Camera Geometry summary

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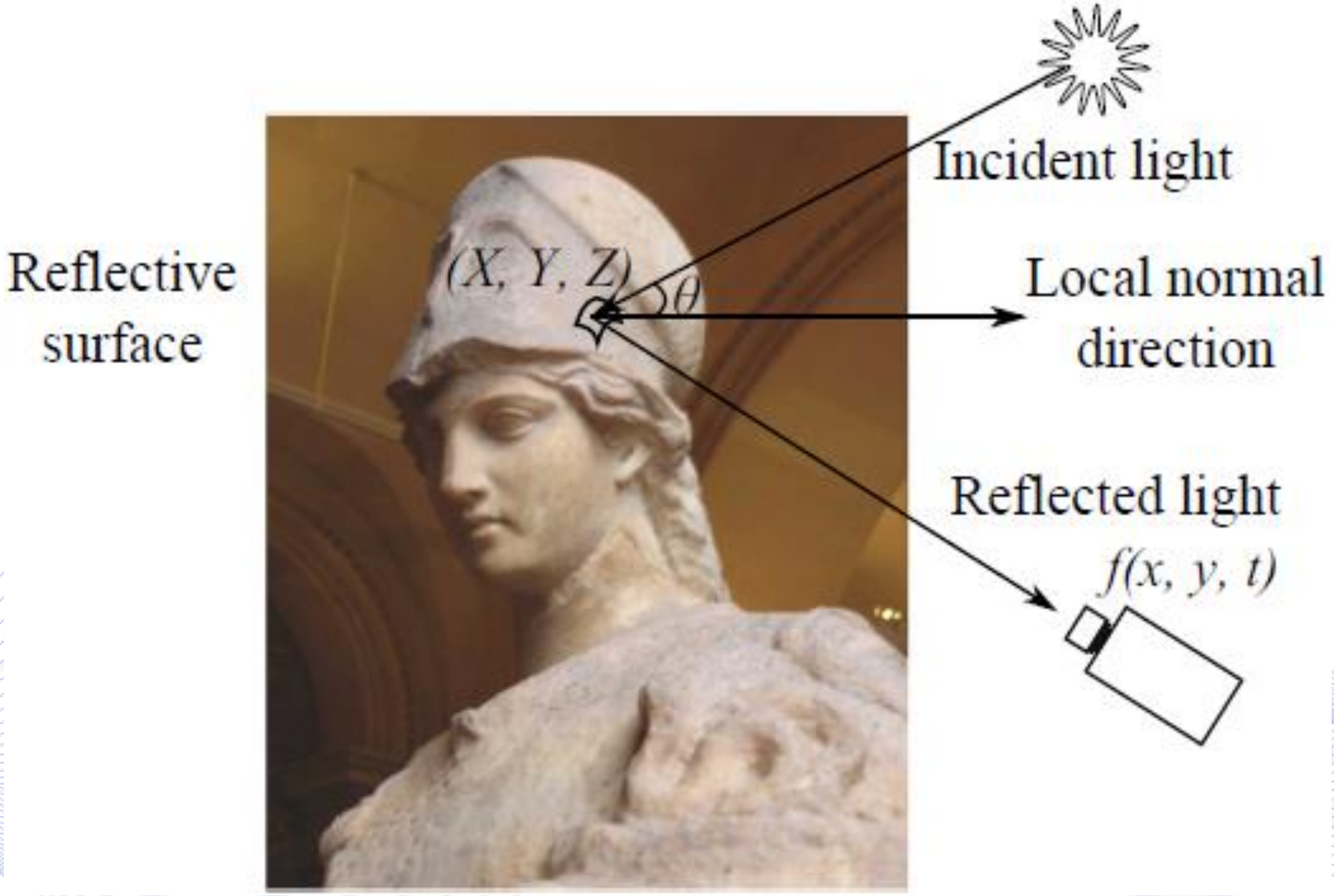
Artificial Intelligence & Information Analysis Lab

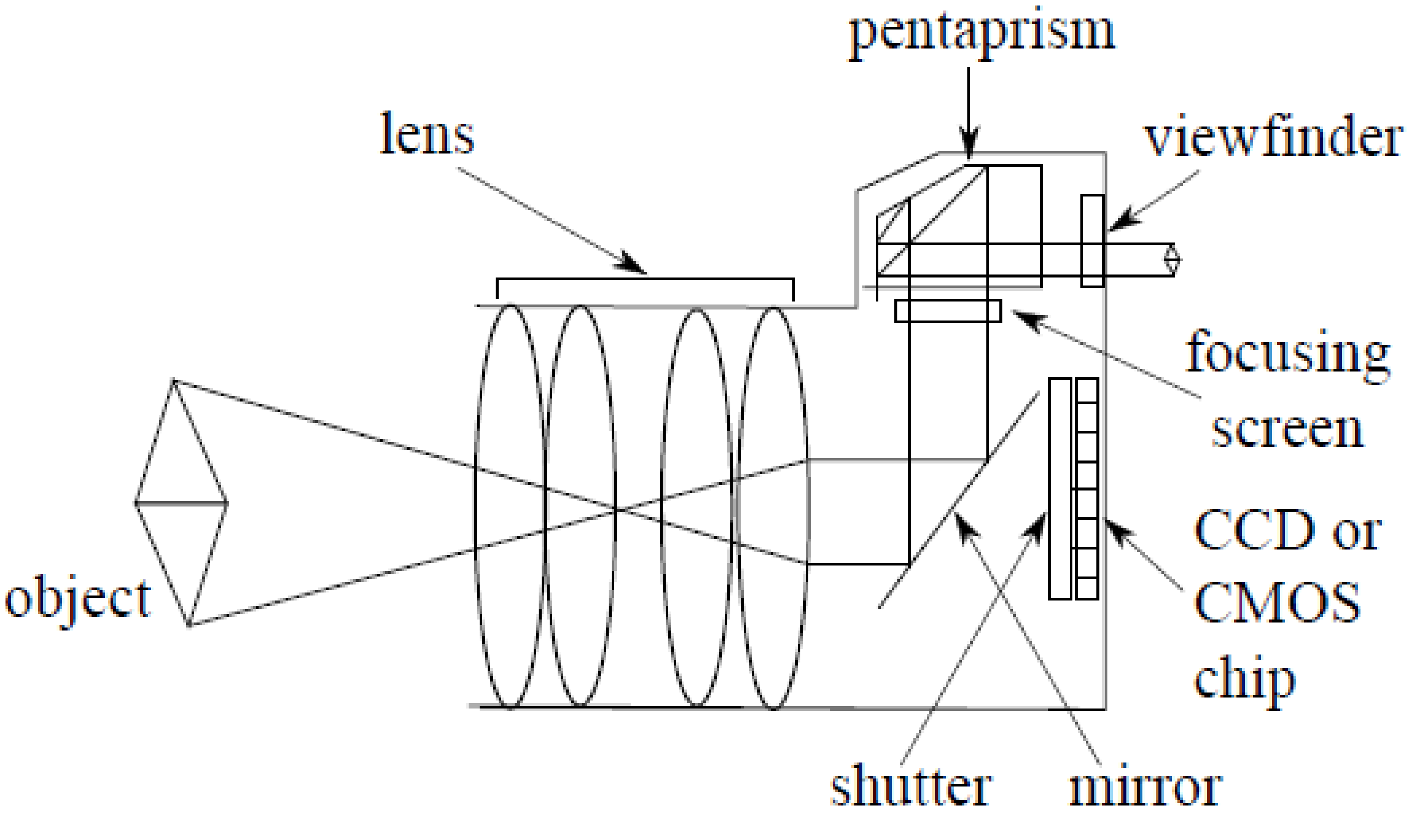
Image acquisition

- A still image visualizes a still object or scene, using a still picture camera.
- A video sequence (moving image) is the visualization of an object or scene illuminated by a light source, using a video camera.
- The captured object, the light source and the video camera can all be either moving or still.
- Thus, moving images are the projection of moving 3D objects on the camera image plane, as a function of time.
- Digital video corresponds to their spatiotemporal sampling.

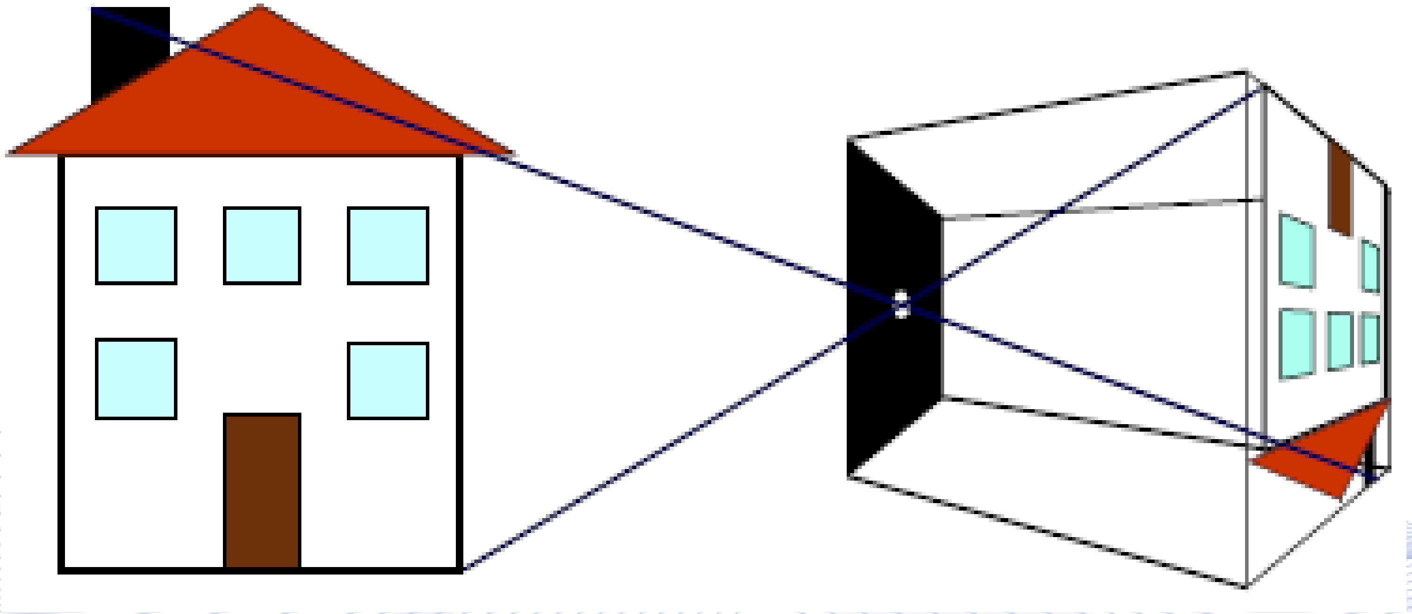
Light reflection



Camera structure

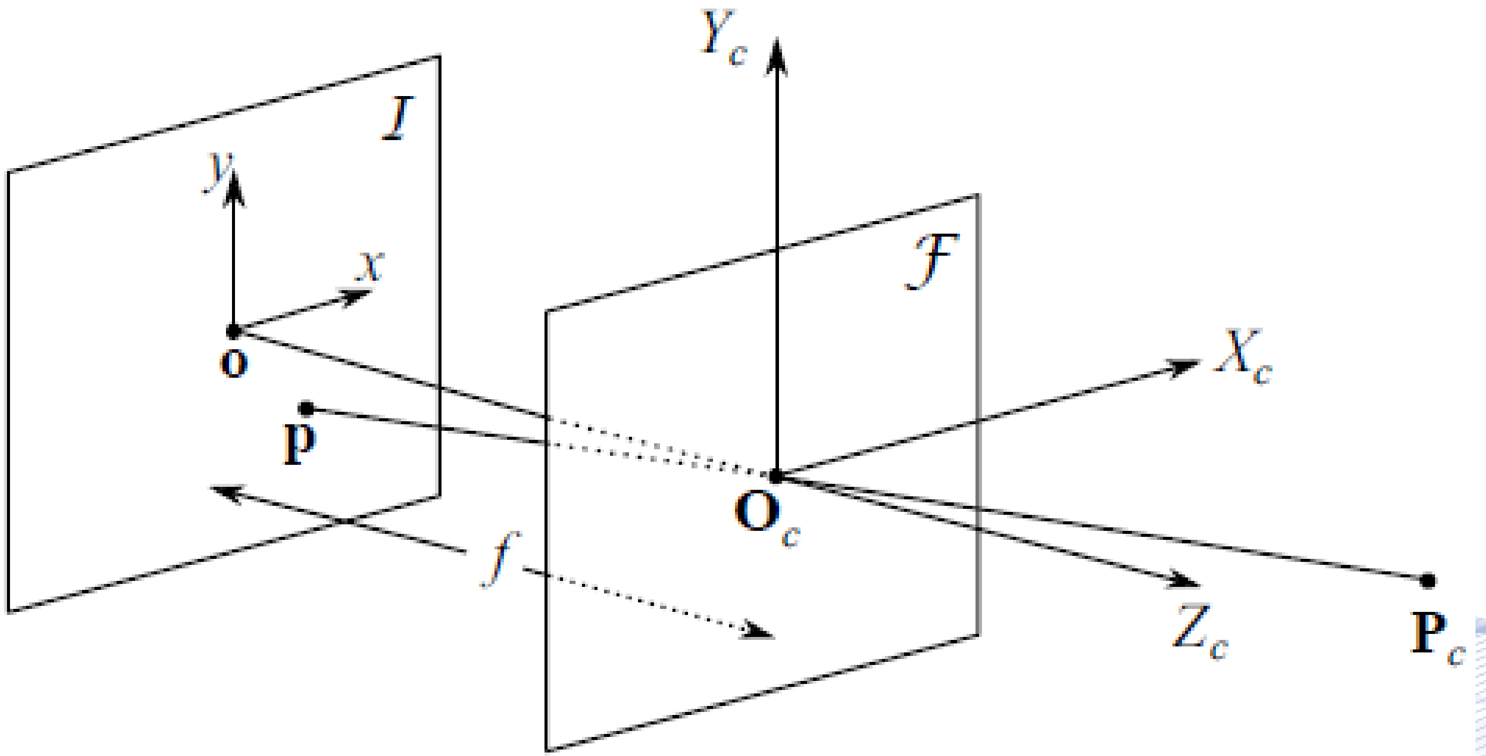


Pinhole Camera and Perspective Projection



Pinhole camera geometry.

Pinhole Camera and Perspective Projection



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Pinhole Camera and Perspective Projection

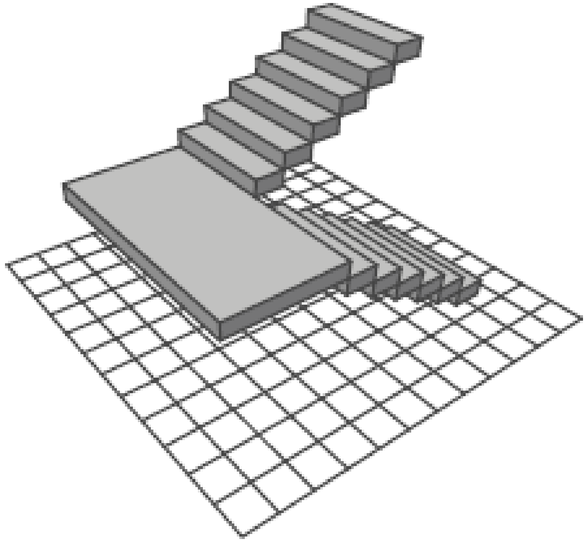
- We want to derive the equations that connect a 3D point (3D vector) $\mathbf{P}_c = [X_c, Y_c, Z_c]^T$ referenced in the camera coordinate system with its projection point (2D vector) $\mathbf{p}' = [x', y']^T$ on the virtual image plane.
- By employing the similarity of triangles $\mathbf{O}_c\mathbf{o}'\mathbf{p}'$ and $\mathbf{O}_c\mathbf{Z}_c\mathbf{P}_c$:

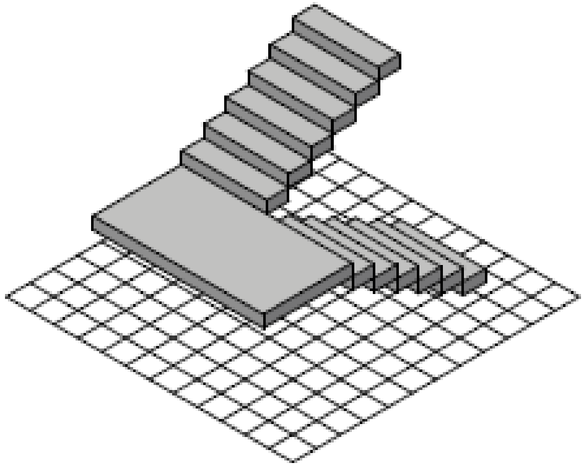
$$\frac{x'}{X_c} = \frac{y'}{Y_c} = \frac{f}{Z_c}, \qquad x' = f\frac{X_c}{Z_c}, \qquad y' = f\frac{Y_c}{Z_c}.$$

- Coordinates on the real image plane are given by the same equations, differing only by a minus sign.

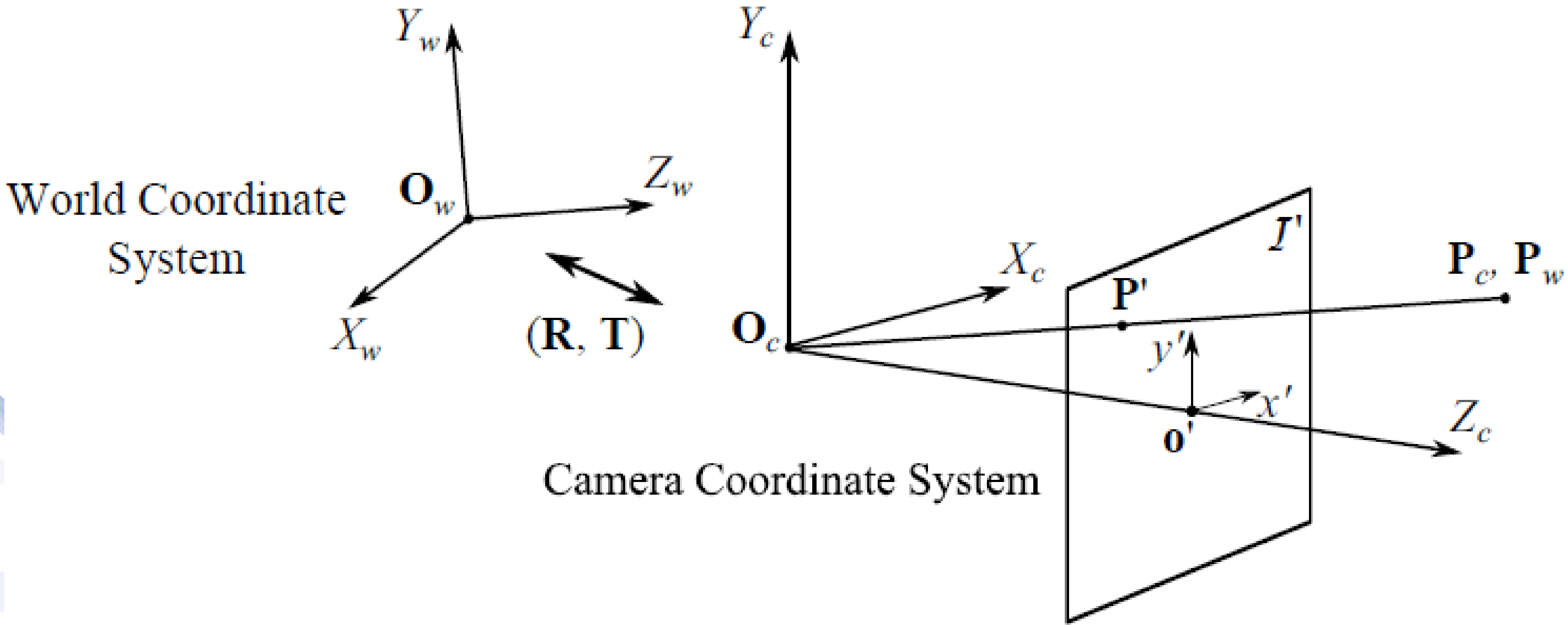
The Weak-Perspective Camera Model

- While a weak-perspective camera preserves parallelism in the projected lines, as orthographic projection does (b), perspective projection (a) does not.

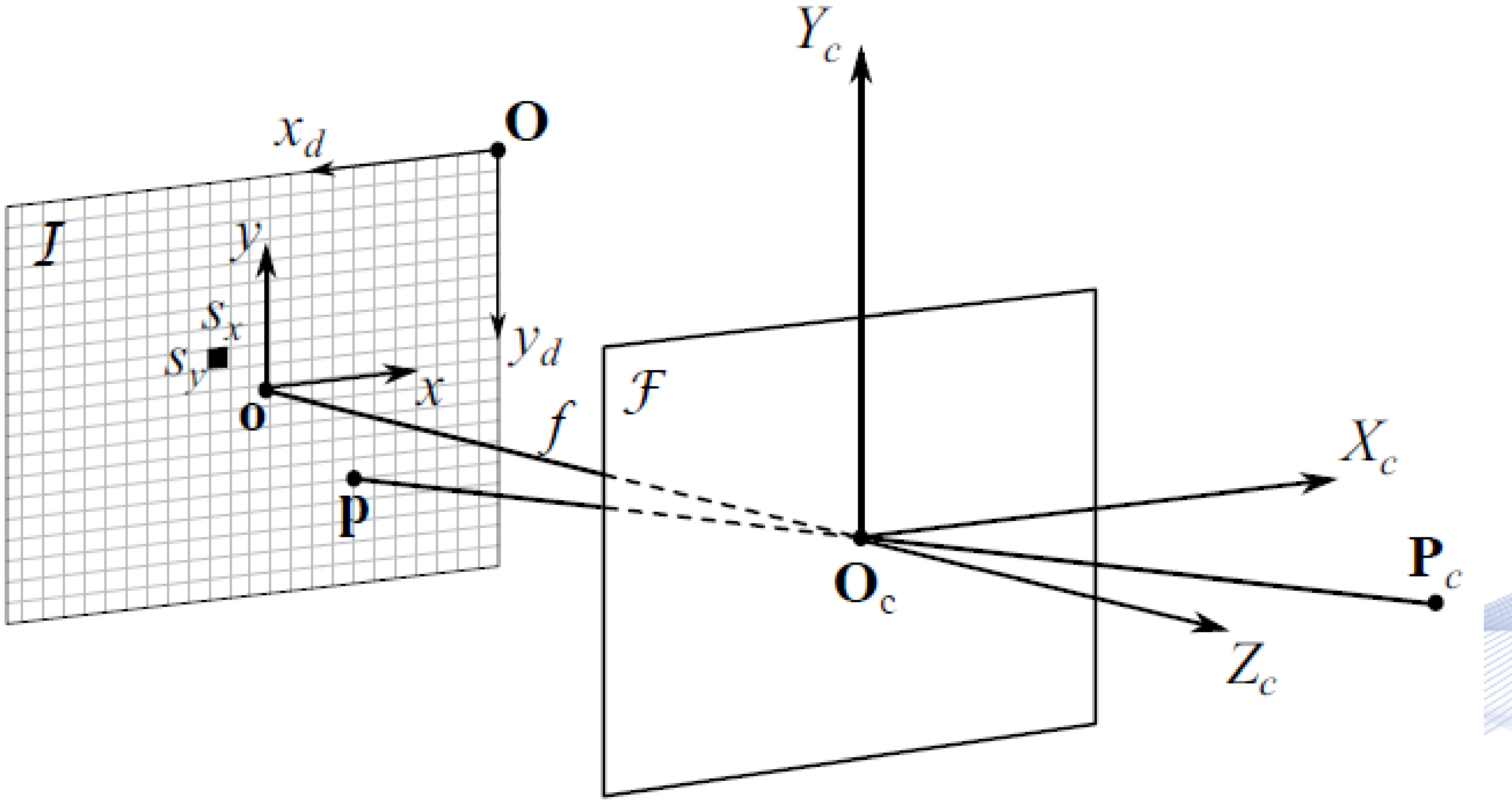




Camera Parameters and Projection Matrix



Camera Parameters and Projection Matrix



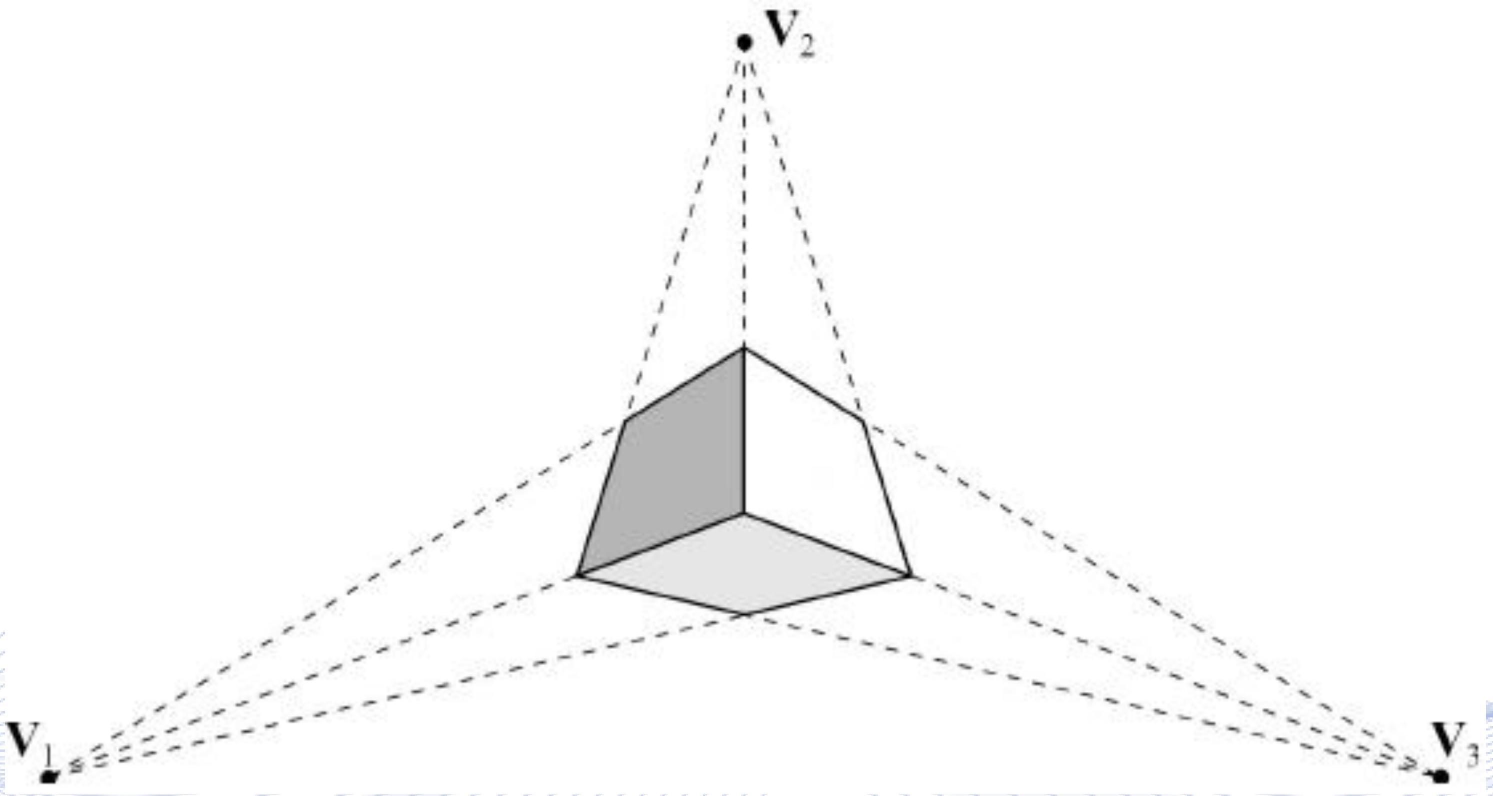
Camera Parameters and Projection Matrix

- $\boldsymbol{\mathcal{P}} = \mathbf{P}_I \ \mathbf{P}_E$ is the 3×4 ***camera projection matrix***.
- Also called *camera calibration matrix*:

$$\boldsymbol{\mathcal{P}} = \begin{bmatrix} -\frac{f}{s_x} & 0 & o_x \\ 0 & -\frac{f}{s_y} & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & -\mathbf{R}_1^T\mathbf{T} \\ r_{21} & r_{22} & r_{23} & -\mathbf{R}_2^T\mathbf{T} \\ r_{31} & r_{32} & r_{33} & -\mathbf{R}_3^T\mathbf{T} \end{bmatrix}.$$

- Ihe camera coordinate system is first translated and then rotated $\mathbf{P}_E$ has the form $\mathbf{P}_E = [\mathbf{R}_1|\mathbf{R}_2|\mathbf{R}_3| - \mathbf{RT}]$.
- Otherwise it would be $\mathbf{P}_E = [\mathbf{R}_1|\mathbf{R}_2|\mathbf{R}_3|\mathbf{T}]$.

Properties of the Projective Transformation



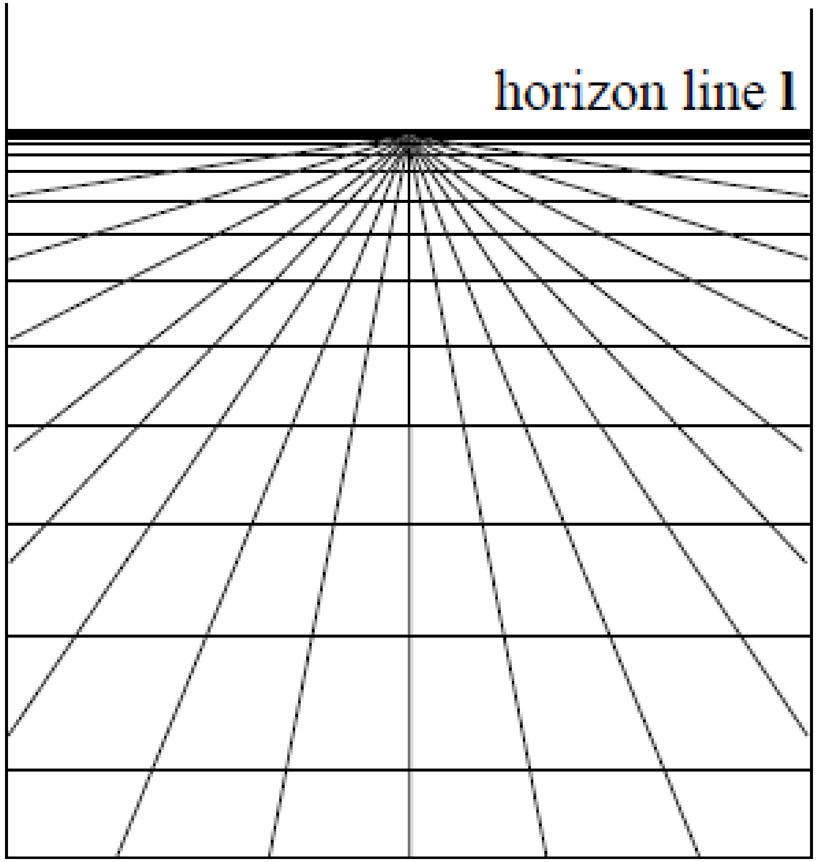
Vanishing points.

A bit of History…

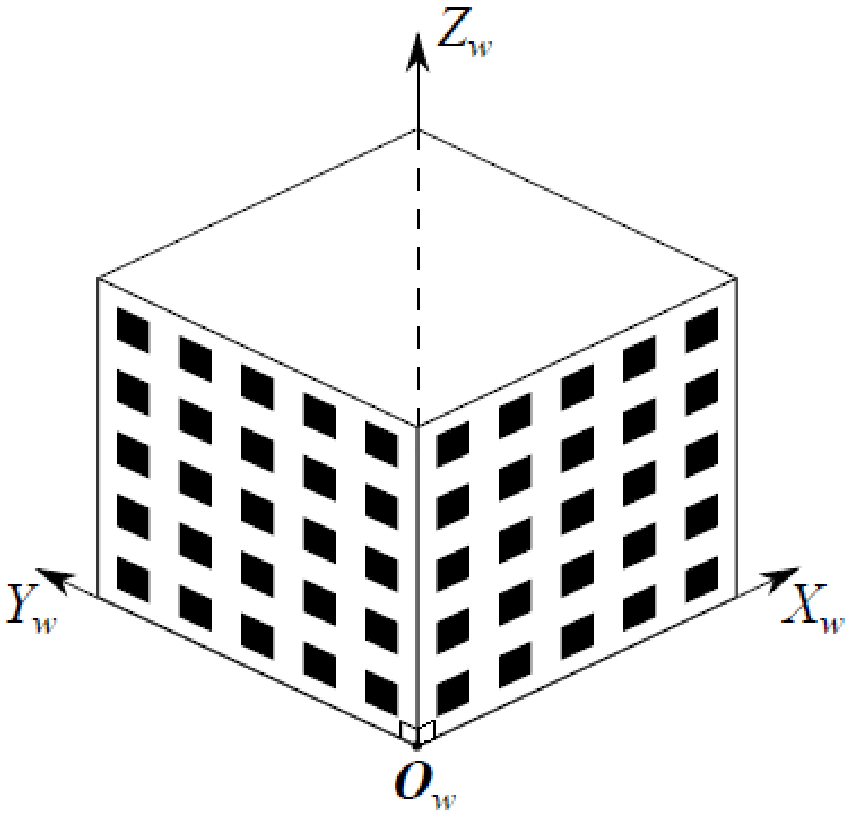
a) Byzantine icon; b) Canaletto painting.

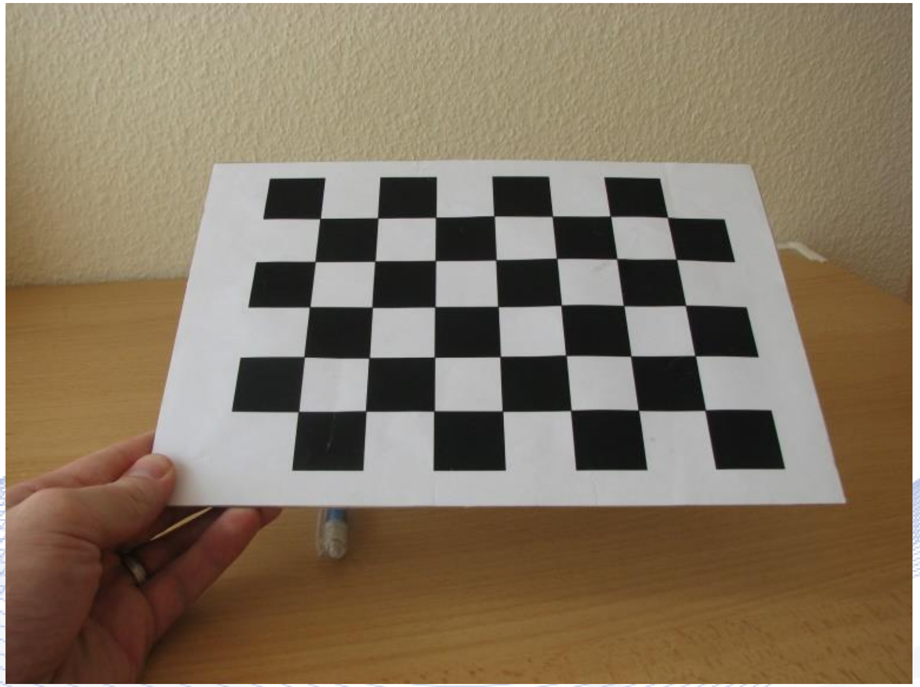
Properties of the Projective Transformation

- *Chirp effect*: the increase in local image spatial frequency proportionally to the distance of the projected scene area from the camera.
- It is evident in 2D image regions where distant and close-up scene parts are projected.



Camera Calibration





Calibration patterns.

Direct camera parameter estimation

$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} = \mathbf{R} \begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} + \mathbf{T} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}.$$

- It can be decomposed into:

$$X_c = r_{11}X_w + r_{12}Y_w + r_{13}Z_w + T_x$$
$$Y_c = r_{21}X_w + r_{22}Y_w + r_{23}Z_w + T_y$$
$$Z_c = r_{31}X_w + r_{32}Y_w + r_{33}Z_w + T_z.$$

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Q & A

Thank you very much for your attention!

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