Robot Kinematics and Dynamic Modeling summary

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Robot Kinematics and Dynamic Modeling

- **Robot Kinematics**
- Kinematic Equations
- Forward kinematics
- Inverse kinematics
- Rotations
- Dynamic Modeling
- Robot Dynamics
- Foundations from Classical Mechanics
- Newton-Euler Method
- Lagrange Method

Robot Kinematics

Definition:

- ***Robot kinematic*** (***RB***) is the science that studies the movement of multi degree of freedom kinematic chains that describes the structure of robotic systems.

- Rigid bodies describe the links of the robot, and its joints provide rotations.

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Kinematic equations

Definition:

- ***Kinematics equations*** are characterized as constraint equations of a mechanical system just as a robot manipulator that determines how input movement at one or more joints specifies the configuration of the device, with the purpose of defining a task position or an end-effector location.

- With ***kinematics equations,*** we can analyze and plan articulated systems ranging from four-bar linkages to serial and parallel robots.

Kinematic equations

Robot kinematics **include two categories of** ***kinematic equations*****:**

- *Forward kinematics*
- *Inverse kinematics*

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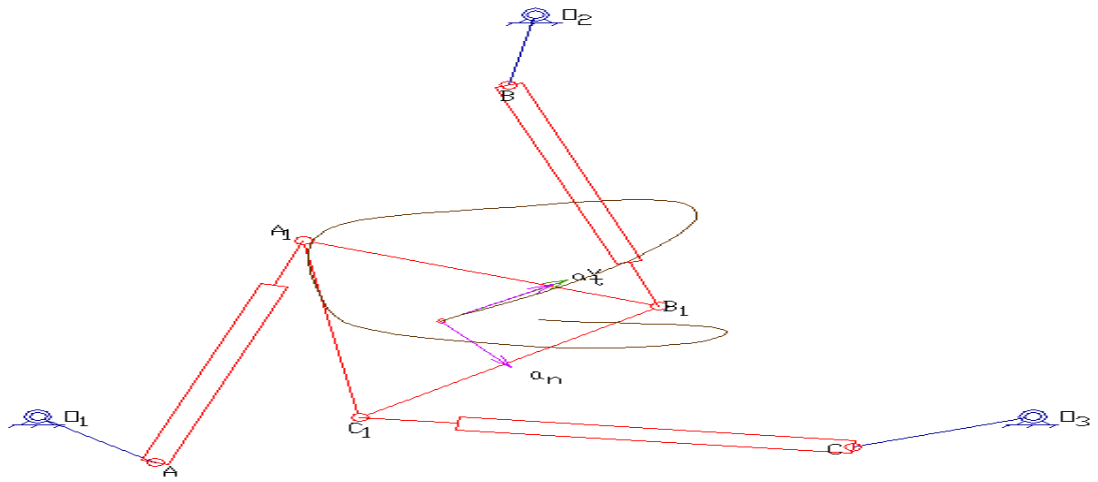
Forward kinematics

Definition:

- ***Forward kinematics*** by using ***kinematic equations*** of a robot, can compute the location of the end-effector from specified values for the joint parameters.



Forward kinematics: planar parallel manipulator (from [WikiPDR])

Forward kinematics

2D – presentation of robotic arm:

- This presentation uses simple trigonometry for moving each joint.

- The first joint is stable for example in a table and link $\mathbf{l}_1$ connects the first with the second joint that is able to rotate and move. Now a second link $\mathbf{l}_2$ placed on the second joint to connect with fixed end effector. At (0,0) is the 1st joint and $\mathbf{l}_1$, $\mathbf{l}_2$ describe the lengths of the two links (Fig 1).

Forward kinematics

- Now we try to rotate the second joint at end of $\mathbf{l}_1$ and rotate it by θ_1.
- The end effector now is located at (x, y) position, which can be described like before (Fig 2):

$$x'' = \mathbf{l}_2 \cdot \cos(\theta_1 + \theta_2),$$

and,

$$y'' = \mathbf{l}_2 \cdot \sin(\theta_1 + \theta_2).$$

- In this case $\theta_1 + \theta_2$ describes the angle between x'' and the second link $\mathbf{l}_2$ ($\theta_2 < 0$).

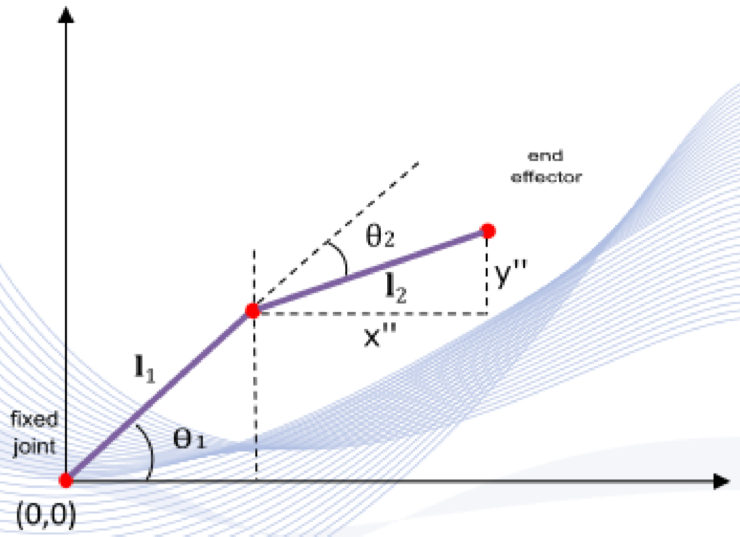


Fig 2: second rotation

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Inverse kinematics

Definition:

- **Inverse kinematics** describe the mathematical procedure of computing the variable joint parameters that must be placed in the end of a kinematic chain.

- There are many ways(solutions) to reach a specified position in **inverse kinematics**.

Inverse kinematics

- Fig 4 shows the set of positions that the end effector can reach when ($\mathbf{l}_1 < \mathbf{l}_2$).
- Point "a" is the furthest position of the arm with maximum length $\mathbf{l}_1 + \mathbf{l}_2$.
- Closest position to the origin is point "b" with length $\mathbf{l}_1 - \mathbf{l}_2$, cause the second link is bending back on the first link.
- We can also have position "c" which our robotic arm can reach with two ways as showed in Fig 3.

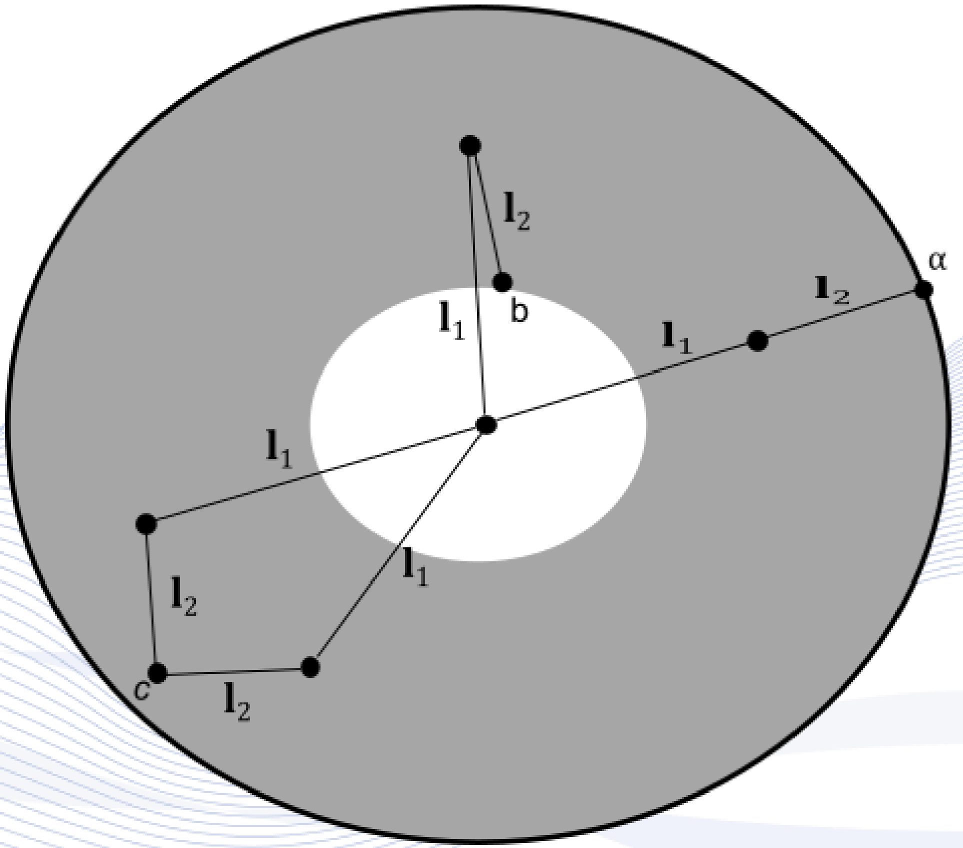

Fig 3: *workspace of a robotic arm*

Inverse kinematics

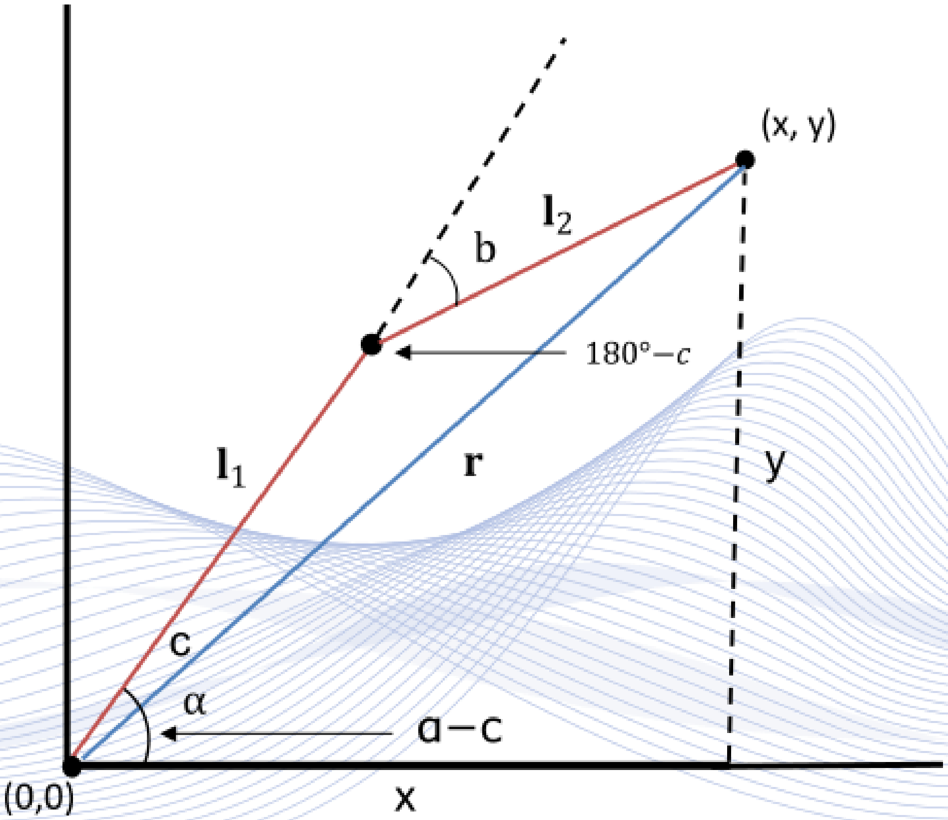

Fig 4: *Inverse kinematics of a two-link arm*

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Rotations

- ***Rotation matrices*** is used to describe the rotational motion of a robotic arm
- A ***rotation matrix*** has three interpretations:
  - Rotating a vector
  - Rotating a coordinate frame
  - Transforming a vector from coordinate frame to another

Rotations

Rotating a vector:

- We have a vector **r** in (x, y) position with polar coordinates (**r**, φ) (Fig 5 (a)).
- We rotate the vector **r** in (x', y') position, so (x', y') has polar coordinates $(\mathbf{r}, \varphi + \theta)$,where $\varphi + \theta$ is the new angle of vector **r** (Fig 5 (b)).

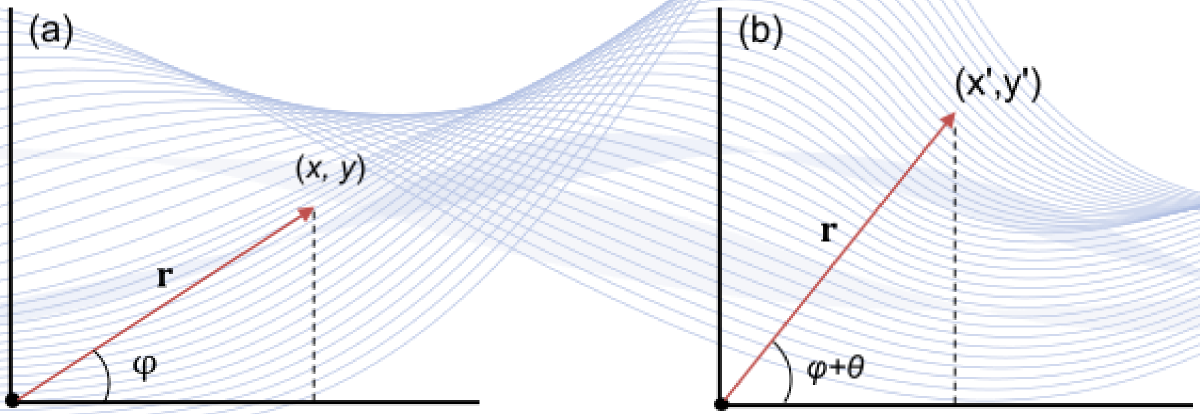

Fig 5: (**a**) *A vector* $\mathbf{r}$. (**b**) *The vector* $\mathbf{r}$ *rotated by* θ

Rotations

Rotating a Coordinate Frame (CF):

- A coordinate frame can be described by two orthogonal unit vector:

$$\mathbf{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad \mathbf{y} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

- In **Fig** 7 (b) we have the rotated CF by θ degrees.

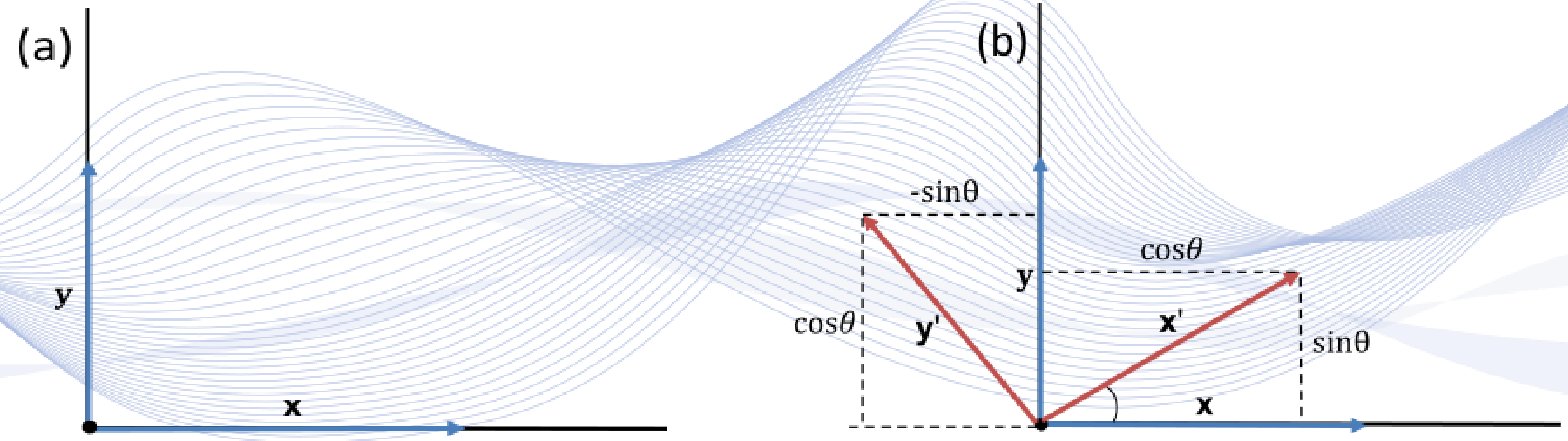

Fig 6: (**a**) *A coordinate frame.* (**b**) *Rotated coordinate frame (red).*

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Rotations

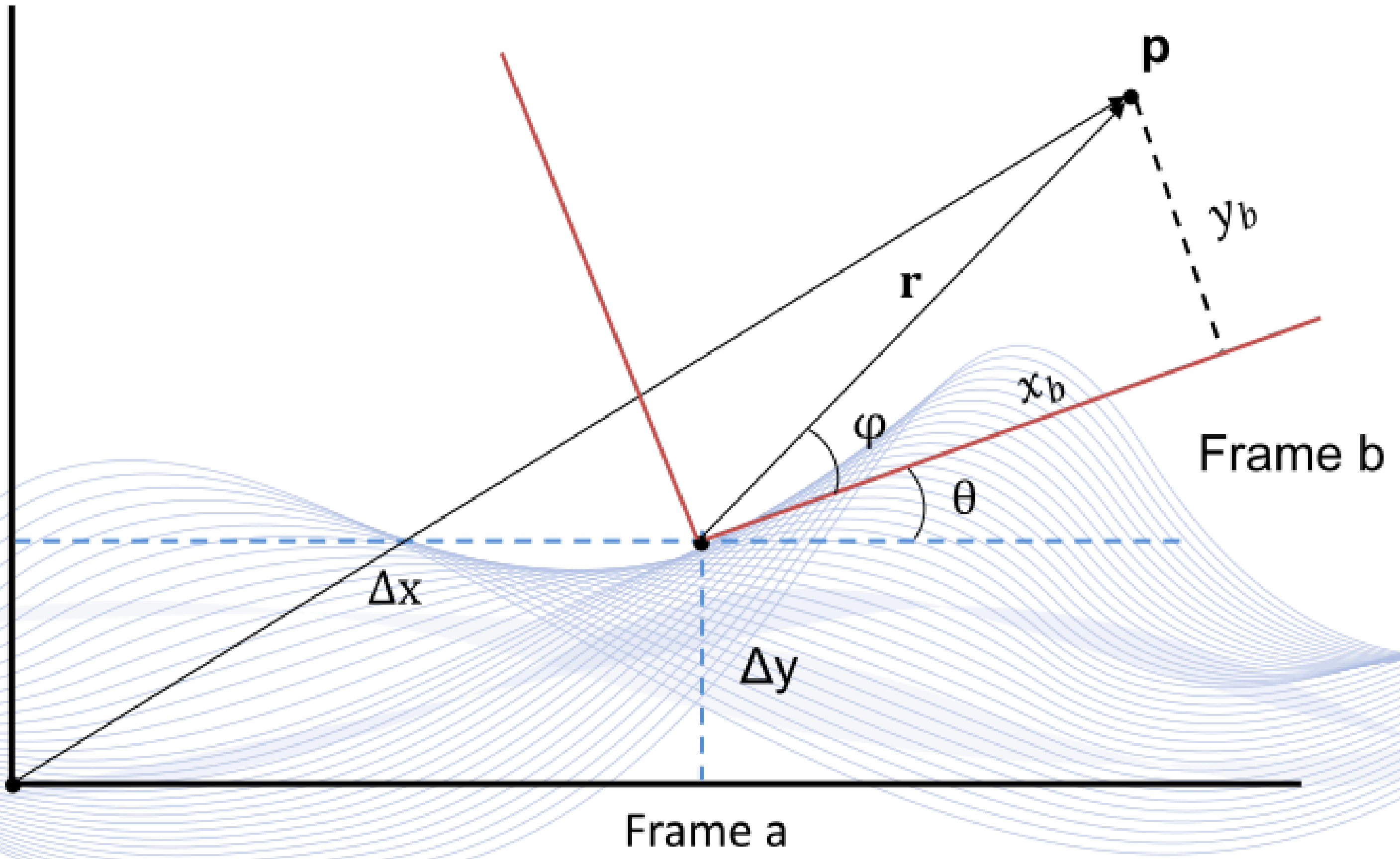

Fig 7: *Rotating and Translating A Coordinate frame*

Rotations

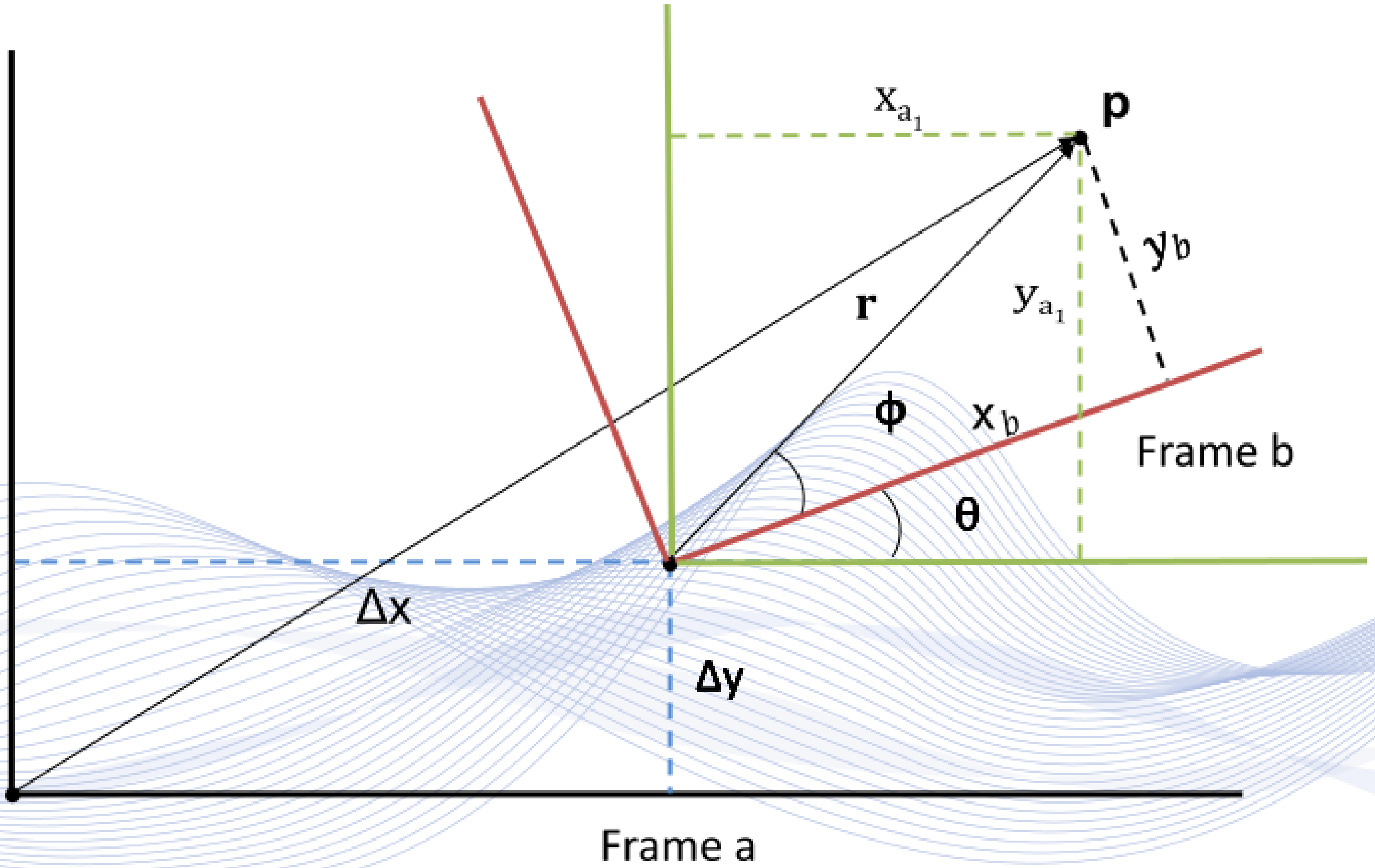

Fig 8: *adding frame a1*

Rotations

- Finally, to acquire the coordinates in frame a, we need to multiply the translation matrix with point (x_{a_1}, y_{a_1}):

$$\begin{bmatrix} x_a \\ y_a \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & \Delta x \\ 0 & 1 & \Delta \mathrm{y} \\ 0 & 0 & 1 \end{bmatrix}}_{\textit{Translation matrix}} \cdot \begin{bmatrix} x_{a1} \\ y_{a1} \\ 1 \end{bmatrix}.$$

- To provide the rotation and translation at same time, need to multiply the above two transforms, so we have as single homogenous transform:

$$\begin{bmatrix} 1 & 0 & \Delta x \\ 0 & 1 & \Delta \mathrm{y} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & \Delta x \\ \sin\theta & \cos\theta & \Delta \mathrm{y} \\ 0 & 0 & 1 \end{bmatrix}.$$

Robot Kinematics

- We can expand the above 2D-representation in a 3D model, but the mathematics is going to be more complicated.

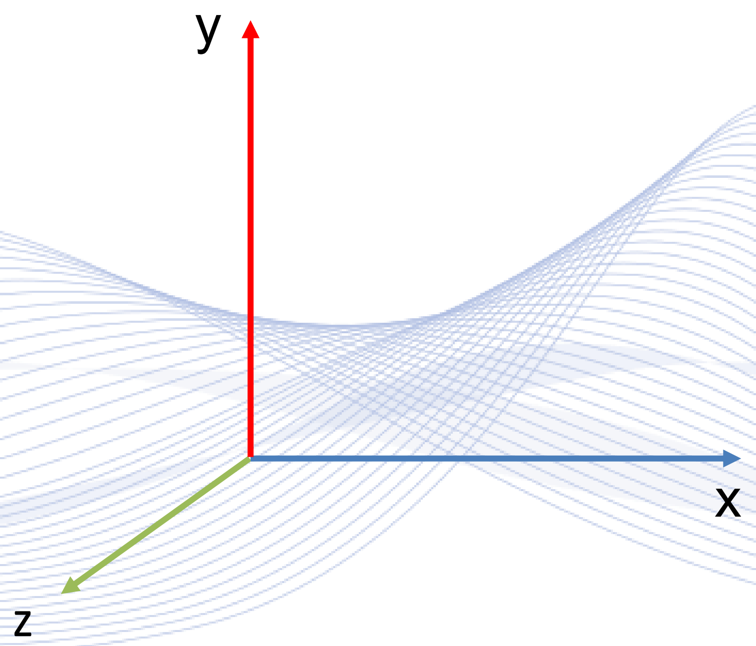

Fig 9: *3D space*

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Dynamic Modeling

Definition:

- In physics, a ***dynamical system*** is defined as a set of particles whose have time-varying behavior, so differential equations are typically used to describe these systems.
- ***Dynamic modeling*** is the use of a computer program to model a ***dynamical system***.

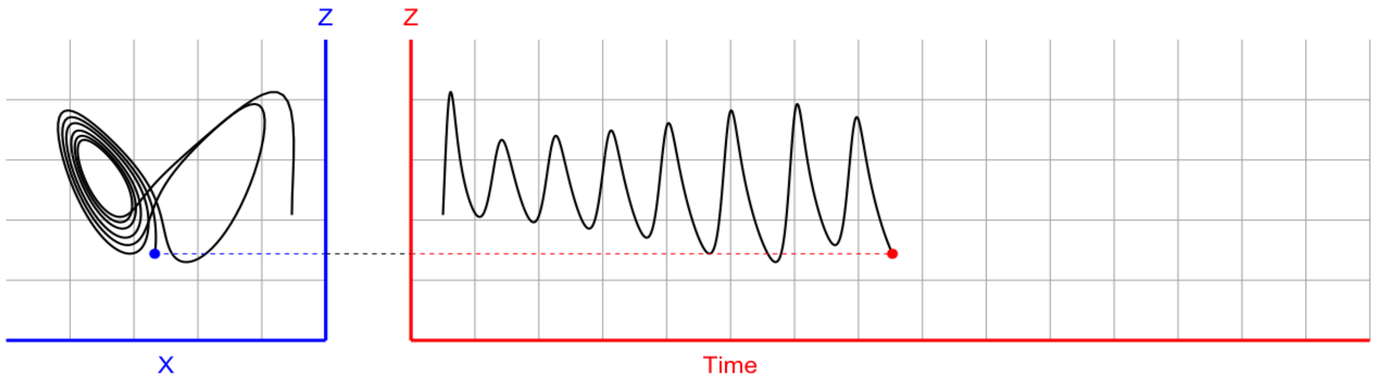

Fig 9: *Dynamic System with time varying-behavior* (from [WikiEDM])

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Dynamics

- For many robots with stable base, we have to look into a multi-body dynamics with form:

$$\boldsymbol{M}(\boldsymbol{q}) \cdot \boldsymbol{q}'' + \boldsymbol{b}(\boldsymbol{q}, \boldsymbol{q}') = \boldsymbol{\tau} + \boldsymbol{J}_c(\boldsymbol{q})^T \cdot \boldsymbol{F}_c$$

Dynamics

Where:

- $\boldsymbol{M}(\boldsymbol{q}) \in \mathbb{R}^{n_q \times n_q}$ Generalized mass matrix(orthogonal).
- $\boldsymbol{q}, \boldsymbol{q}', \boldsymbol{q}'' \in \mathbb{R}^{n_q}$ Generalized position, velocity and acceleration vectors.
- $\boldsymbol{b}(\boldsymbol{q}, \boldsymbol{q}') \in \mathbb{R}^{n_q}$ Coriolis and centrifugal terms.
- $\boldsymbol{g}(\boldsymbol{q}) \in \mathbb{R}^{n_q}$ Gravitational terms.
- $\boldsymbol{\tau} \in \mathbb{R}^{n_q}$ External generalized forces.
- $\boldsymbol{F}_c \in \mathbb{R}^{n_q}$ External Cartesian forces (e.g., from contacts).
- $\boldsymbol{J}_c(\boldsymbol{q}) \in \mathbb{R}^{n_q}$ Geometric Jacobian corresponding to the external forces.

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Foundations from Classical Mechanics

Newton's Law for Particles

- We have particles with mass *m*, where has infinitely small dimensions, which can be defined by the position vector **r**.
- The whole mass is concentrated to a single point.
- To describe the motion of the system we will use the Newton's second law:

$$\mathbf{r}''m = \mathbf{F}.$$

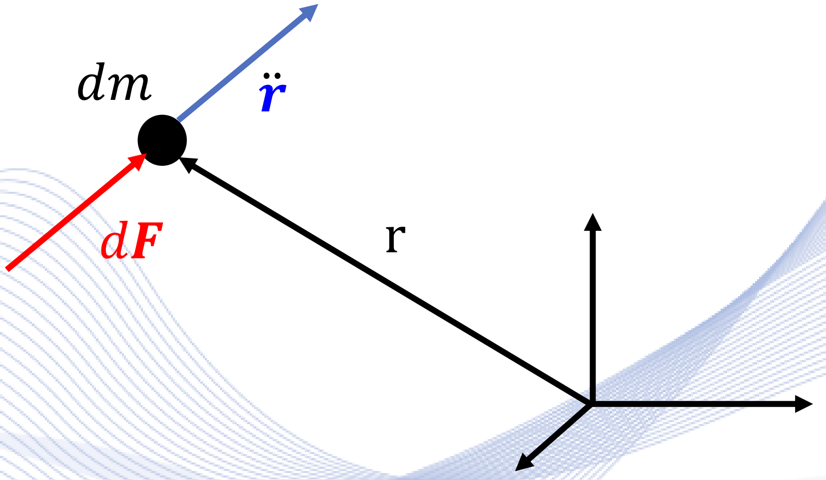

Fig 10: *Force acting on single particle*

Foundations from Classical Mechanics

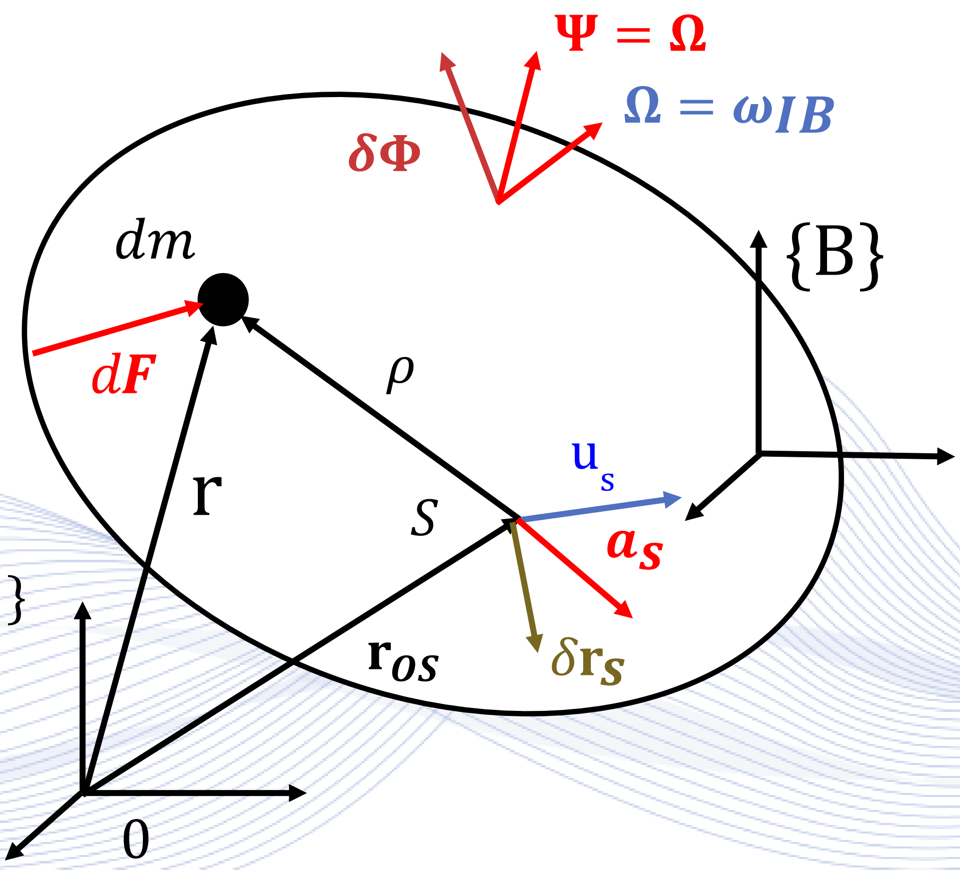

Fig 11: *Kinematics of a single body*

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Newton-Euler Method

Newton-Euler for Single Bodies

- For a single body, the evaluation of the principle of VT(6) results to:

$$0 = \delta W = \int_{\mathcal{B}} \begin{pmatrix} \delta \mathbf{r}_s \\ \delta \boldsymbol{\Phi} \end{pmatrix}^{\mathrm{T}} \cdot \begin{bmatrix} \mathbb{I}_{3\times 3} \\ [\boldsymbol{\rho}]_{\times} \end{bmatrix} \cdot \left([\mathbb{I}_{3\times 3} - [\boldsymbol{\rho}]_{\times}] \cdot \begin{bmatrix} a_s \\ \Psi \end{bmatrix} \cdot dm + [\boldsymbol{\Omega}]_{\times}^2 \cdot \boldsymbol{\rho} dm - d\mathbf{F}_{\text{ext}} \right.$$

$$= \begin{pmatrix} \delta \mathbf{r}_s \\ \delta \boldsymbol{\Phi} \end{pmatrix}^{\mathrm{T}} \cdot \int_{\mathcal{B}} \begin{bmatrix} \mathbb{I}_{3\times 3}\, dm & [\boldsymbol{\rho}]_{\times}^{T}\, dm \\ [\boldsymbol{\rho}]_{\times}\, dm & [\boldsymbol{\rho}]_{\times}^{2}\, dm \end{bmatrix} \cdot \begin{bmatrix} a_s \\ \Psi \end{bmatrix} + \begin{bmatrix} [\boldsymbol{\Omega}]_{\times}^2 \cdot \boldsymbol{\rho} dm \\ [\boldsymbol{\rho}]_{\times} \cdot [\boldsymbol{\Omega}]_{\times}^2 \cdot \boldsymbol{\rho} dm \end{bmatrix} - \begin{bmatrix} d\mathbf{F}_{\text{ext}} \\ [\boldsymbol{\rho}]_{\times}\, d\mathbf{F}_{\text{ext}} \end{bmatrix}. \quad (7)$$

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Lagrange Method

- The ***Lagrange method*** is focused on about three basic ideas:
  1. The definition of generalized coordinates $\boldsymbol{q}$ and generalized velocities $\boldsymbol{q}'$, that might or might not encode the information about the constraints applicable to the system.
  2. We have Lagrangian operation L and for mechanical systems is equal to

$$\mathrm{L} = T - U$$

where, T is total kinetic energy and U is total potential energy.

Lagrange Method

3. The ***Euler-Lagrange*** equation or ***Euler-Lagrange*** of the second kind as also known, that applies to the Lagrangian operation L and to the total exterior generalized forces $\boldsymbol{\tau}$:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial q'}\right) - \left(\frac{\partial L}{\partial q}\right) = \boldsymbol{\tau}.$$

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Q & A

Thank you very much for your attention!

More material in
http://icarus.csd.auth.gr/cvml-web-lecture-series/

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