Mathematical Morphology summary

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Artificial Intelligence & Information Analysis Lab

Contents

- Mathematical Morphology
- Greyscale Morphology
- Skeletons
- Shape Decomposition
- Voronoi tessellation
- Watershed transform

Mathematical morphology

Mathematical morphology uses a set theoretic approach to image analysis.

The morphological transformations must possess the following properties:

1. ***Translation invariance***:

$$\Psi(X_z) = [\Psi(X)]_z.$$

2. ***Scale invariance***:

$$\Psi_\lambda(X) = \lambda\Psi(\lambda^{-1}X).$$

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Mathematical morphology

3. ***Local knowledge***. Transformation $\Psi(X)$ must require only information within a local neighborhood for its operation.

4. ***Semicontinuity***. The morphological transformation $\Psi(X)$ must possess certain continuity properties.

Morphological transformations using ***structuring element*** (set) B:

Dilation:

$$X \oplus B^s = \bigcup_{b \in B} X_{-b} = \left\{ z \in E : B_z \bigcap X \neq \emptyset \right\}.$$

Erosion:

$$X \ominus B^s = \bigcap_{b \in B} X_{-b} = \{ z \in E : B_z \subset X \}.$$

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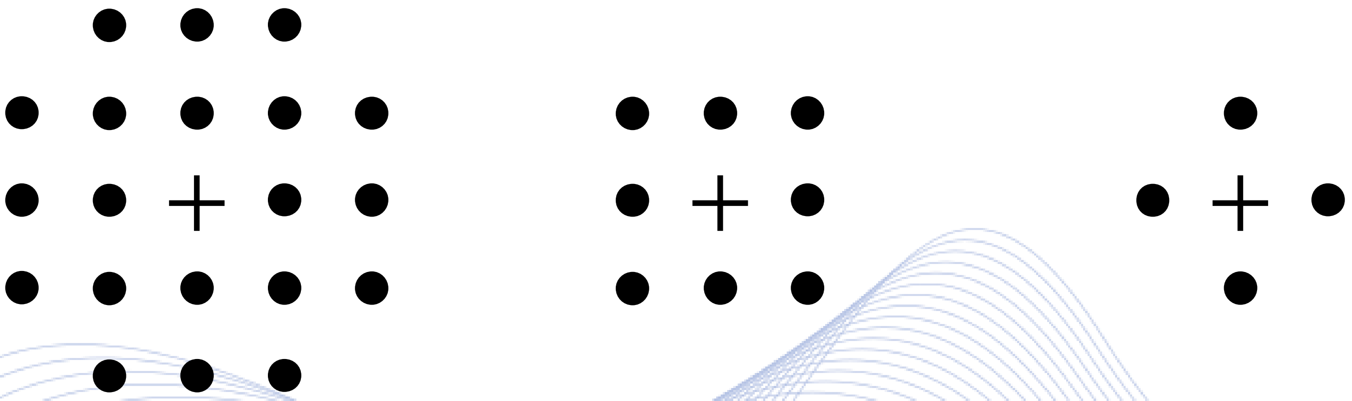
Mathematical morphology



Structuring elements B: CIRCLE, SQUARE, RHOMBUS.

Mathematical morphology

Dilation and erosion and are special cases of ***Minkowski set addition*** and ***Minkowski set subtraction***.

$$X \oplus B = \bigcup_{b \in B} X_b, \qquad X \ominus B = \bigcap_{b \in B} X_b.$$

(a) (b) (c)

a) Thresholded image; b) eroded and c) dilated image by the structuring elements SQUARE.

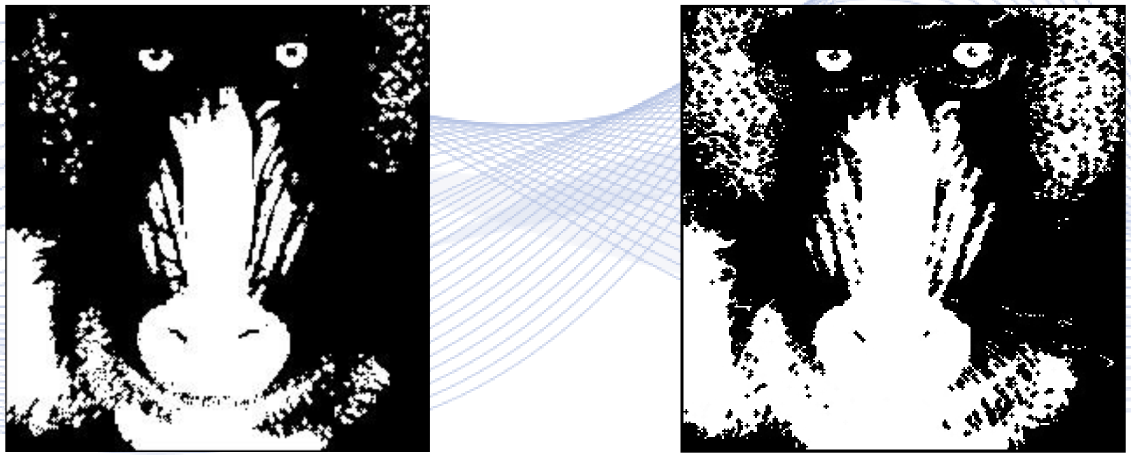
Mathematical morphology

Opening X_B:

$$X_B = (X \ominus B^s) \oplus B = \bigcup \{B_z : B_z \subset X\}.$$

Closing X^B:

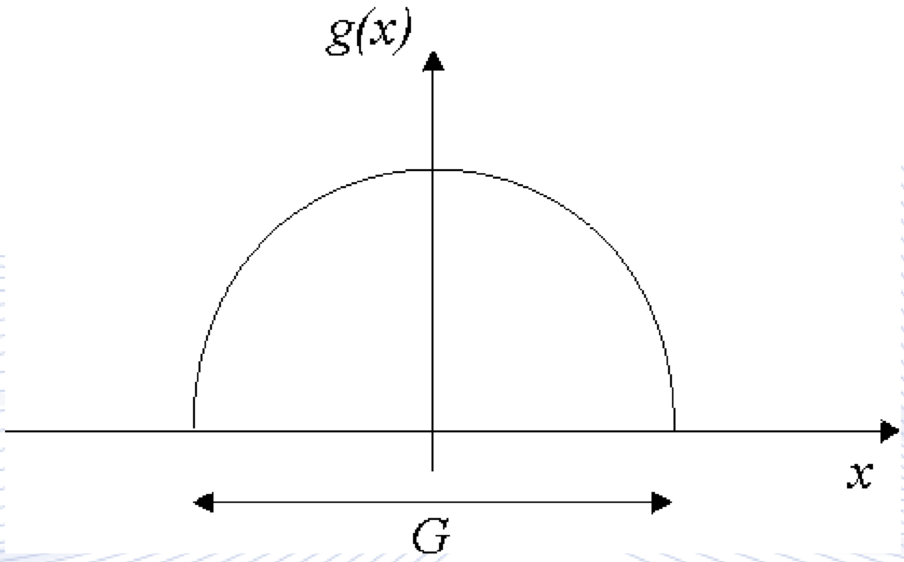
$$X^B = (X \oplus B^s) \ominus B = \bigcap \{B_z^c : B_z \subset X^c\}.$$



a) opened image; b) closed image.

Greyscale morphology

The tools for greyscale morphological operations are simple functions $g(x)$ having domain G. They are called ***structuring functions***.



An example of a structuring function.

Greyscale morphology

Greyscale dilation and erosion of a function $f(x)$ by $g(x)$:

$$[f \oplus g^s](x) = \max_{z \in D, z-x \in D} \{f(z) + g(z-x)\},$$

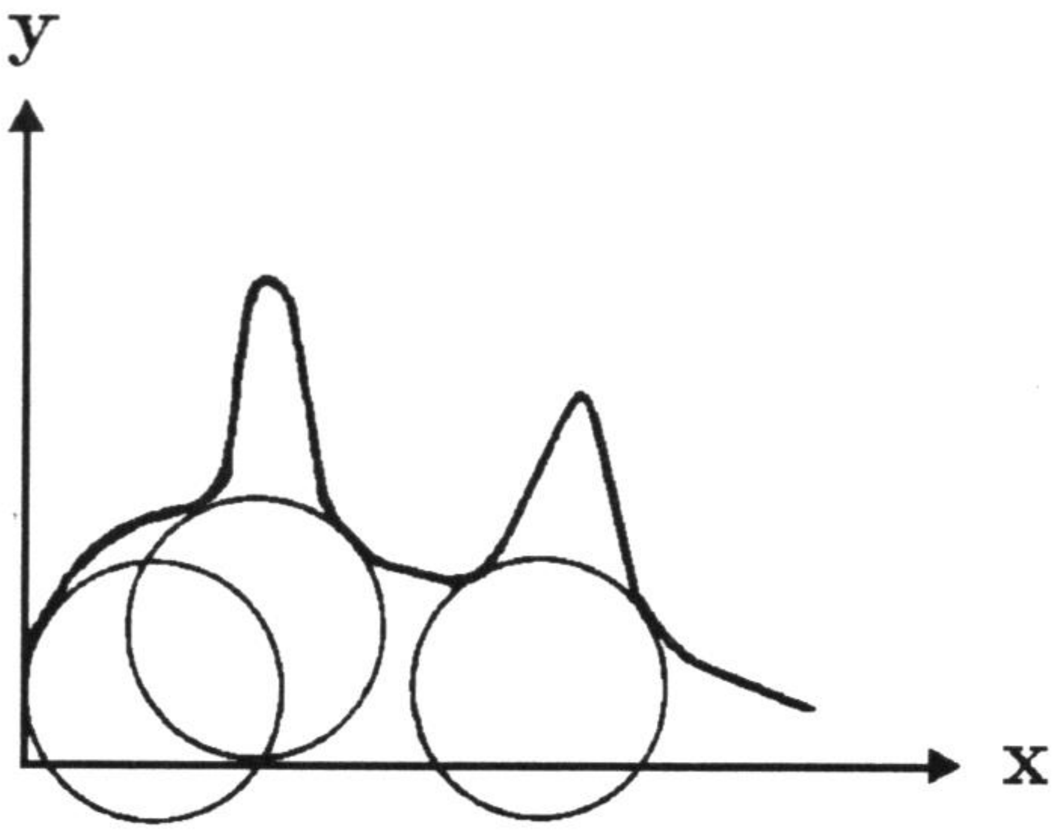
$$[f \ominus g^s](x) = \min_{z \in D, z-x \in D} \{f(z) - g(z-x)\}.$$

Greyscale opening and closing:

$$f_g(x) = [(f \ominus g^s) \oplus g](x) = [f(x) \ominus g(-x)] \oplus g(x),$$

$$f^g(x) = [(f \oplus g^s) \ominus g](x) = [f(x) \oplus g(-x)] \ominus g(x).$$

Greyscale morphology



Opening as a rolling ball transformation.

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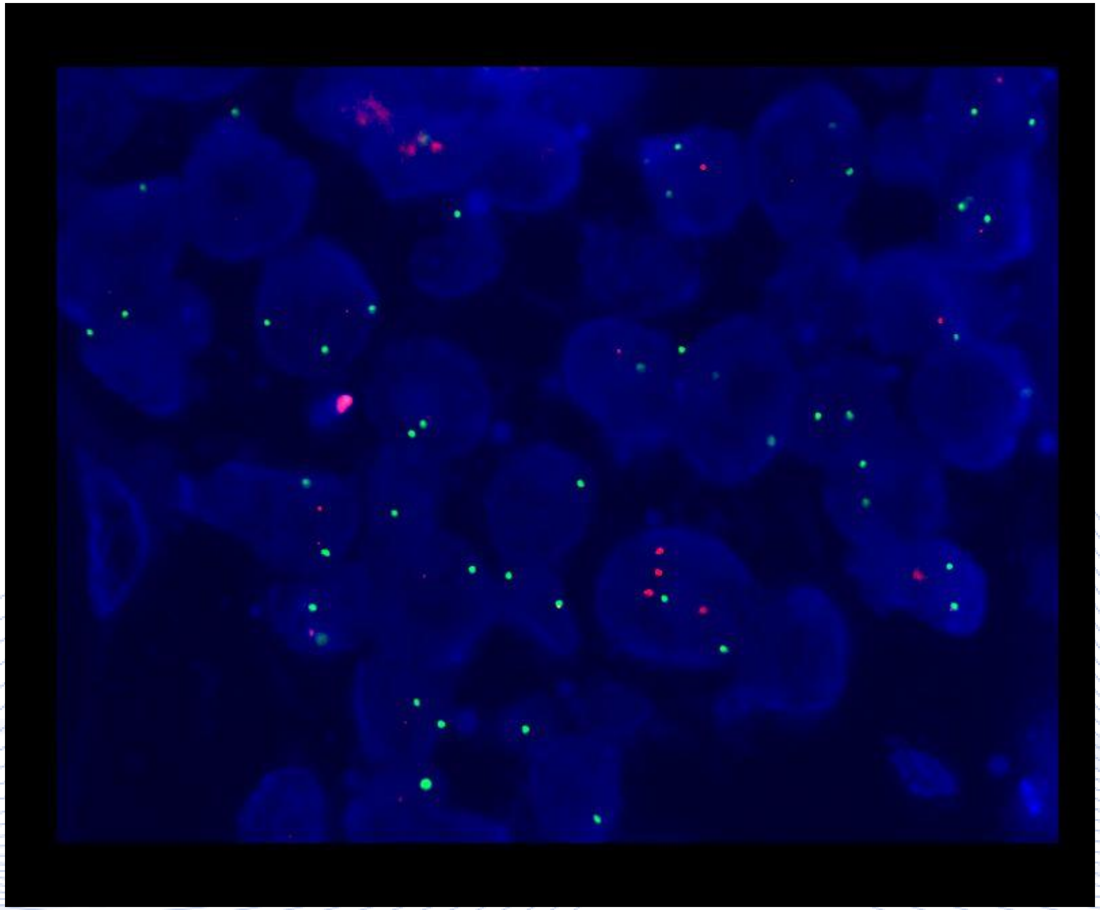
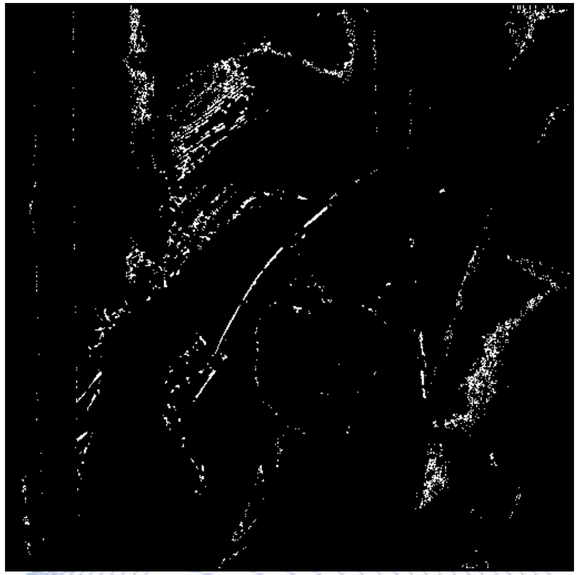
Greyscale morphology



Use of top-hat transform for identifying green and red spots in a FISH image.

Greyscale morphology

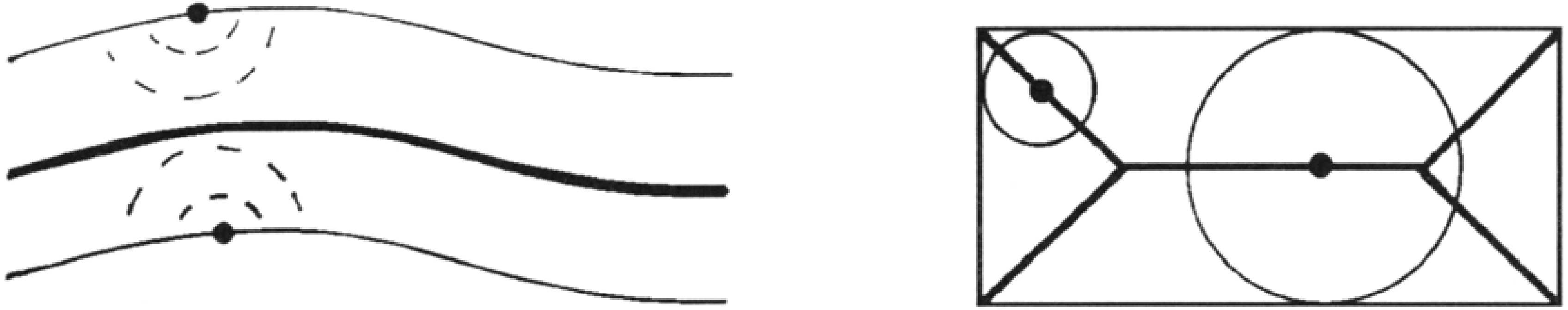


(a) (b)

a) Thresholded image; b) Result of top-hat filtering.

Skeletons

Object ***skeleton*** is an important topological descriptor of a two-dimensional binary object.



(a) (b)

a) Grassfire propagation model of medial axis; b) Maximal disk definition of skeleton.

Skeletons

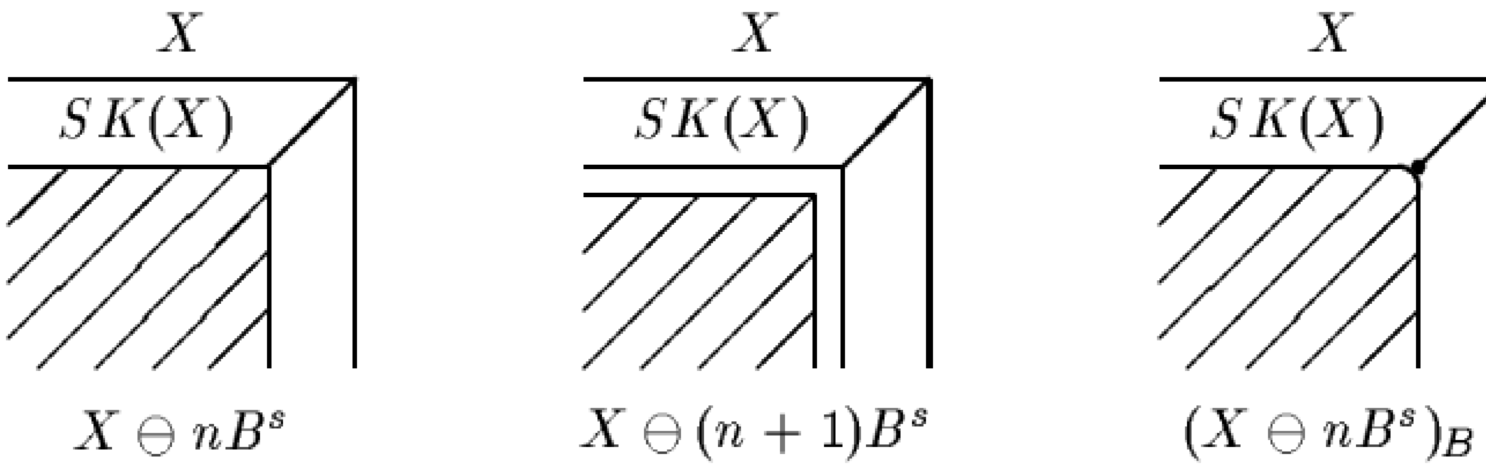
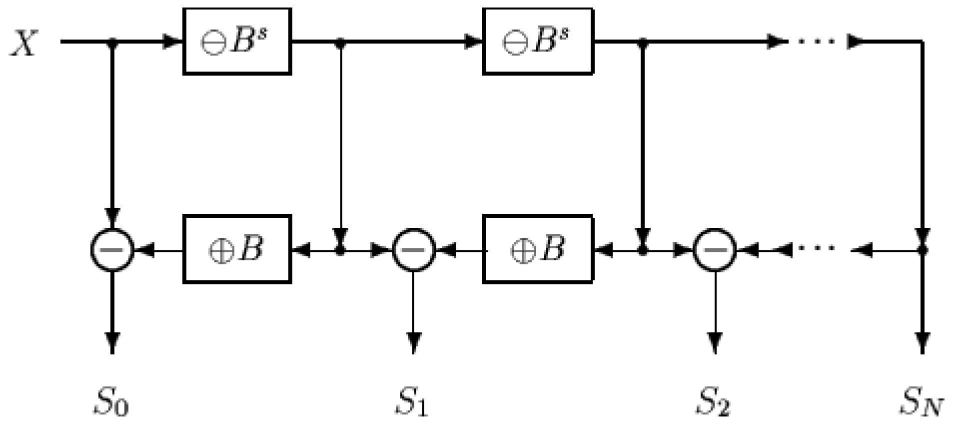
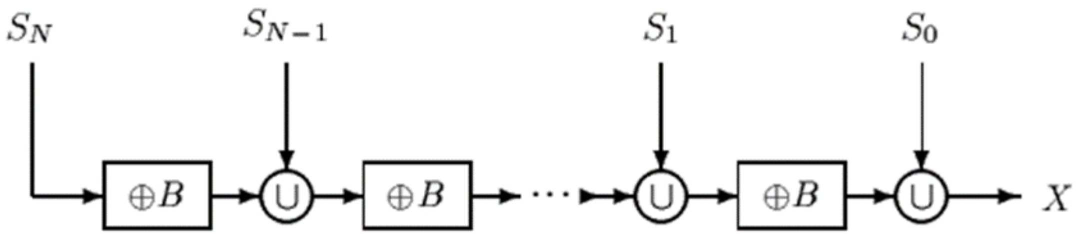


Illustration of morphological skeletonization.

Skeletons



(a)



(b)

a) Fast skeletonization algorithm;

b) Fast object reconstruction from skeleton subsets.

Shape decomposition

- A complex object X can be decomposed into a union of 'simple' subsets $X_1, \ldots, X_n$, thus providing an intuitive object description scheme called shape decomposition.

- Shape decomposition must use simple geometrical primitives in order to conform with our intuitive notion of simple shapes.

- The complexity of the decomposition must be small compared with the original description of X.

- A small noise sensitivity is desirable.

Shape decomposition

(a) (b)

a) Original binary image; b) First 16 components of its morphological shape decomposition.

Voronoi Tesselation

- A basic problem in computational geometry
- A tool in image analysis
- Input: an image containing a set of classified points (seeds or markers)
- Goal: to classify the rest of the image points to the nearest seed
- Dilation operation can be used to perform Voronoi tesselation

Voronoi Tesselation

- Basic idea: to find proximity properties of points in the Euclidean plane $\mathcal{R}^2$
- Let $X = \{x_1, x_2, \ldots, x_m\}$ be a set of m points in $\mathcal{R}^2$ and $d(.)$ a distance function between two points

$$V(i) = \{p \in \mathcal{R}^2 : d(p, x_i) \leq d\left(p, x_j\right), j \neq i\} \text{ (Voronio region)}$$

$$\mathrm{Vor}(X) = \bigcup_{i=1}^{m} V(i) \text{ (Voronoi diagram)}$$

Voronoi Tesselation

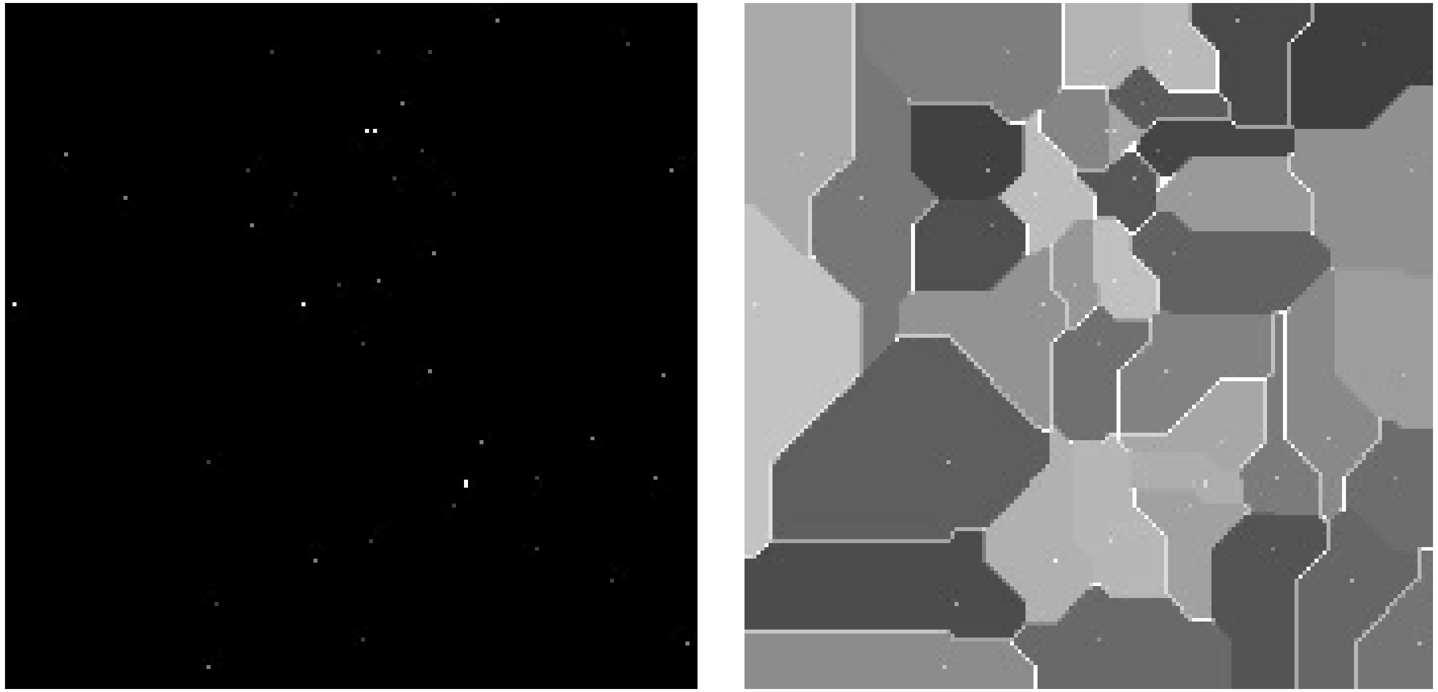


Image domain containing seed points and a corresponding Voronoi diagram using the city block distance

Watershed transform

- Treats the image as a relief and traces the flow of an inertialess liquid originating in a point $\mathbf{p}$, until it reaches a minimum of the relief
- Result: image segmentation into catchment basins, one for each regional minimum of the image
- Regional minimum: a connected set of points in a specific graylevel surrounded only by points of higher graylevels
- Graylevels: a series of binary images $f_i = \{\mathbf{p} \in D, f(\mathbf{p}) = i\}$ in which a discrete grayscale image f in domain $D \subset \mathcal{Z}^2$ can be decomposed

Q & A

Thank you very much for your attention!

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