Signal Sampling summary

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Artificial Intelligence & Information Analysis Lab

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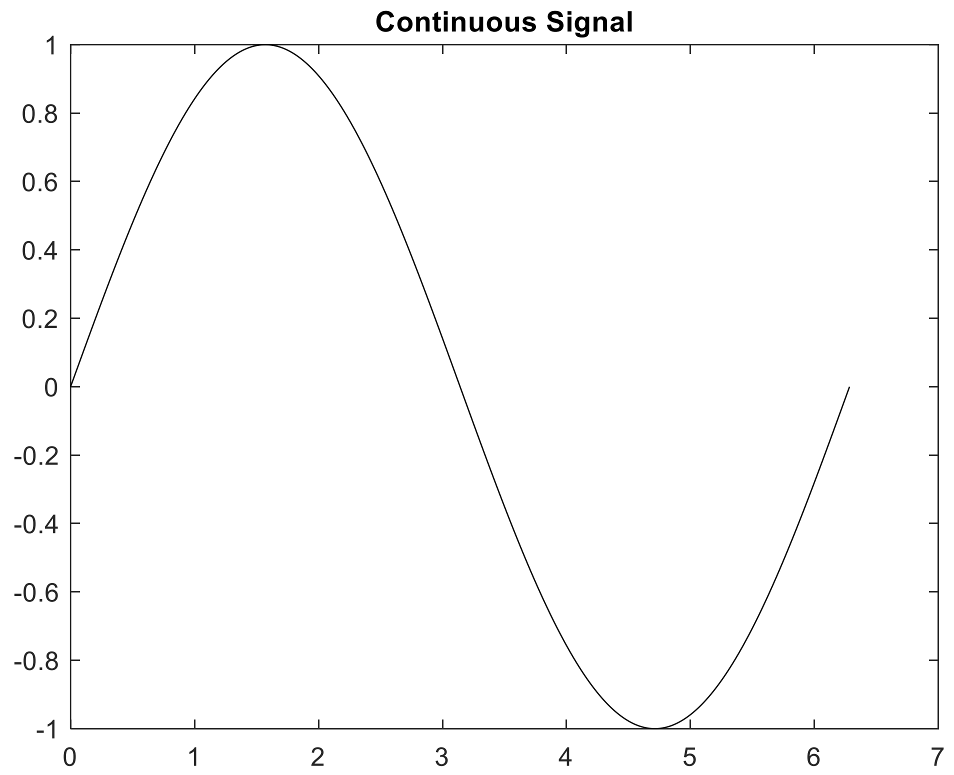
Continuous Signals

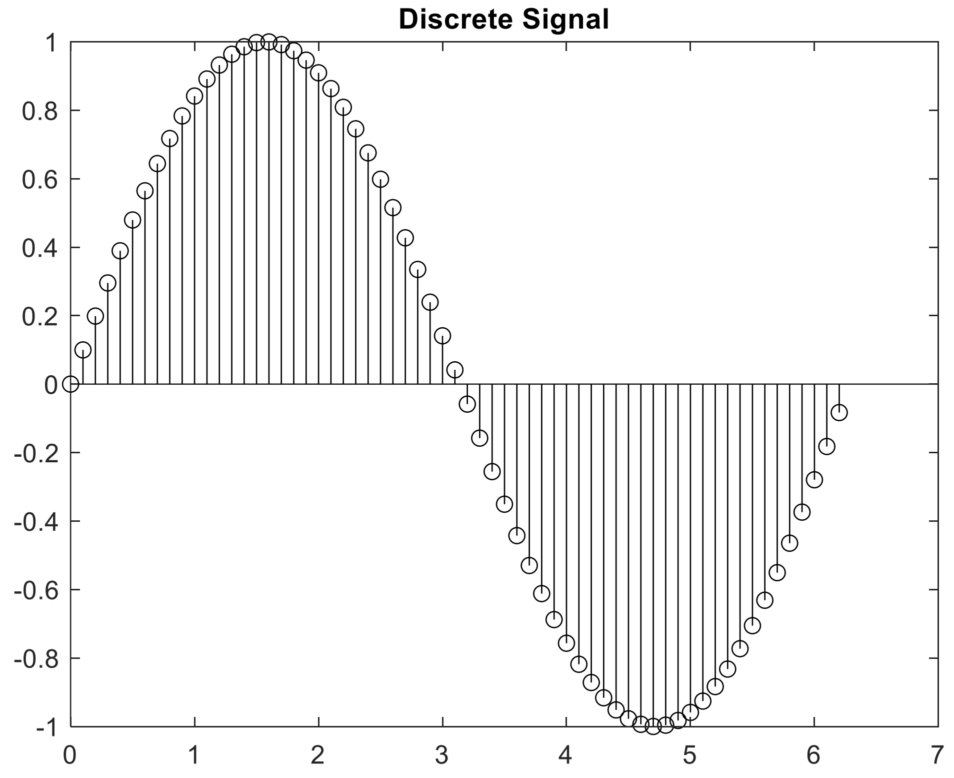
Let the values of a signal $x(t)$ be known for a continuous period of time, then we have a ***continuous-time signal*** $x(t): \mathbb{R} \rightarrow \mathbb{R}$.



Discrete Signals

A ***discrete-time*** (***discrete***) signal is a time series (function) $x(n)$: $\mathbb{Z} \rightarrow \mathbb{R}$, whose values are known only for some certain, discrete-time moments nT:

$$x(n) = x(nT),\ n \in \mathbb{Z}.$$



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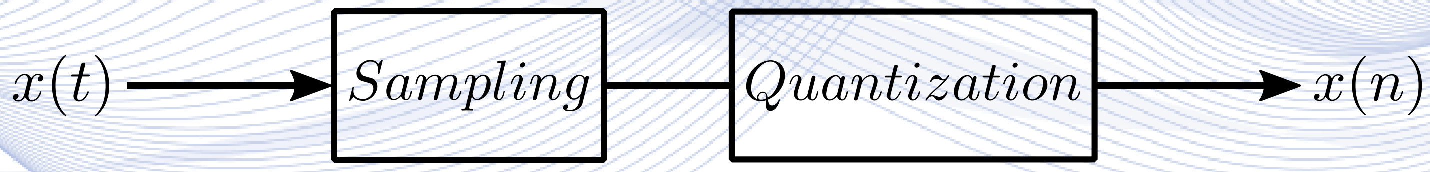
Signal Digitization

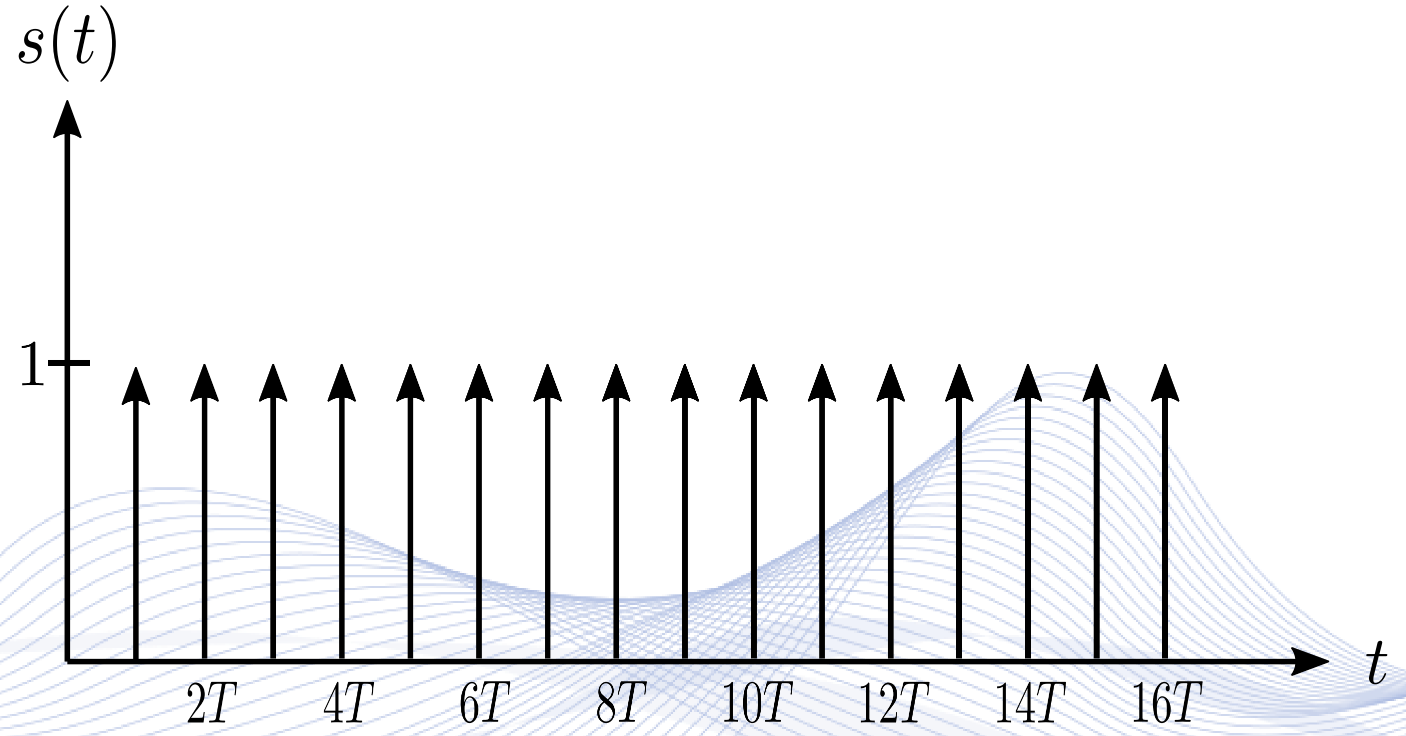
Signal digitization has two steps:

- ***Signal sampling*** at sampling frequency F.
- ***Signal quantization*** at b bit/sample.

Signal digitization is performed by A/D converters.



Sampling Function



Sampling function is a series of delta functions.

Signal Sampling

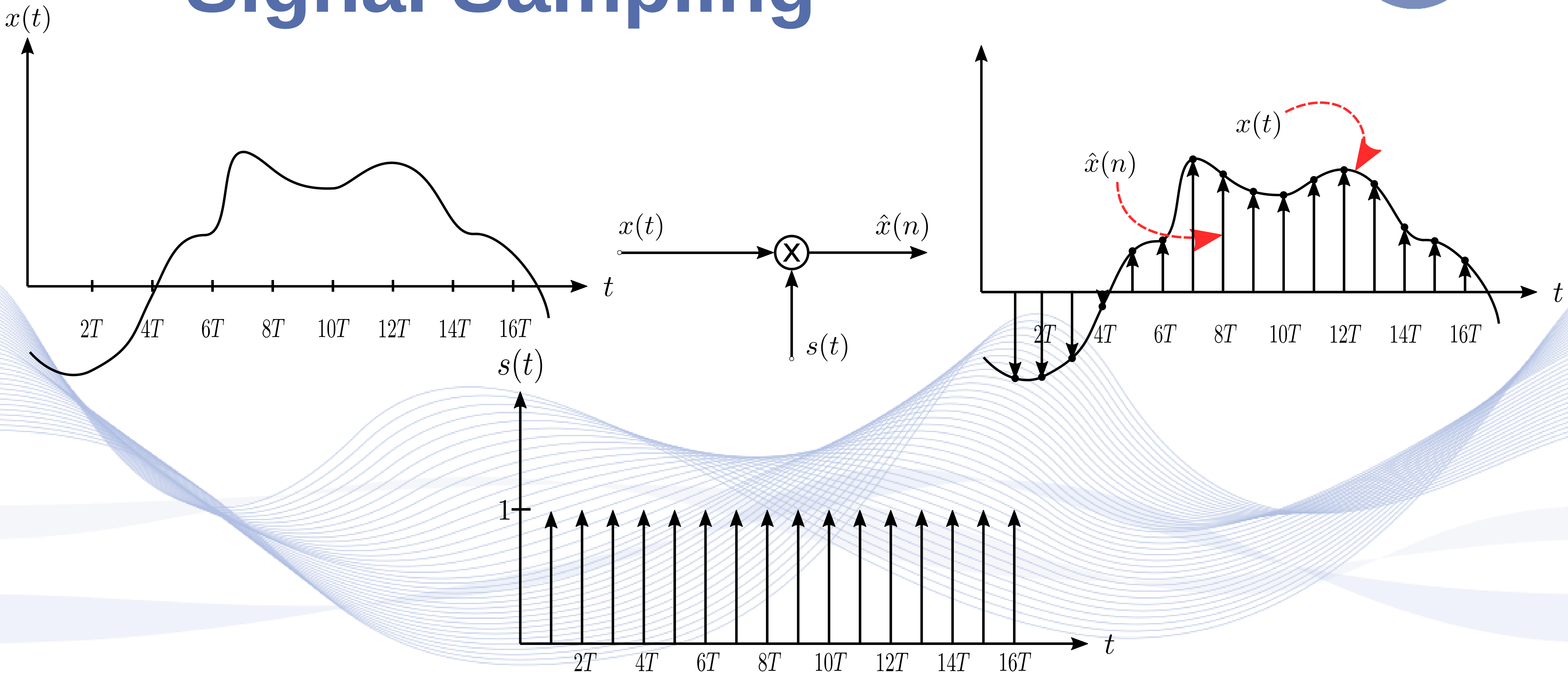
The output of a 1D ***signal sampling*** system is the discrete function $\hat{x}(t)$:

$$\hat{x}(t) = x(t)s(t) = \sum_{n=-\infty}^{\infty} x(t)\delta(t - nT) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT).$$

Equivalently, a discrete-time sampled signal is a function $\mathbb{Z} \rightarrow \mathbb{R}$:

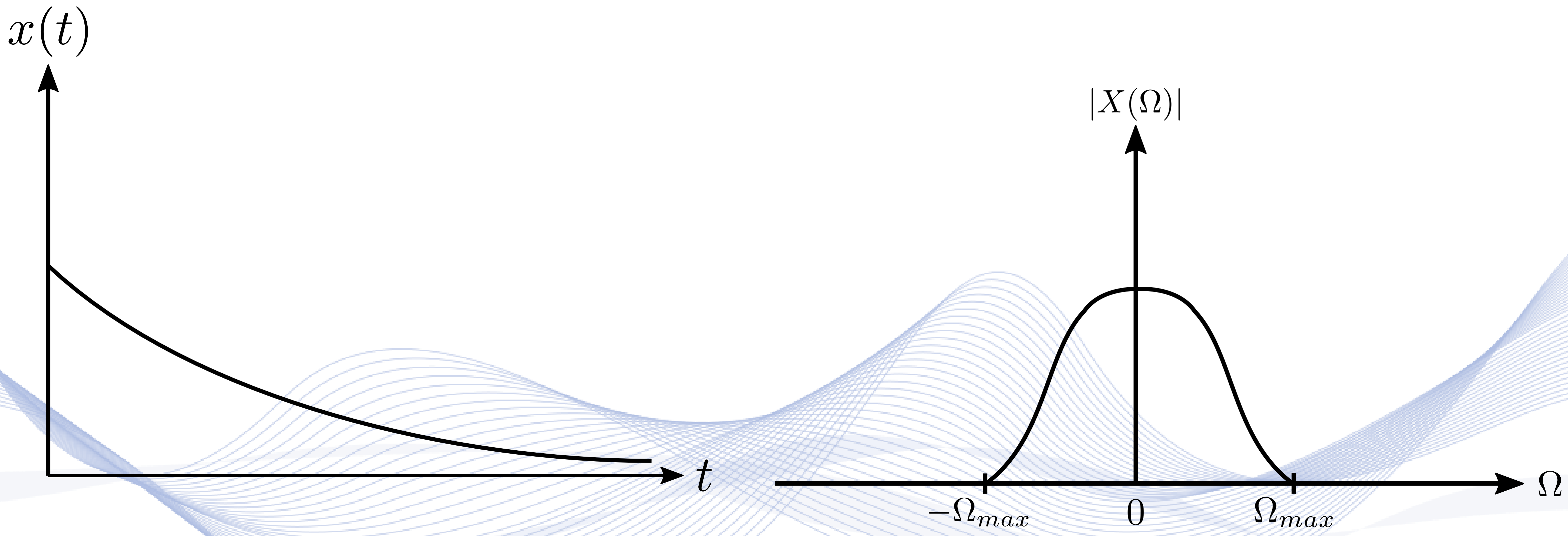
$$\hat{x}(n) = x(nT), \qquad n \in \mathbb{Z}.$$

Signal Sampling



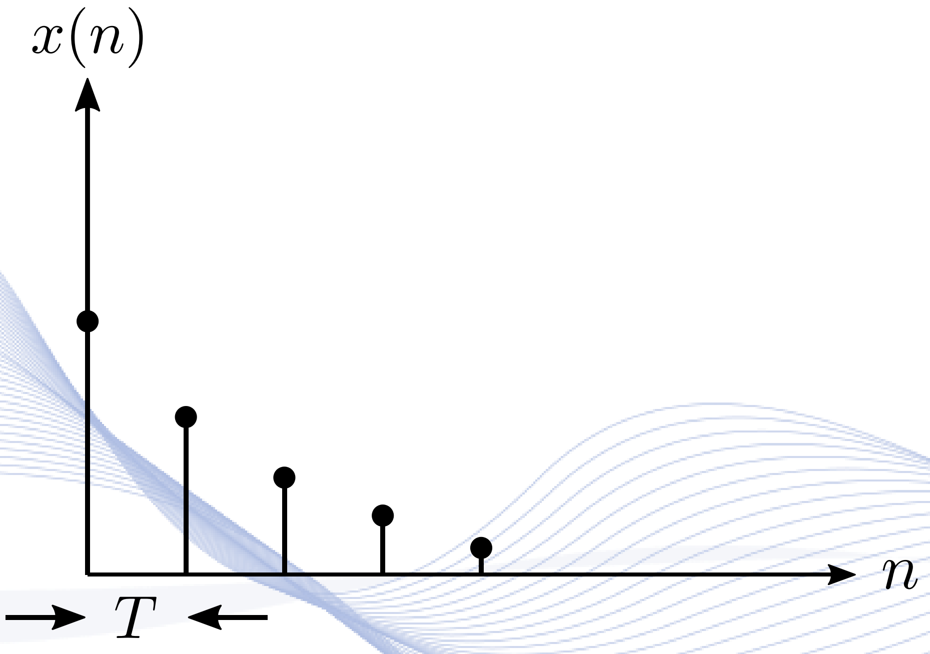
Signal sampling.

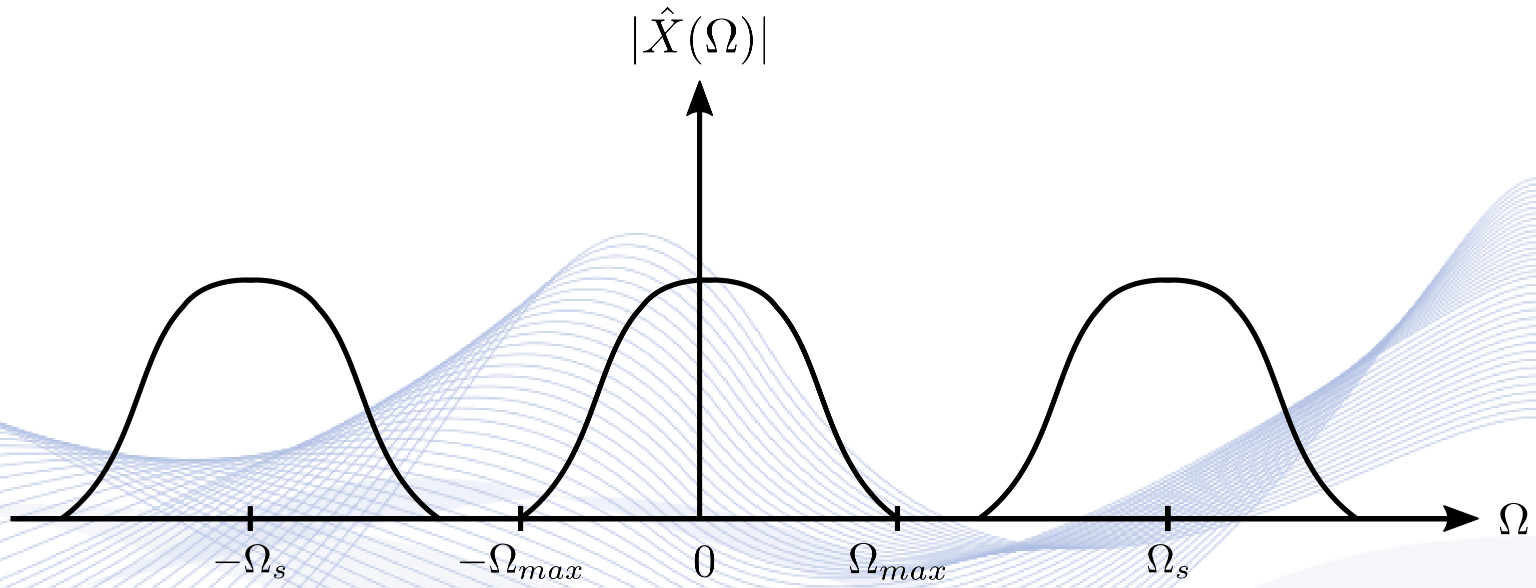
Shannon Theorem



Spectrum $X(\Omega)$ of signal $x(t)$.

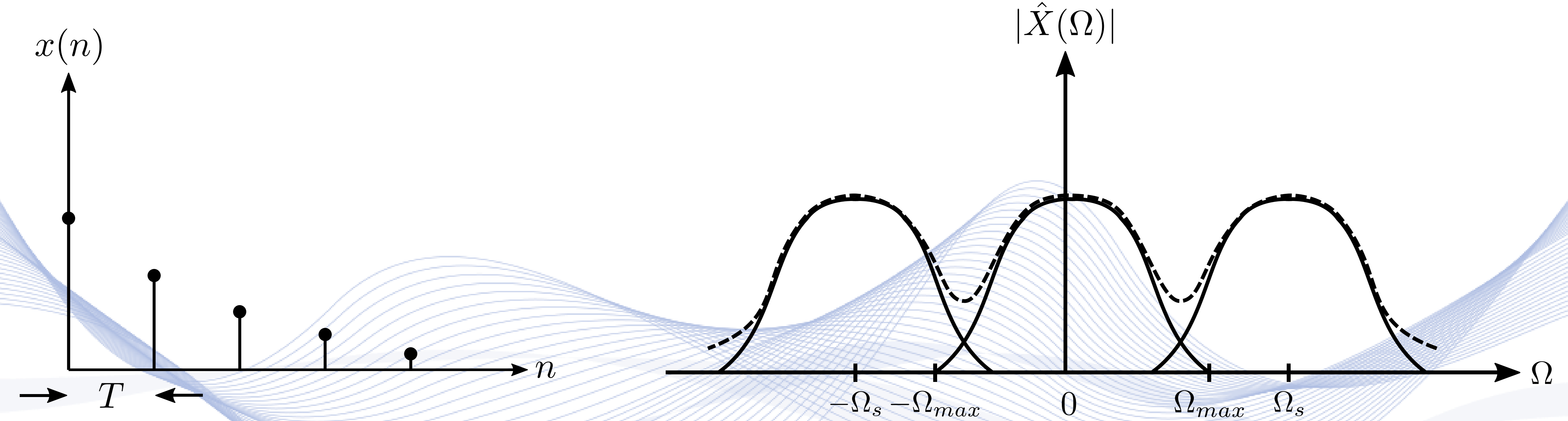
Sampling without Aliasing





Sampled signal spectrum for $\Omega_s \geq 2\Omega_{max}$.

Sampling with Aliasing



Sampled signal spectrum for $\Omega_s < 2\Omega_{max}$.

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Shannon Theorem

- The function $x(t)$, of which the spectrum $X(\Omega)$ has the highest frequency Ω_{max}, is described absolutely with the values of $x(nT)$, $n = 0, \pm 1, \pm 2, \ldots, T = \frac{2\pi}{\Omega_s}$, if and only if Ω_s is greater than $2\Omega_{max}$.
- This special frequency ($2\Omega_{max}$), is called ***Nyquist frequency***.

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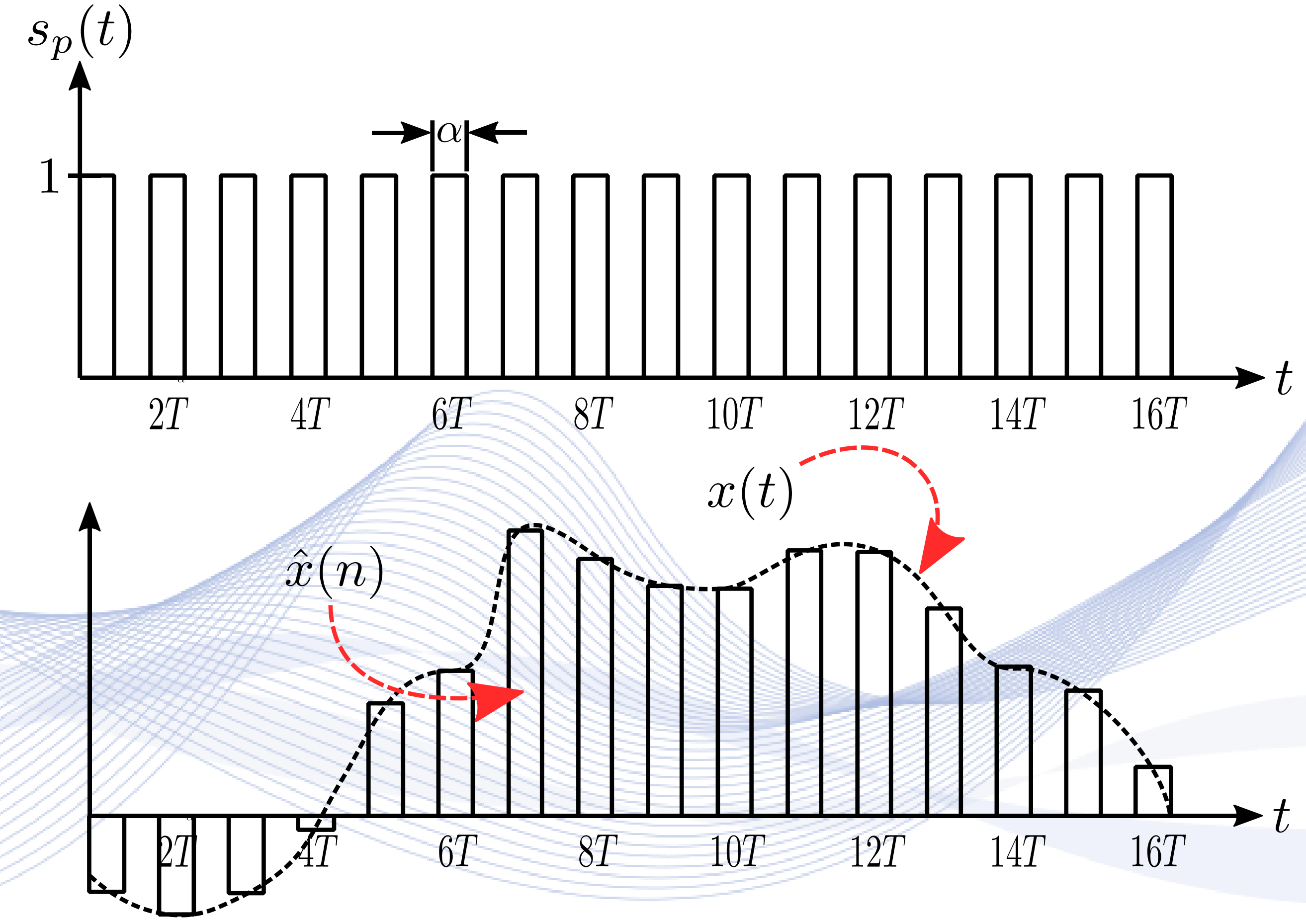
Applied Signal Sampling

It is practically impossible to implement ideal delta functions. Instead, sampling can be done using impulses of width α.



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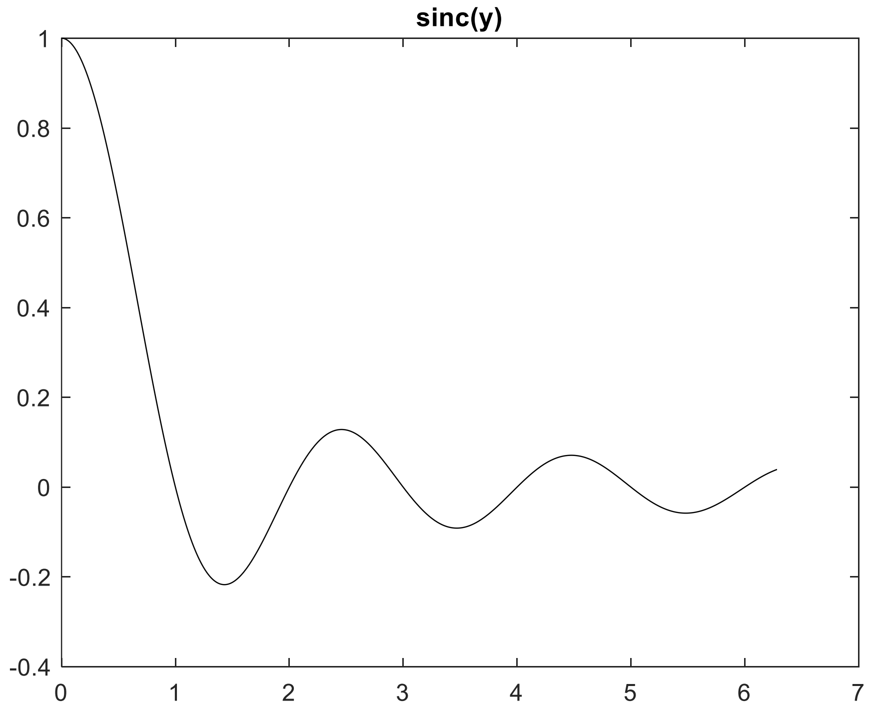
Applied Signal Sampling

- In that case, the spectrum of $\hat{X}(\Omega)$ is also periodically repeated, where coefficients C_n are given by:

$$C_n = \frac{1}{T}\int_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}} e^{-in\Omega_s t}dt = \frac{\alpha}{T}\frac{\sin\left(\frac{n\Omega_s\alpha}{2}\right)}{\frac{n\Omega_s\alpha}{2}}.$$

- It is a sinc function: $\mathrm{sinc}(y) = \frac{\sin(y)}{y}$ for: $y = \frac{n\Omega_s\alpha}{2}$.

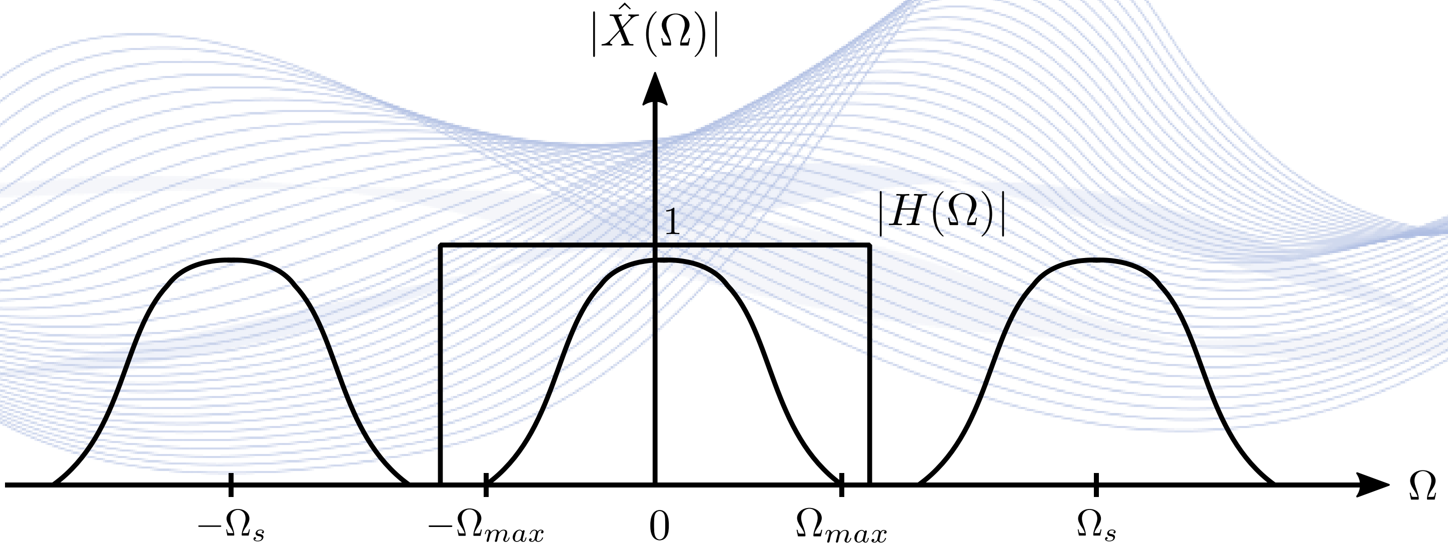


Signal Reconstruction

If the sampling interval T is small enough, so that the spectrum $X(\Omega)$ satisfies the relation:

$$X(\Omega) = 0, \qquad |\Omega| \geq \frac{\pi}{T},$$

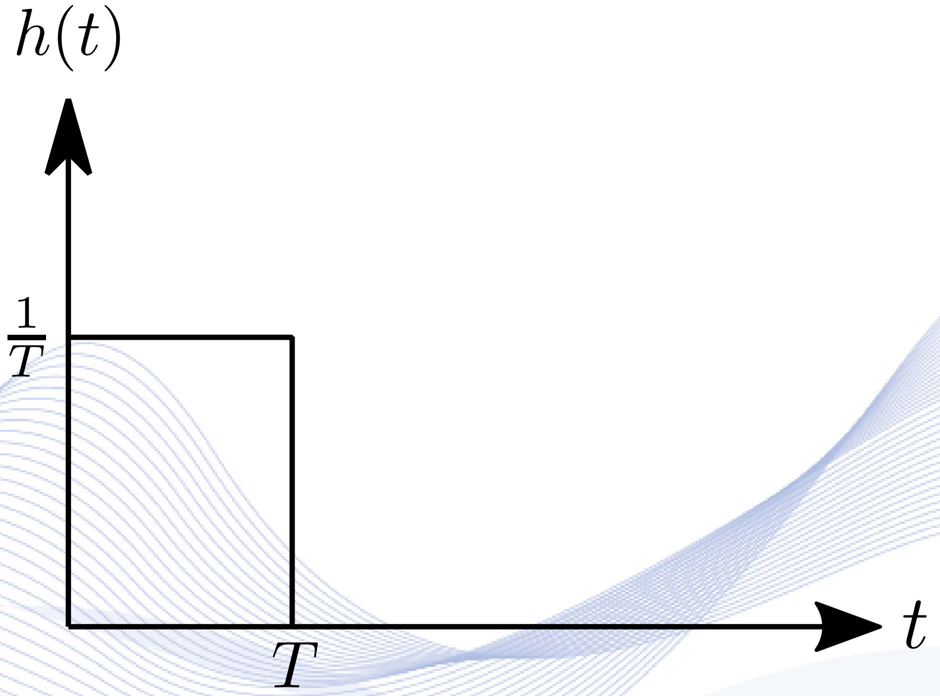
a square low-pass filter $H(\Omega)$ can be used to recover $X(\Omega)$ from the discrete 2D image spectrum $\hat{X}(\Omega)$ by retaining only its basic period.



Signal Reconstruction

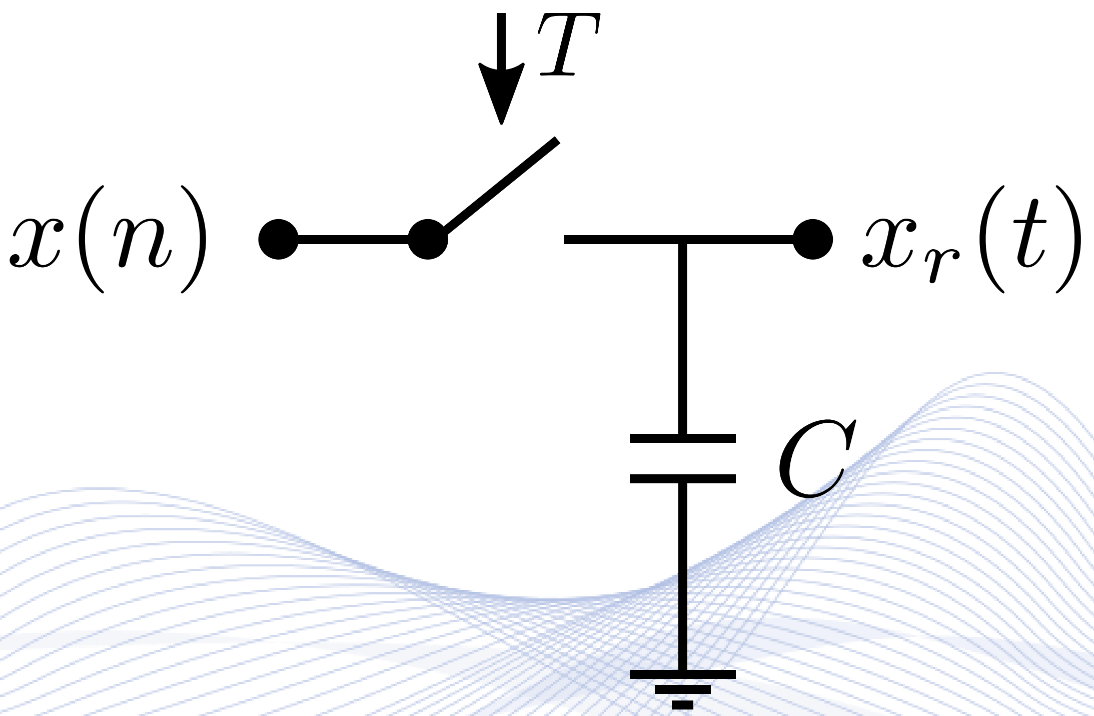
- The simplest ***reconstruction filter*** is the ***sample-and-hold filter***, its impulse response is the following:

$$h(t) = \begin{cases} \frac{1}{T}, & 0 \leq t \leq T \\ 0, & t < 0, t > T. \end{cases}$$



- Its use corresponds to 0-order polynomial interpolation.

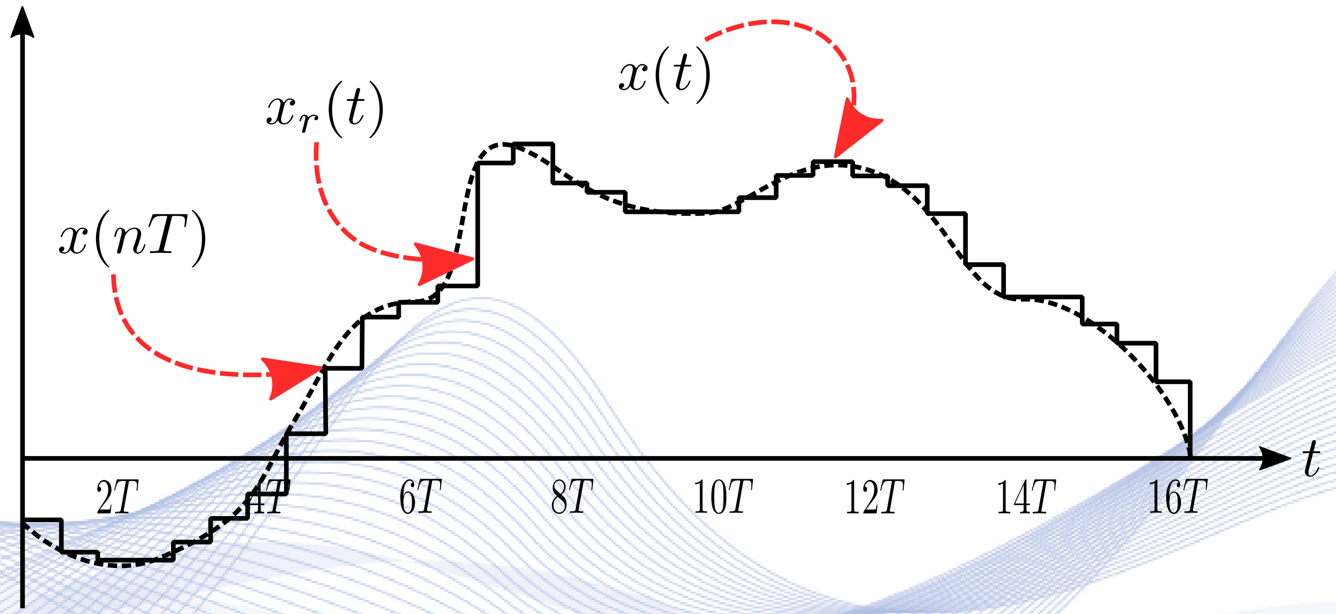
Signal Reconstruction



Elementary sample-and-hold circuit.

Signal Reconstruction

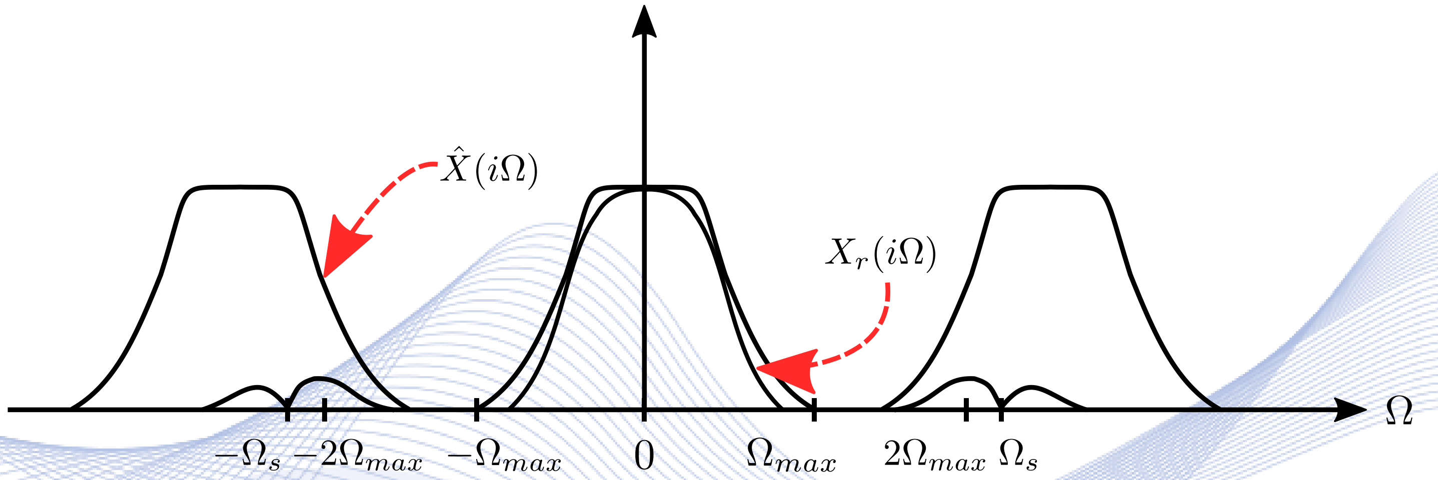
- The reconstructed signal $x_r(t)$ has high frequencies due to discontinuities.
- It converges to the original signal $x(t)$, as sampling period T tends to zero.



Signal Reconstruction

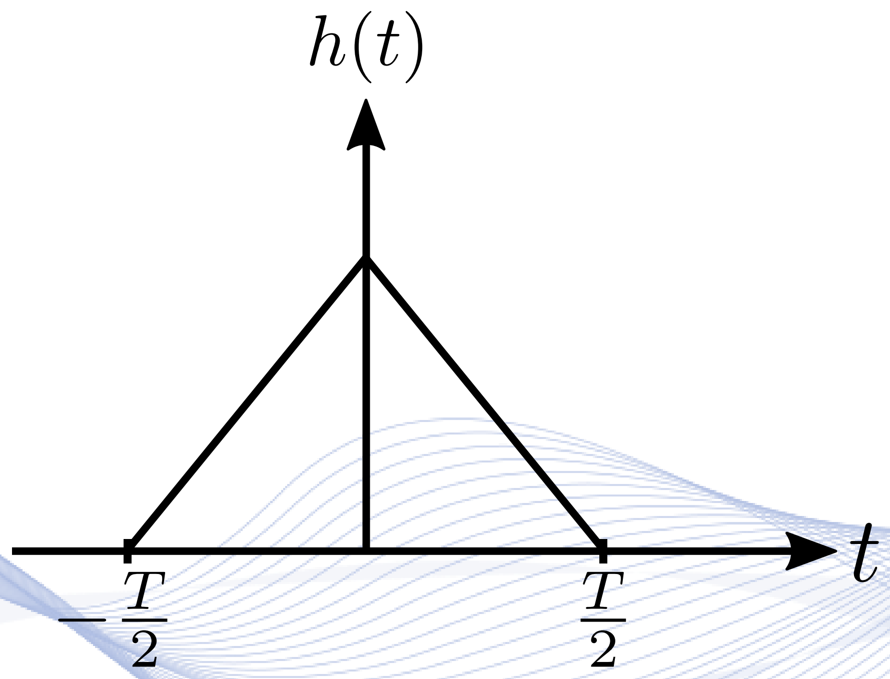
- The spectrum $X_r(i\Omega)$ of the reconstructed signal contains residual high frequencies.

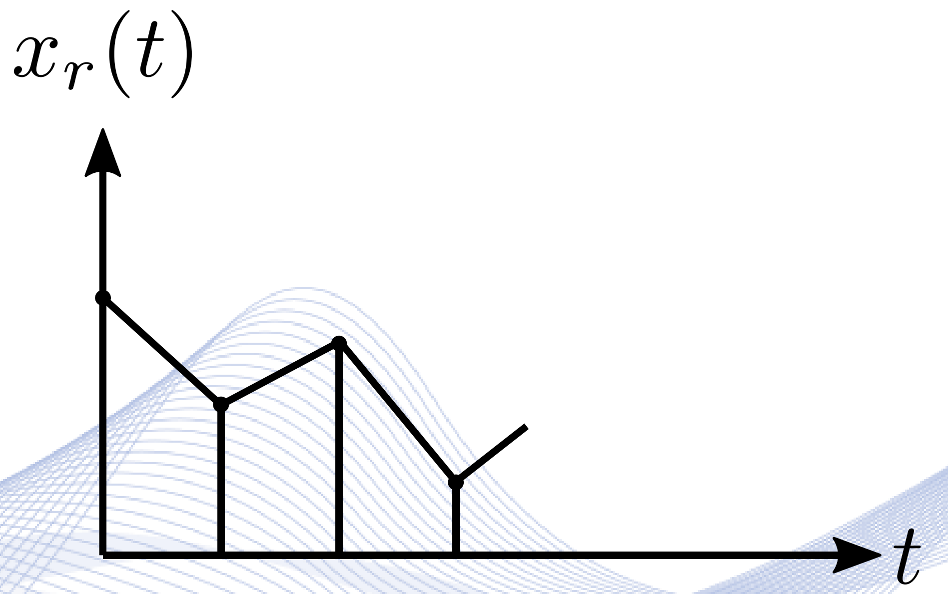
- Therefore, signal $x_r(t)$ is usually filtered with a low-pass filter in order, to cut off the high frequency residues of $X_r(i\Omega)$.



Spectrum $X_r(i\Omega)$ of the reconstructed signal.

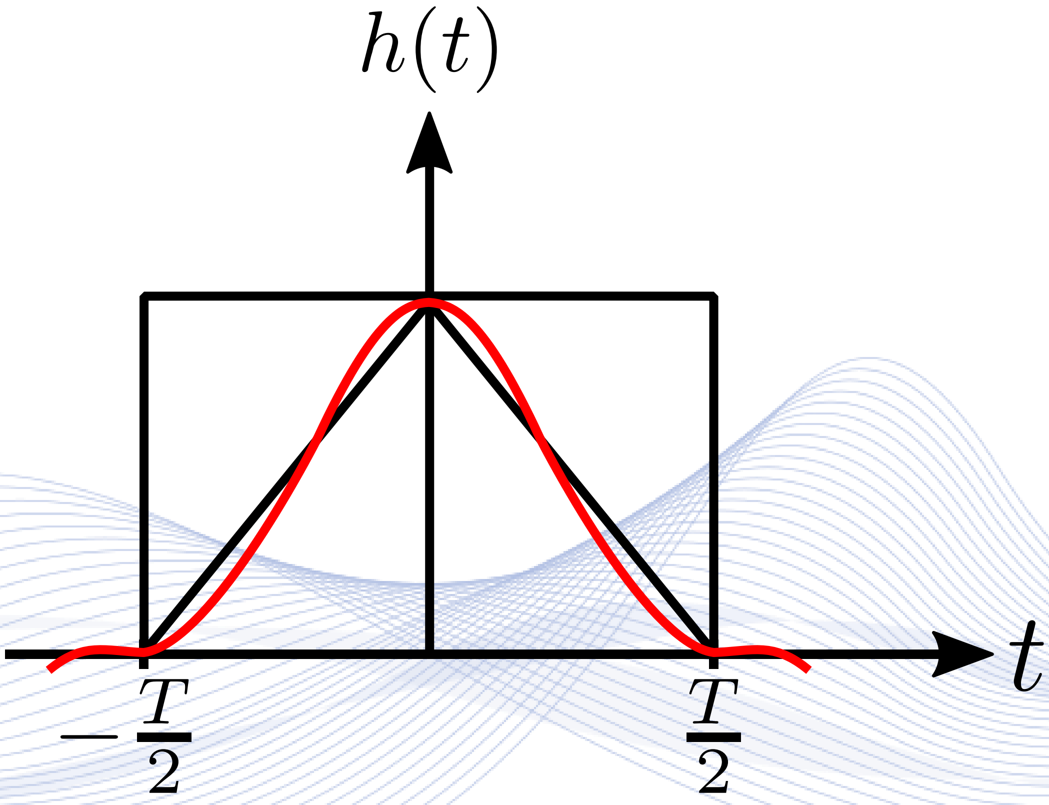
Signal Reconstruction





Linear interpolation kernel and linear interpolation.

Signal Reconstruction

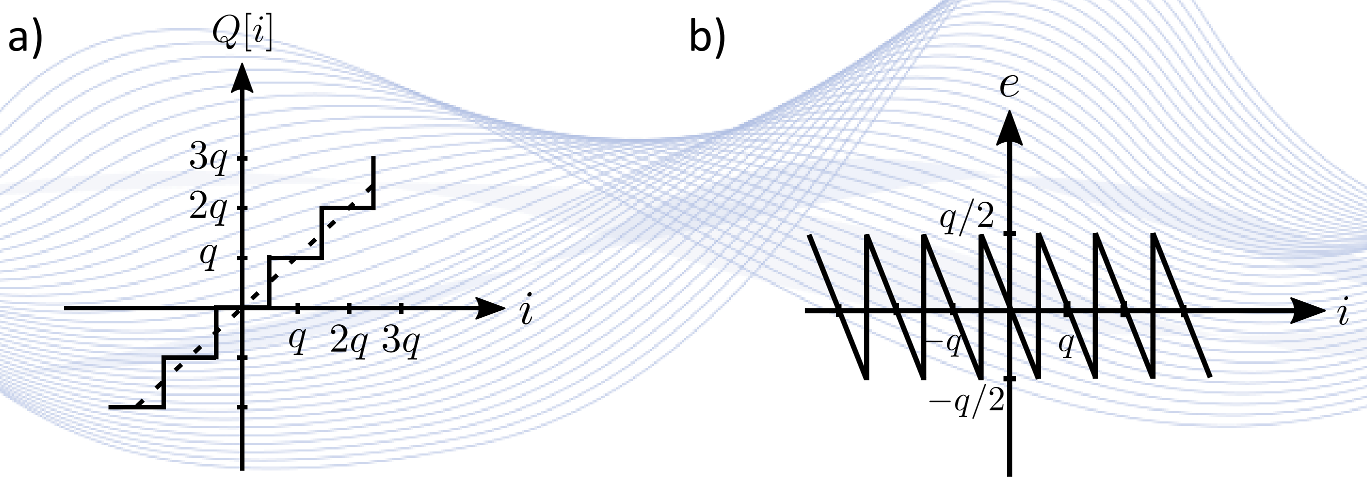


Spline interpolation kernel and interpolation.

Signal Quantization

Quantization is performed by an A/D converter.

- When b bits are allocated by signal sample, ***quantization step*** q is given by $q = \frac{1}{2^b}$;
- Quantized signal levels: $kq, k = 0,1,2, \dots, 2^b - 1$.



a) Input-output curve of quantizer; b) Quantization error.

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Q & A

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