Introduction to Signals and Systems

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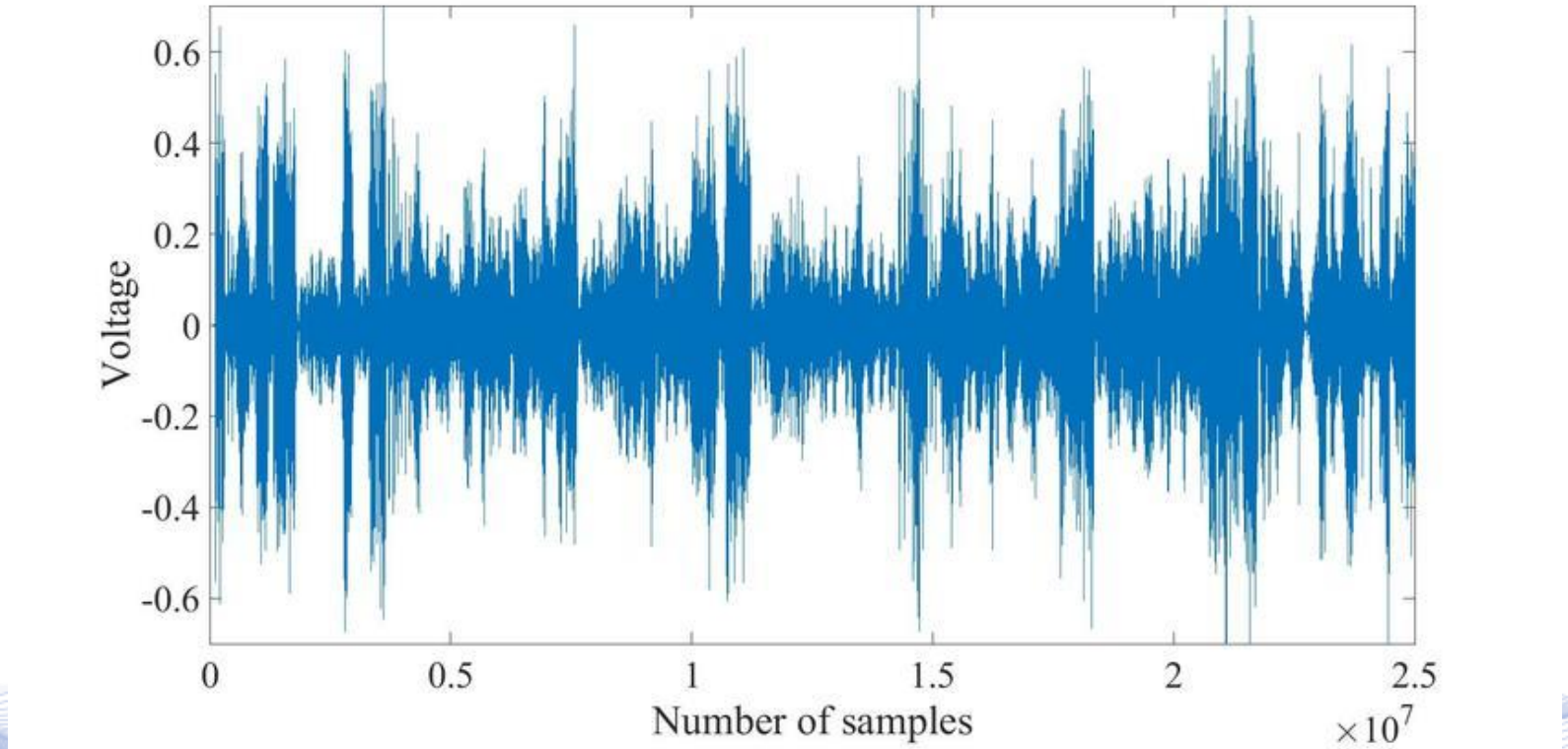
Artificial Intelligence & Information Analysis Lab

Signals and systems

- **1D signals** of the form: $x(t)$: $\mathbb{R} \rightarrow \mathbb{R}$
  - domain$\mathbb{R}$ and codomain $\mathbb{R}$.
- Audio signals, music signals,
- Biomedical signals: ECG. EEG, EMG
- 1D spatial signals of the form: $f(x)$: $\mathbb{R} \rightarrow \mathbb{R}$
  - Traffic load along a street.

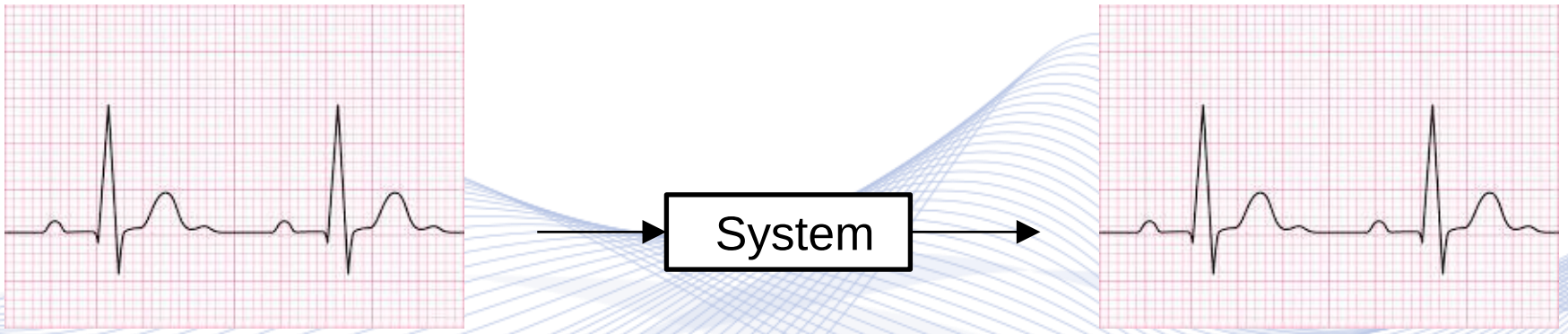
Signals and systems



Music and audio signals [ResearchGate].

Signals and Systems

- **1D signals** of the form: $x(t)\colon \mathbb{R} \to \mathbb{R}$, $y(t)\colon \mathbb{R} \to \mathbb{R}$
- input: signal $x(t)$ output: signal $y(t)$

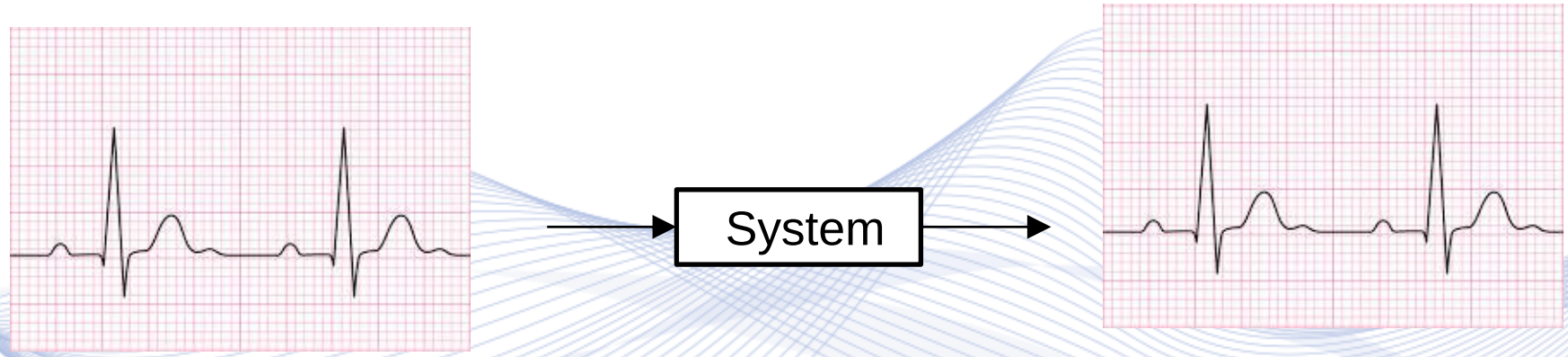


Definition of signal processing

- Digital signal processing system

input: signal

output: signal



Definition of signal analysis

- Input: signal

output: symbolic description

1D data types: 1D signals

- **1D signals**:
- temporal 1D signals of the form: $f(t): \mathbb{R} \rightarrow \mathbb{R}$, having:
  - domain$\mathbb{R}$ and codomain $\mathbb{R}$.
- Audio signals, music signals,
- Biomedical signals: ECG. EEG, EMG
- 1D spatial signals of the form: $f(x): \mathbb{R} \rightarrow \mathbb{R}$
  - Traffic load along a street.

1D data types: 1D signals

- ***Single variable functions*** are 1D signals:
  - Sine/cosine functions/signals $\mathbb{R} \to \mathbb{R}$:

$$f(t) = \cos(\Omega t) = \cos(2\pi F t)\,.$$

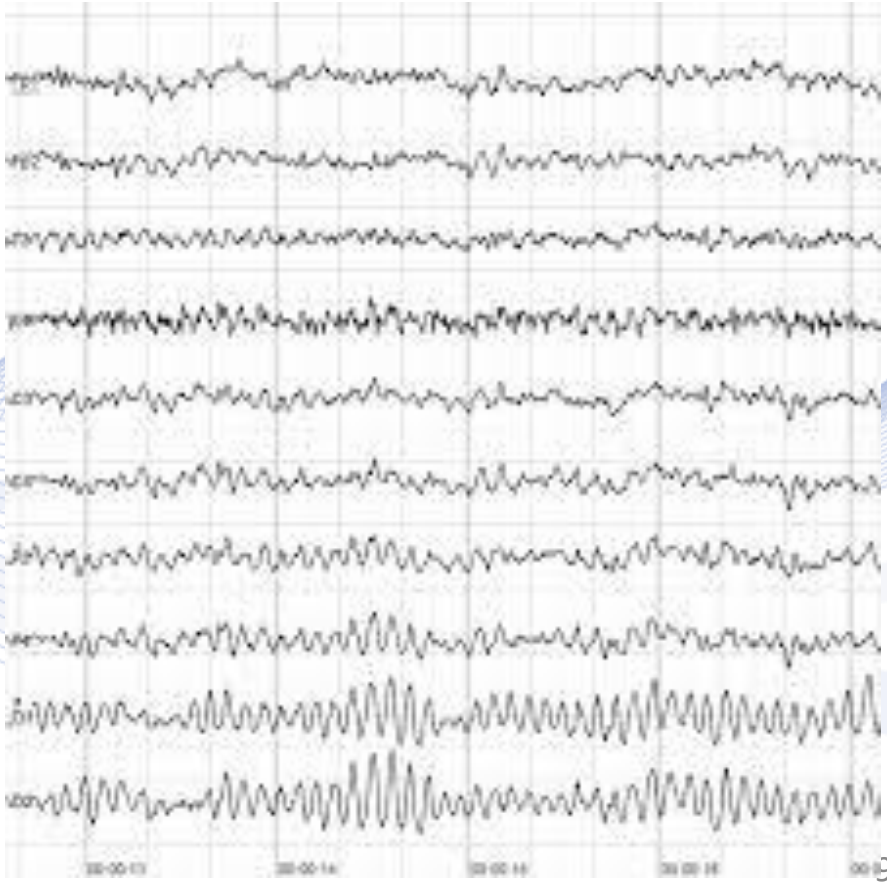
  - Complex-valued 1D signals $\mathbb{R} \to \mathbb{C}$:

$$f(t) = \exp(i\Omega t) = \cos(\Omega t) + i\sin(\Omega t)\,.$$

- ***Multichannel signals*** have the form: $\mathbf{f}(t)\colon \mathbb{R} \to \mathbb{R}^n$
  - Stereo music signal: $\mathbf{f}(t) = [f_l(t), f_r(t)]^T\colon\ \mathbb{R} \to \mathbb{R}^2$
  - 7.1 surround music sound: $\mathbf{f}(t) = [f_0(t), \ldots, f_7(t)]^T\colon\ \mathbb{R} \to \mathbb{R}^8$

1D data types: 1D signals

- Multichannel biomedical recordings $\mathbf{f}(t)\colon \mathbb{R} \to \mathbb{R}^n$.
  - ECG, EEG recordings.



Signal sampling/digitization

- Continuous 1D signal of the form: $x(t): \mathbb{R} \rightarrow \mathbb{R}$,
- ***Signal sampling/digitization*** transforms continuous coordinate signals to ***digital signals***:

$$x(n): \mathbb{Z} \rightarrow [0, \dots, 2^B - 1].$$

- Digital signals consist of signal ***samples***.
- A sample has B bits per pixel. If $B = 8$, each sample has 256 values.

Special signals

Computational linguistics

- ***Text*** as signal:

$$x(i): \quad \mathbb{Z} \rightarrow \{A, B, \ldots, Z\}$$

  - Co-domain: ***set*** of 26 Latin alphabet letters $A, B, \ldots, Z$ (and other characters).

Special signals

Bioinformatics

- ***DNA sequence*** as signal:

$$x(i): \quad \mathbb{Z} \rightarrow \{A, T, G, C\}$$

  - Co-domain: ***set*** of DNA bases A, T, G, C.
- ***Proteomic sequence*** as signal.

Graph signals

Graphs $\mathcal{G}(\mathcal{V}, \mathcal{E})$ consist of a vertex set $\mathcal{V}$ and an edge set $\mathcal{E}$.

- ***Graph signals***:

$$\mathbf{x}(v): \mathcal{V} \rightarrow \mathbb{R}^n.$$

- Number of Covid-19 cases on a city graph:

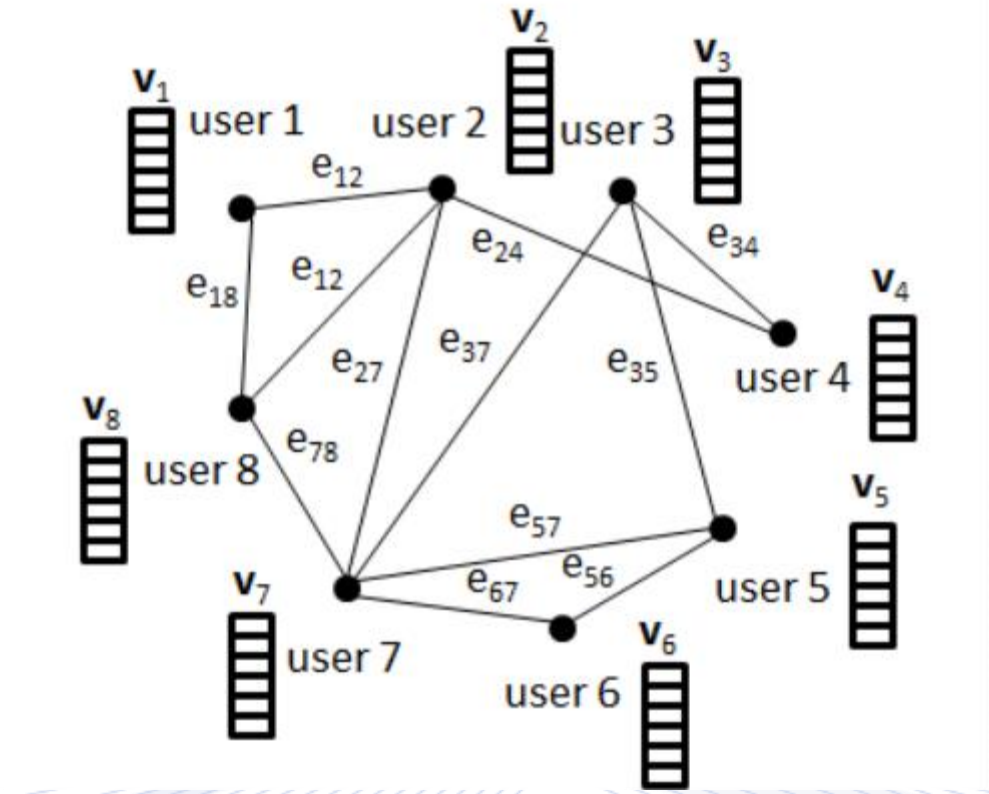
$$x(v): \mathcal{V} \rightarrow \mathbb{R}.$$

- Coordinates of body joints on a body skeleton graph:

$$\mathbf{x}(v): \mathcal{V} \rightarrow \mathbb{R}^3.$$

- Ideas spread on a friend graph (Facebook): $x(v): V \rightarrow \{I_1, \dots I_n\}$.

Graph signals



Social network vectorial graph signal.

2D data types: images

Spatial coordinates x, y.

2D data types: images

- ***Still images/pictures***: spatial 2D signals of the form $f(x, y): \mathbb{R}^2 \rightarrow \mathbb{R}$, having:
  - domain $\mathbb{R}^2$ and codomain $\mathbb{R}$.
  - two spatial coordinates x, y.
- ***Image sampling/digitization*** transforms continuous coordinates images to ***digital images***:

$$f(i, j): \mathbb{Z}^2 \rightarrow [0, \dots, 2^B - 1].$$

2D data types: images

a) Grayscale image b) binary image.

2D data types: images

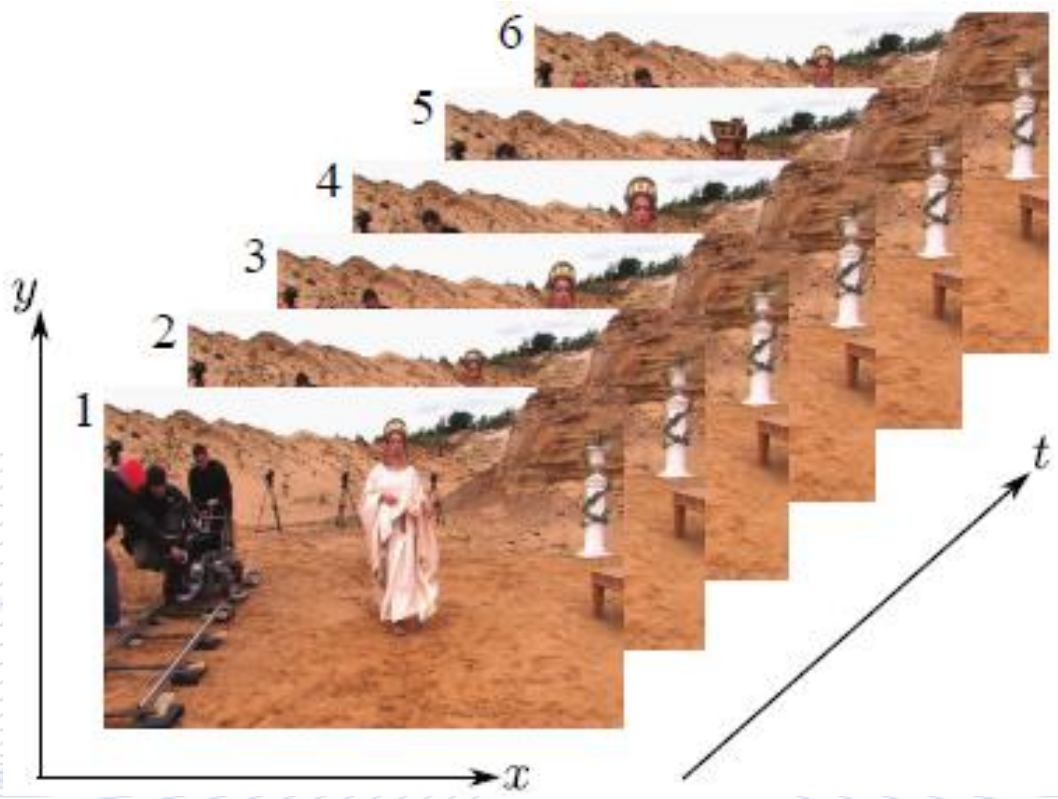
- Digital images consist of ***pixels*** (picture elements).
- A grayscale image has B bits per pixel. If $B = 8$, each pixel has 256 gray shades:
  - 0 black, 255 'white': ***Black and white images (BW)***.
- A binary image has 1 bit per pixel: black/white.
- ***Color digital images*** are multichannel (vectorial) images with three channels ***red (R), green (G), Blue (B)***:

$$\mathbf{f}(i,j) = [f_R(i,j), f_G(i,j), f_B(i,j)]^T: \qquad \mathbb{Z}^2 \to [0, \dots, 2^B - 1]^3.$$

2D data types: images

Color image.

3D data types: video



3D data types: video

- Moving images: spatiotemporal 3D signals of the form: $f(x, y, t)$: $\mathbb{R}^3 \rightarrow \mathbb{R}$, having:
  - domain $\mathbb{R}^3$ and codomain $\mathbb{R}$.
  - the time t coordinate has a different nature than the spatial coordinates x, y.
- *Video scanning*: the process for obtaining an 1D analog video signal, by sampling the time-varying images (luminance or RGB channels) along the vertical axis y and time t.

3D data types: video

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Image Processing overview

- Image processing
- Image analysis
- Image data types
- **Related disciplines**

Related disciplines

- Digital image processing (2D signals)
- Audio/music signal processing
- Machine Learning and Pattern Recognition
  - Application in signal analysis
- Language engineering
- Computational linguistics
- Signal communication
- Multimedia data bases
- Digital television

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Q & A

Thank you very much for your attention!

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http://icarus.csd.auth.gr/cvml-web-lecture-series/

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