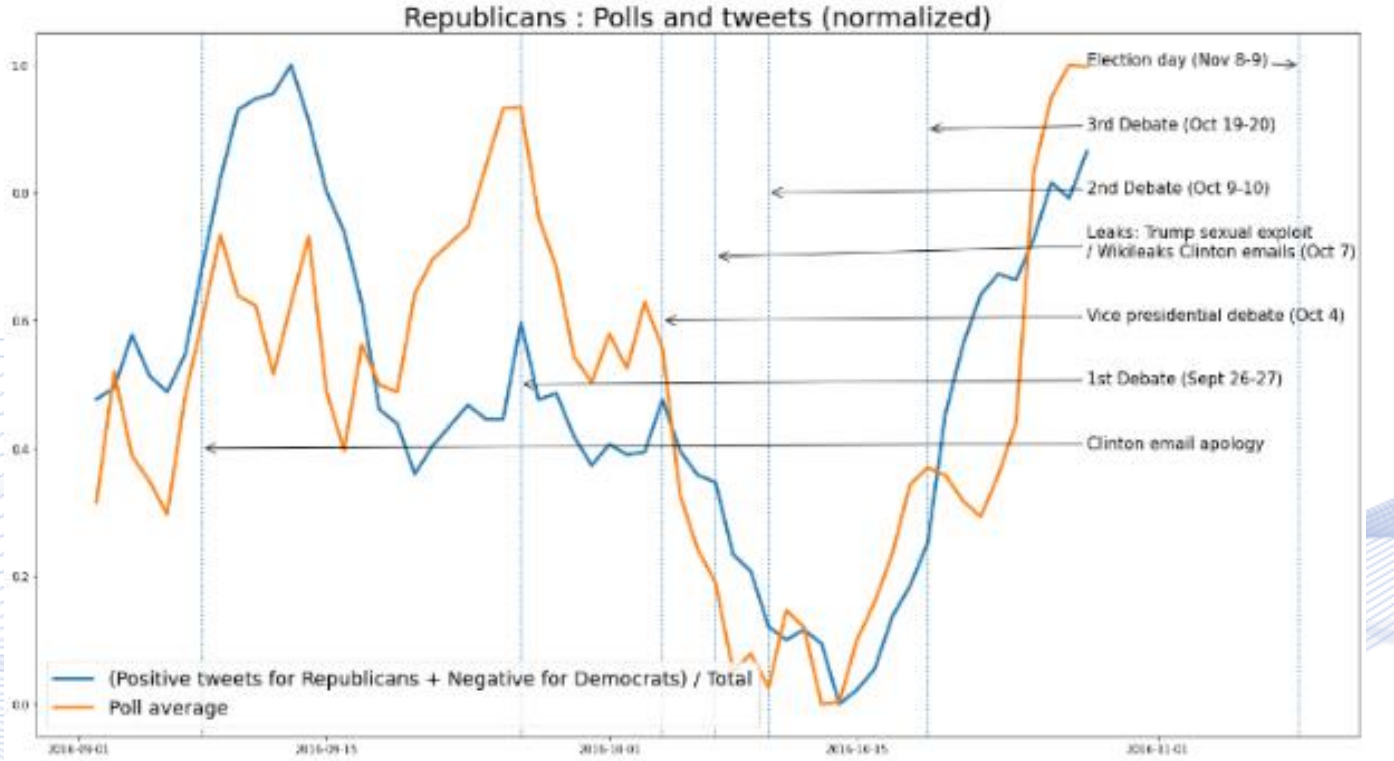
AI and Society

AI and Politics: observing the society.

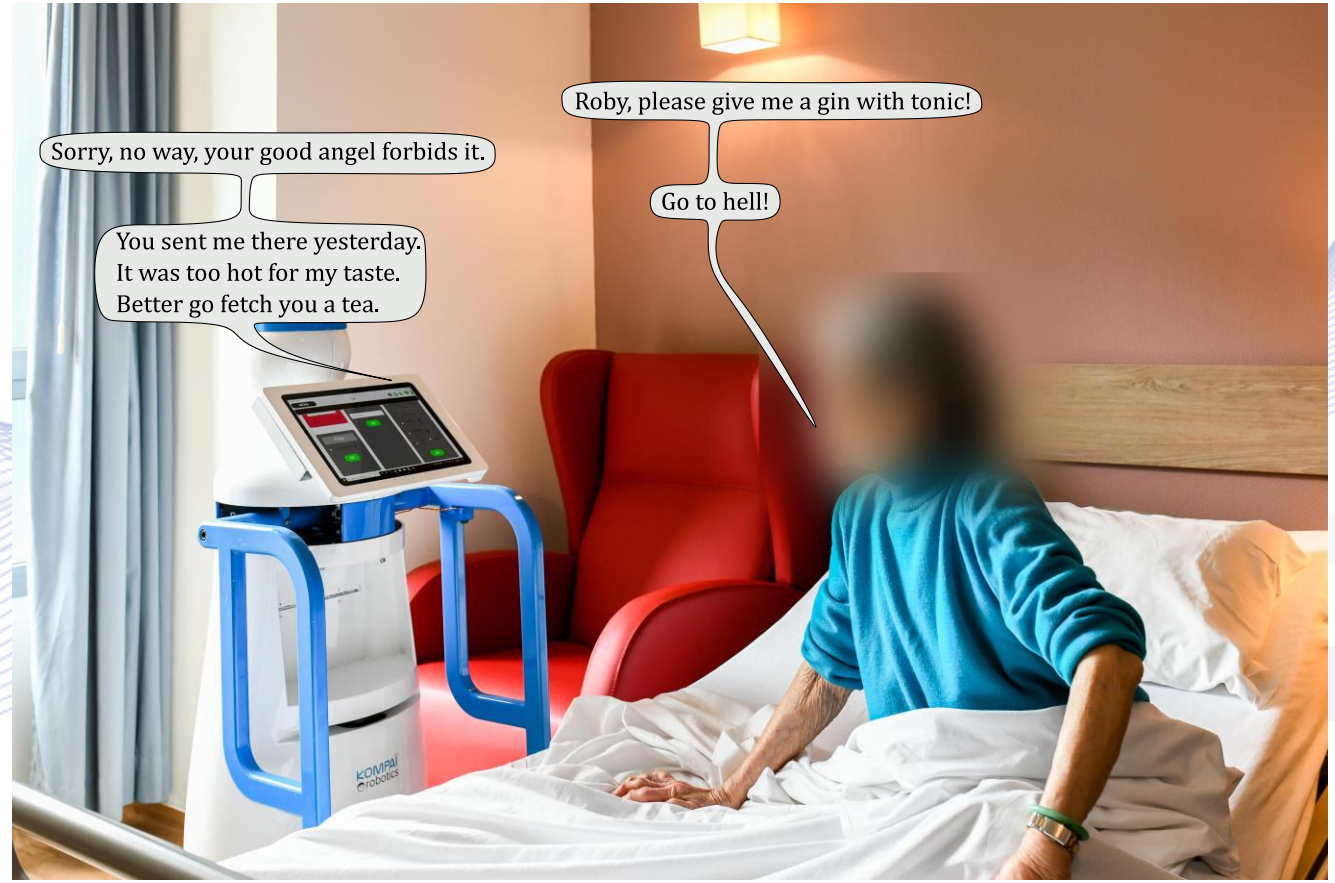
- Are opinion polls redundant?



Poll and tweet sentiment trends for the 2016 US presidential election.

AI and Society

- Intelligent systems can be very useful.
- ***Should we be technophobic?***



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AI and the Environment

Law of Complexity

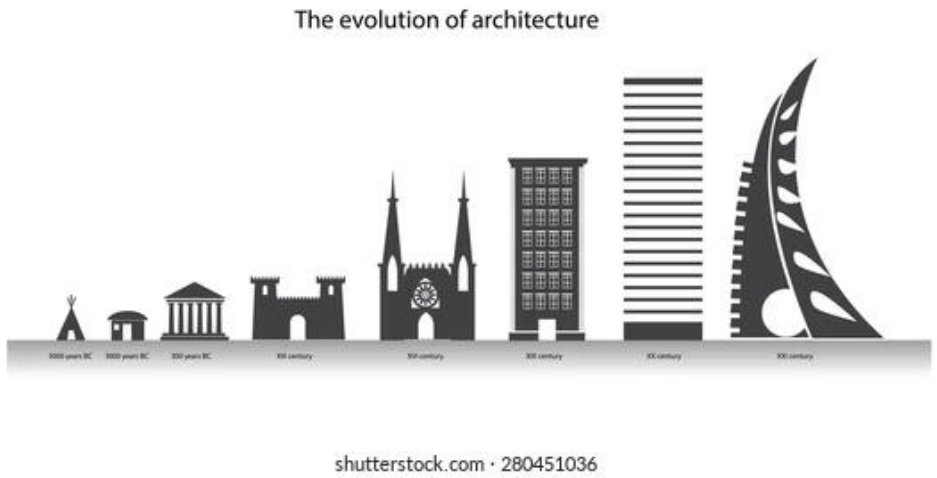
- Is ***matter complexity*** the basis of life and intelligence?

$$Atoms > nucleotides > DNA - RNA - proteins > subcellular\ structures > cells\ (neurons) > organisms > multicelular\ organism > \ colonies, swarms, networks.$$

- ***Can we envisage other complex matter forms?***

AI and the Environment

- Does living ***matter complexity*** ever increases?
- Do we see the same in man-made constructions?
  - Smart buildings, complex societal processes, intelligent machines?



- Do we move from ***life-through-evolution*** to ***life-by- design***?
- ***Is sky the limit in AI advances?***

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Q & A

Thank you very much for your attention!

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