Image Sampling summary

Prof. Ioannis Pitas
Aristotle University of Thessaloniki
pitas@csd.auth.gr
www.aiia.csd.auth.gr

Version 3.6.1

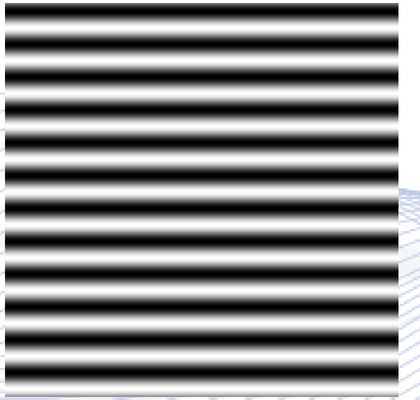
Artificial Intelligence & Information Analysis Lab

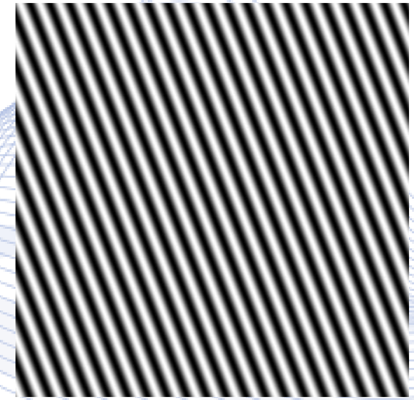
Image Sampling

- Spatial image frequencies
- 2D signal digitization
- 2D discrete signals

Spatial Image frequencies

- A (temporal) ***frequency*** F is linked to ***angular frequency*** $\Omega = 2\pi F = 2\pi/T$.
- Spatial frequencies (video content changes along x, y axes): $\Omega_x = 2\pi F_x$ and $\Omega_y = 2\pi F_y$.





2D sinusoidal signals: a) $(F_x, F_y) = (0,6)$; b) $(F_x, F_y) = (10,4)$.

Spatial Image frequencies

Spatial frequencies F_x, F_y:

- They show spatial luminance changes on the image plane.
- Local frequency vector $\boldsymbol{\Omega} = [\Omega_x, \Omega_y]^T$ is colinear to local image content change (edge) direction.
- Spatial frequencies can be defined along different orthogonal axes than (x, y).
- They are measured in cycles per unit length:
  - e.g., a 2D sinusoidal spatial pattern $f(x, y) = \sin(20\pi y)$ has a frequency $(0,10)$.

Spatial Image frequencies

- Any image can be decomposed in many exponential components on the Fourier Transform domain (Ω_x, Ω_y):

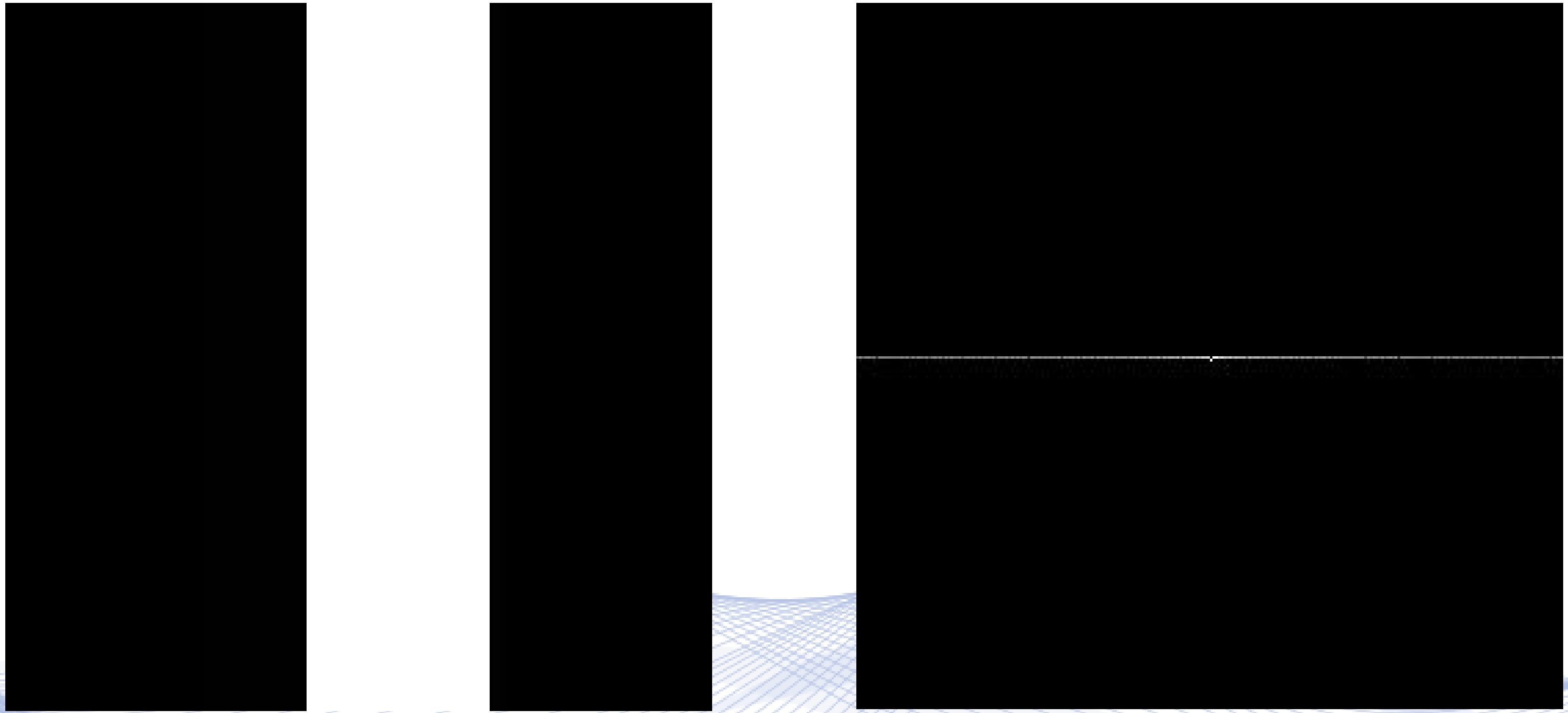
$$F_a(\Omega_x, \Omega_y) = \iint_{-\infty}^{\infty} f_a(x, y) e^{-i(\Omega_x x + \Omega_y y)} dx dy.$$

- Polar representation of 2D spatial frequencies:

$$F_s = \sqrt{F_x^2 + F_y^2}.$$

$$\theta = \arctan\left(\frac{F_y}{F_x}\right).$$

Spatial Image frequencies

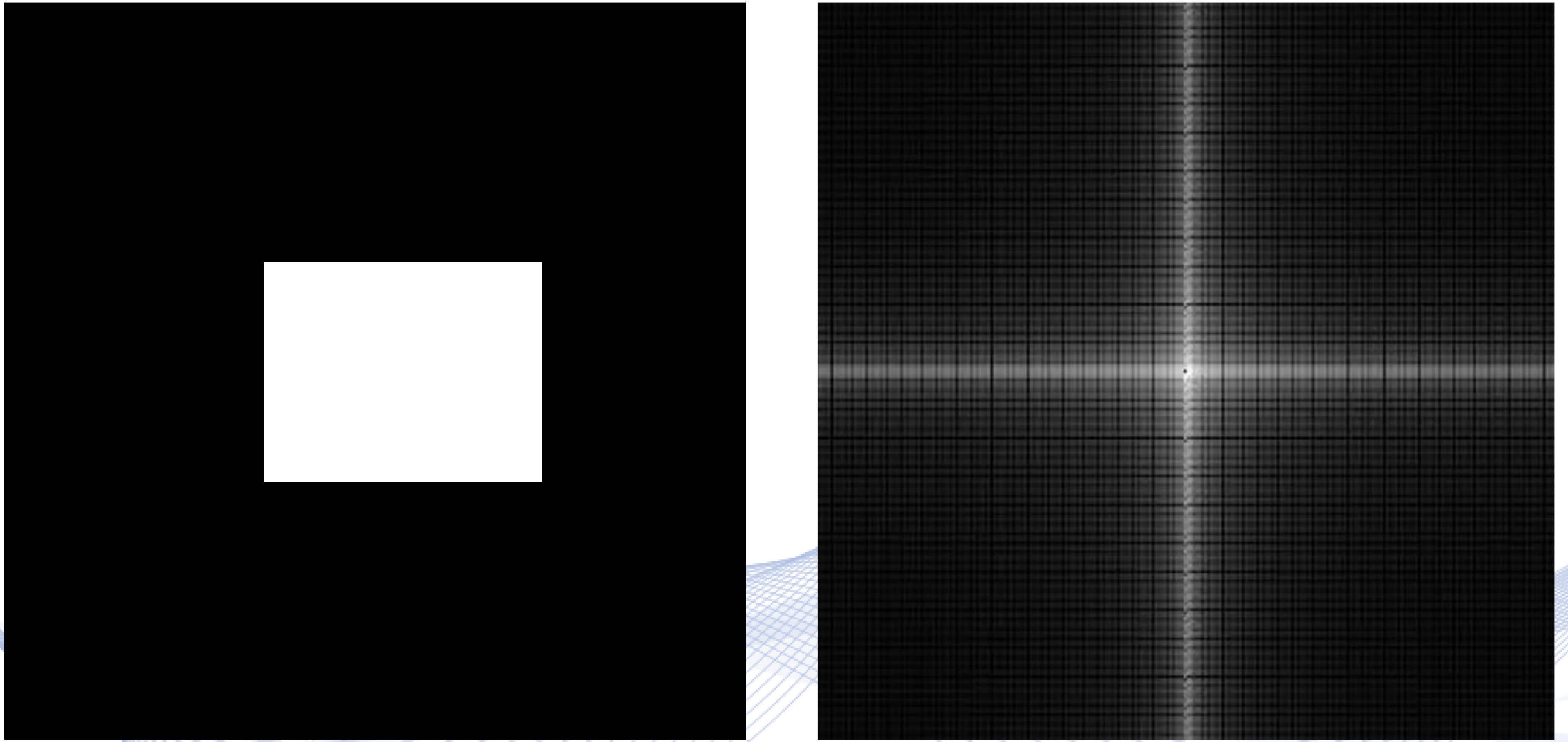


Example of an image spectrum.

Spatial Image frequencies

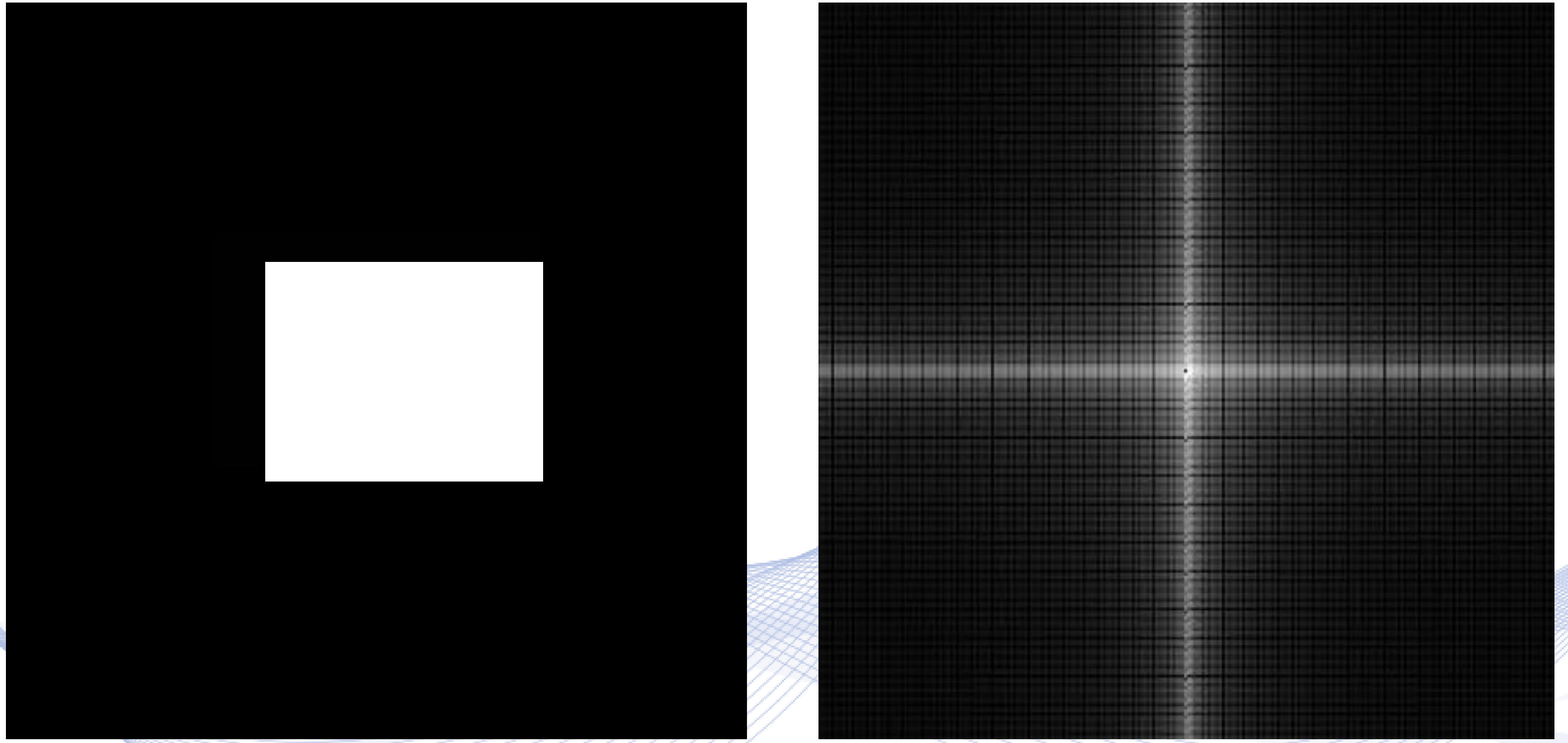
Example of an image spectrum.

Spatial Image frequencies



Example of an image spectrum.

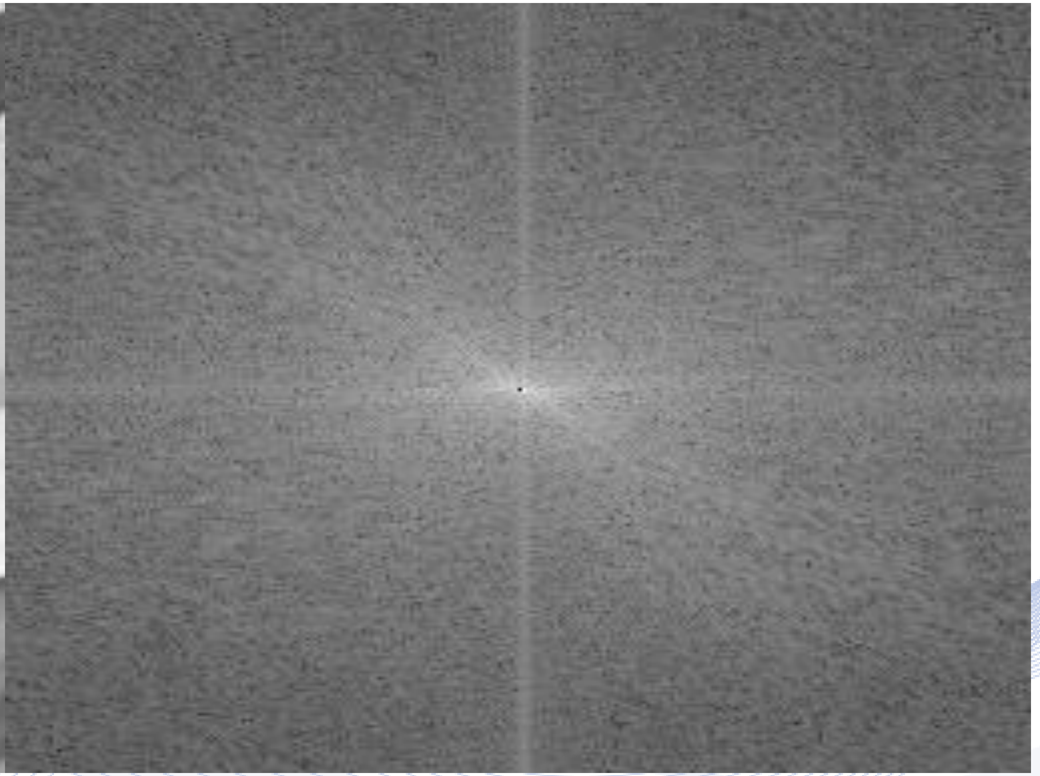
Spatial Image frequencies



Example of an image spectrum.

Spatial Image frequencies

a) Test image LENNA;



b) Periodogram of LENNA.

Spatial Image frequencies


a) Potatoes image.


b) Peas image.

1D Signal Sampling

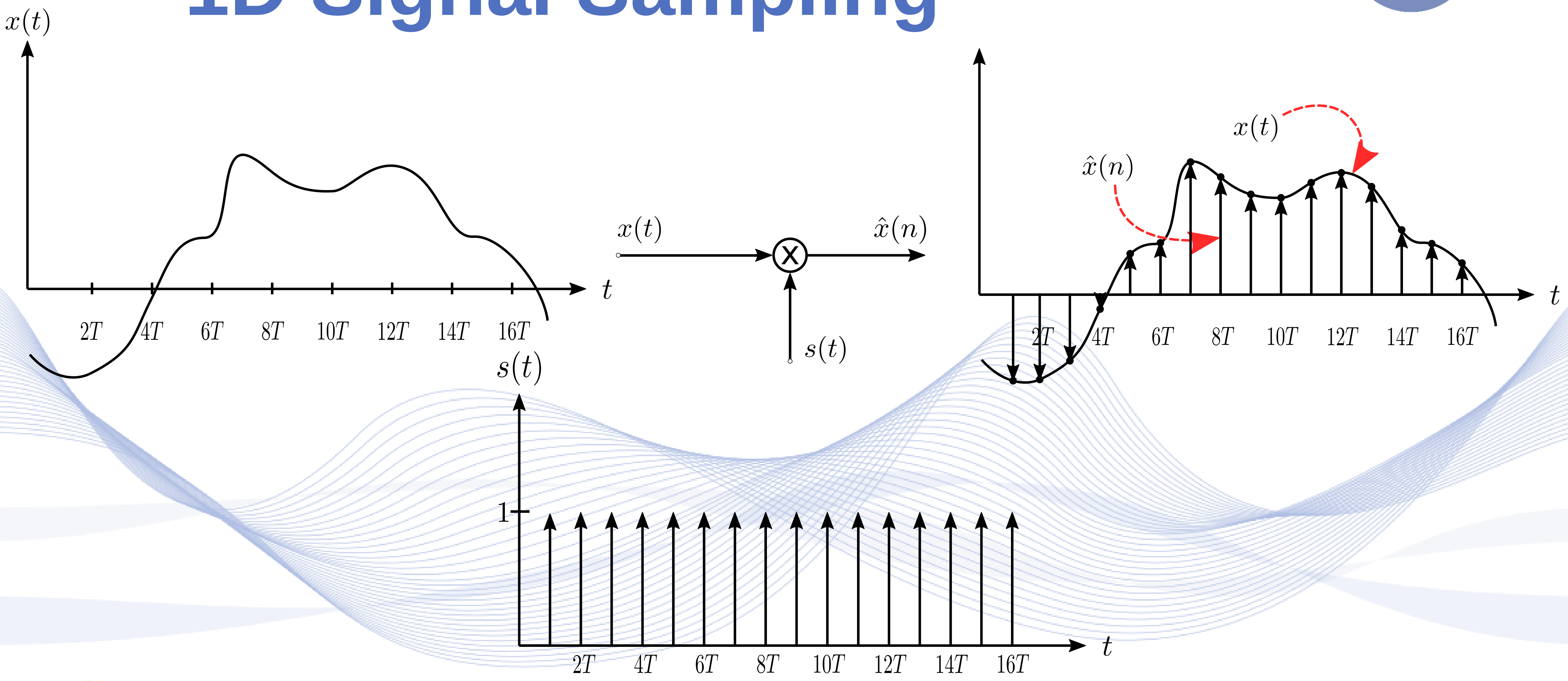
The output of a 1D signal sampling system is the discrete-time function $\hat{x}(t)$:

$$\hat{x}(t) = x(t)s(t) = \sum_{n=-\infty}^{\infty} x(t)\delta(t-nT) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t-nT).$$

Equivalently, a discrete-time sampled signal is a function $\mathbb{Z} \rightarrow \mathbb{R}$:

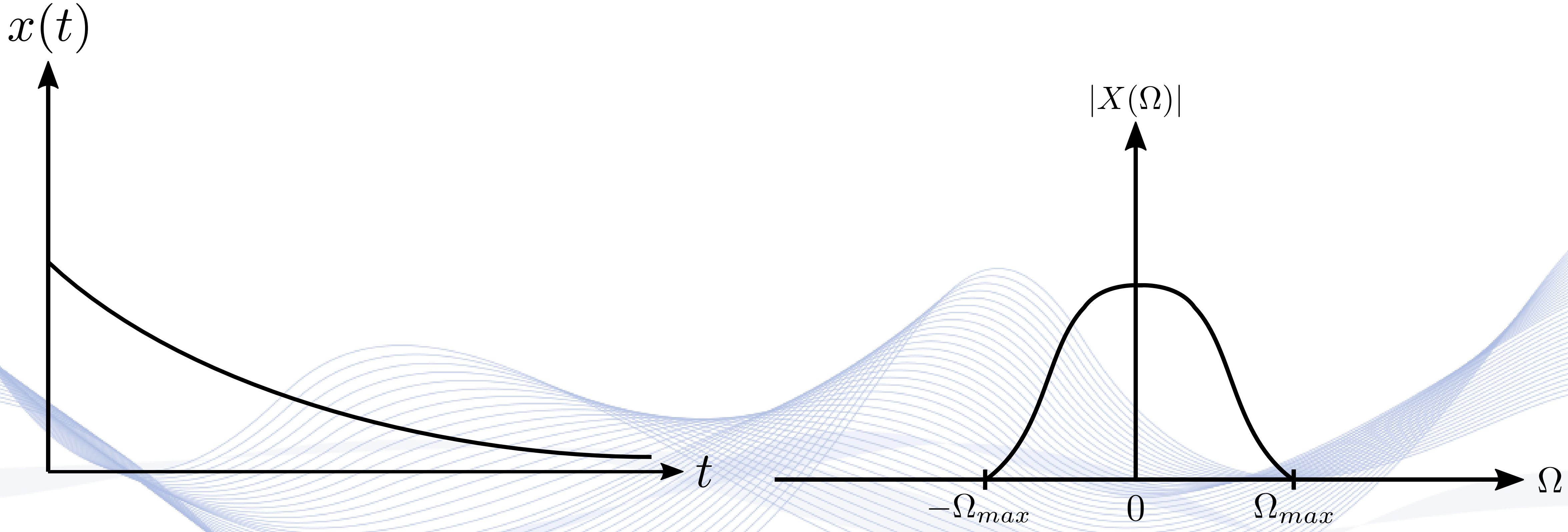
$$\hat{x}(n) = x(nT), \qquad n \in \mathbb{Z}.$$

1D Signal Sampling



Signal sampling.

1D Signal Sampling



Spectrum $X(\Omega)$ of signal $x(t)$.

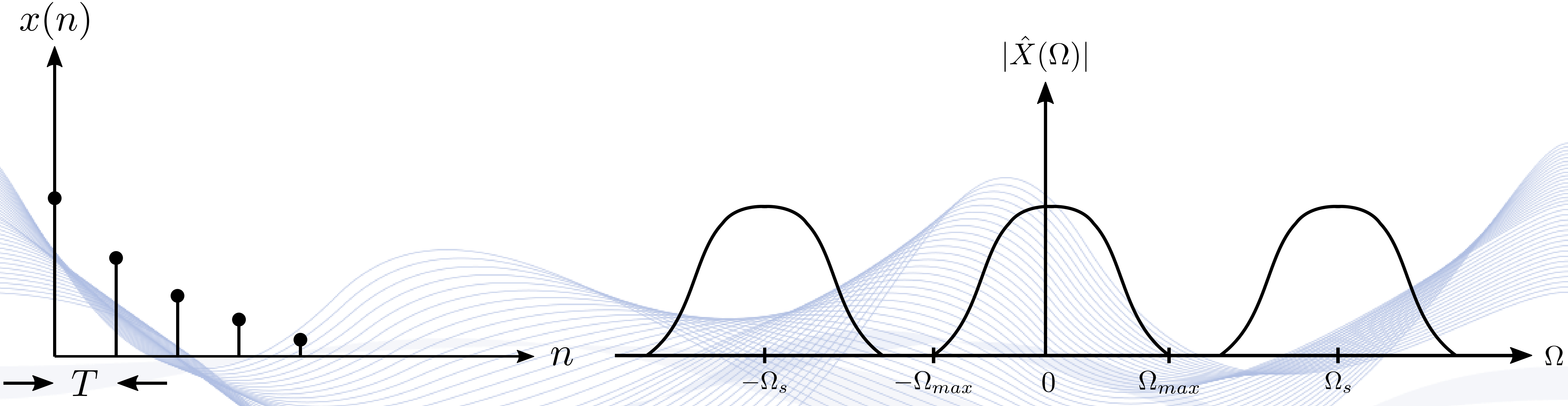
1D Signal Sampling

Shannon Theorem

- If the sampling frequency $\Omega_s \geq 2\Omega_{max}$ is at least double that the highest frequency Ω_{max} of $X(\Omega)$, then there is no overlapping on the spectrum of $\hat{X}(\Omega)$.

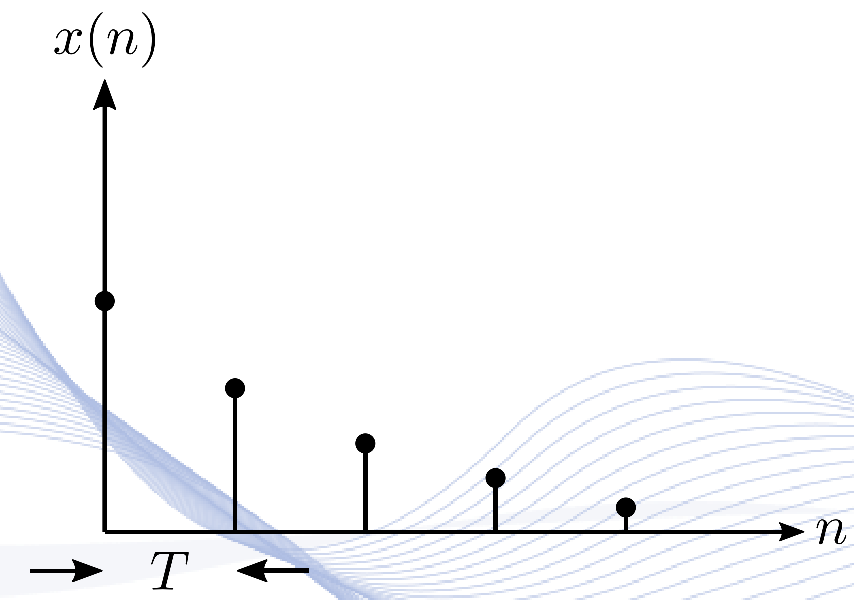
- If the sampling frequency $\Omega_s < 2\Omega_{max}$, then the expected overlapping on the spectrum of $\hat{X}(\Omega)$ will cause ***aliasing errors***.

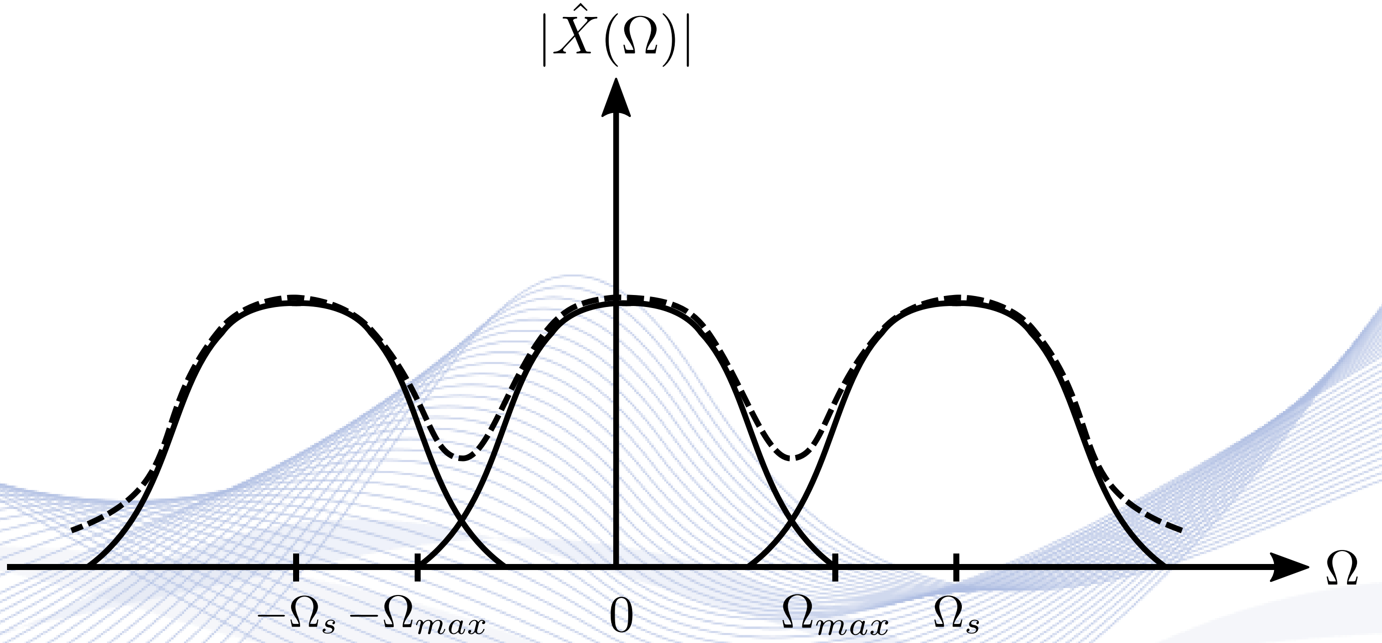
1D Signal Sampling



Sampled signal spectrum for $\Omega_s \geq 2\Omega_{max}$.

1D Signal Sampling





Sampled signal spectrum for $\Omega_s < 2\Omega_{max}$.

2D signal sampling

- Images are two-dimensional (2D) signals $x_a(t_1, t_2)$.

- t_1, t_2 are the horizontal/vertical coordinates of the image plane.

- Analog images have to be:

a) ***sampled*** (to become discrete 2D signals) and

b) ***digitized*** (pixels must become numbers)

to be digitally processed.

2D signal sampling

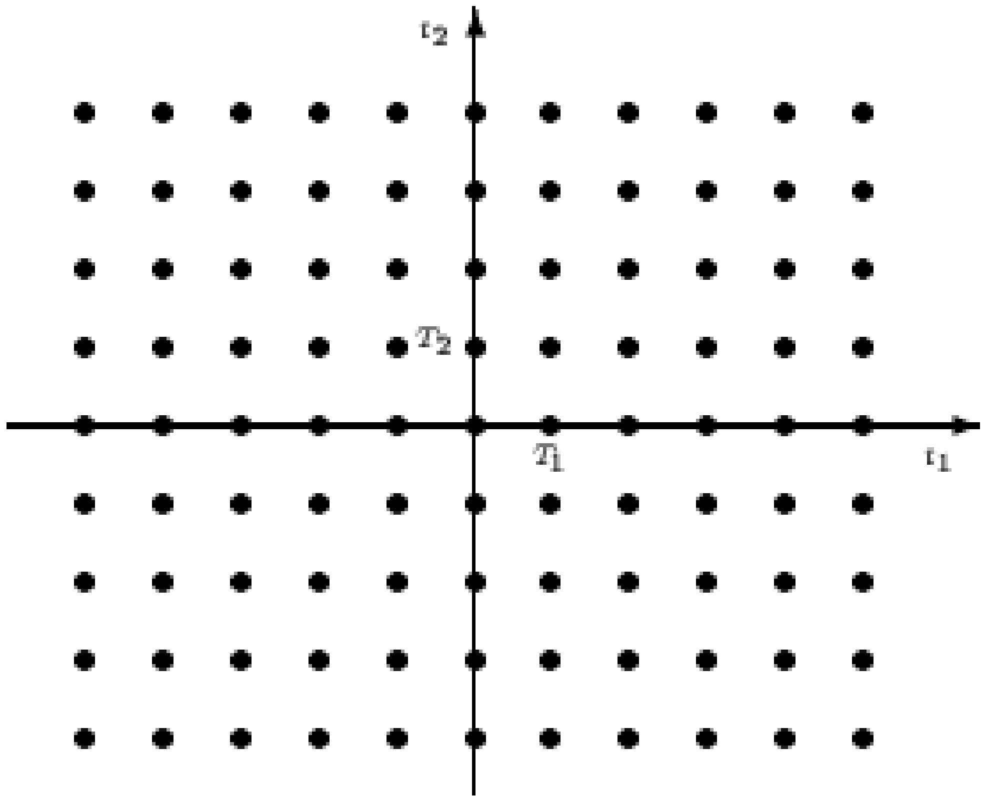
- Uniform 2D signal sampling along t_1, t_2:

$$x(n_1, n_2) = x_a(n_1 T_1, n_2 T_2).$$

- T_1, T_2 : sampling intervals.

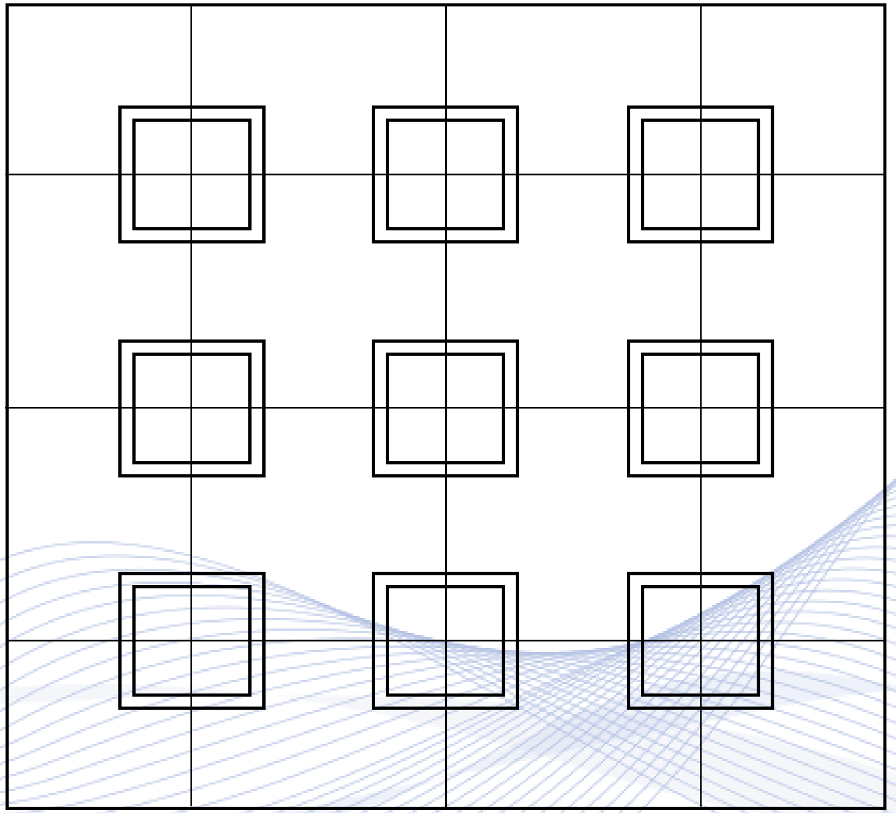
- Sampling issues:
  - Relation between the Fourier spectra of the analog and discrete (sampled) 2D signals (images).
  - Reconstruction of the analog image from a discrete one.

2D signal sampling



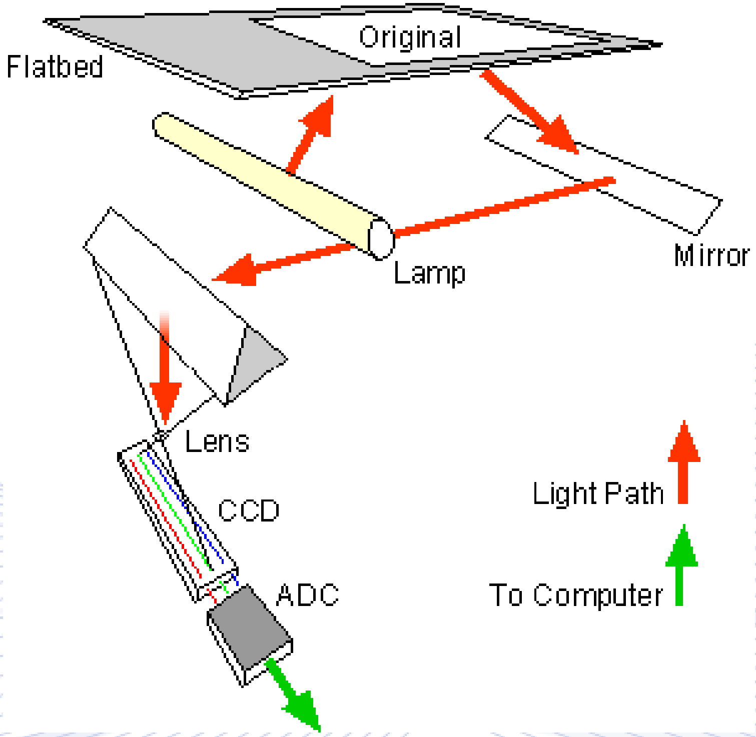
Rectangular image sampling grid.

2D signal sampling



CCD cell grid.

2D signal sampling



Document scanner [CIR].

2D signal sampling

Derivation of $x(n_1, n_2)$ by sampling:

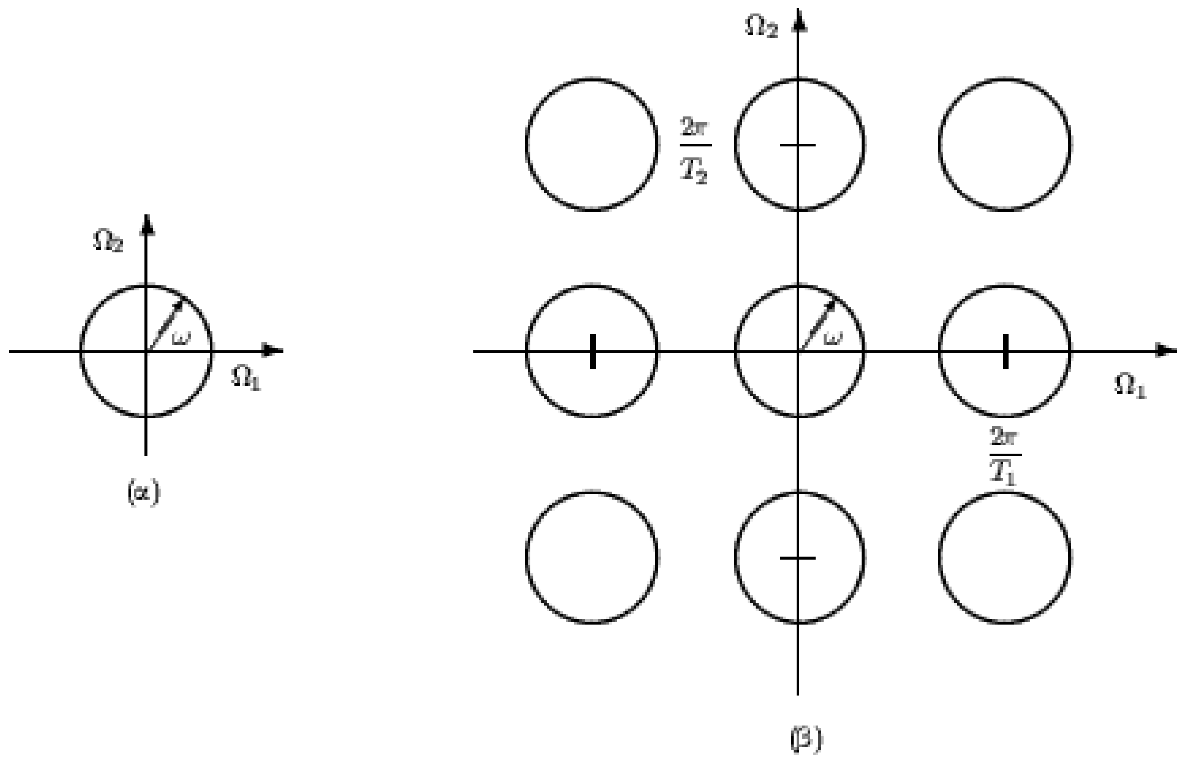
$$x(n_1, n_2) = x_a(n_1 T_1, n_2 T_2).$$

Continuous 2D Fourier transform:

$$X_a(\Omega_1, \Omega_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_a(t_1, t_2)\exp(-i\Omega_1 t_1 - i\Omega_2 t_2)\,dt_1 dt_2\,,$$

$$x_a(t_1, t_2) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} X_a(\Omega_1, \Omega_2)\exp(i\Omega_1 t_1 + i\Omega_2 t_2)\,d\Omega_1 d\Omega_2\,.$$

2D signal sampling



Fourier Transform of a) analog; b) discrete image.

2D signal sampling

2D version of ***Shannon sampling theorem***:

If the sampling frequencies:

$$\Omega_{s_1} = \frac{2\pi}{T_1}, \qquad \Omega_{s_2} = \frac{2\pi}{T_2}$$

satisfy:

$$\Omega_{s_1} \geq 2\,\Omega_{1max}, \qquad \Omega_{s_2} \geq 2\,\Omega_{2max}.$$

periodic spectrum repetition does not result in spectrum overlaps.

2D signal sampling

- If sampling frequencies are less than ***Nyquist frequencies***, spectral ***aliasing*** results.

- Aliasing results in the destruction of image high frequencies:
  - Image details are destroyed during sampling.

2D signal sampling

Image with high and low sampling rate.

Artificial Intelligence & Information Analysis Lab

2D signal reconstruction

If the sampling intervals T_1, T_2 are small enough, so that the spectrum $X_a(\Omega_1, \Omega_2)$ satisfies the relation:

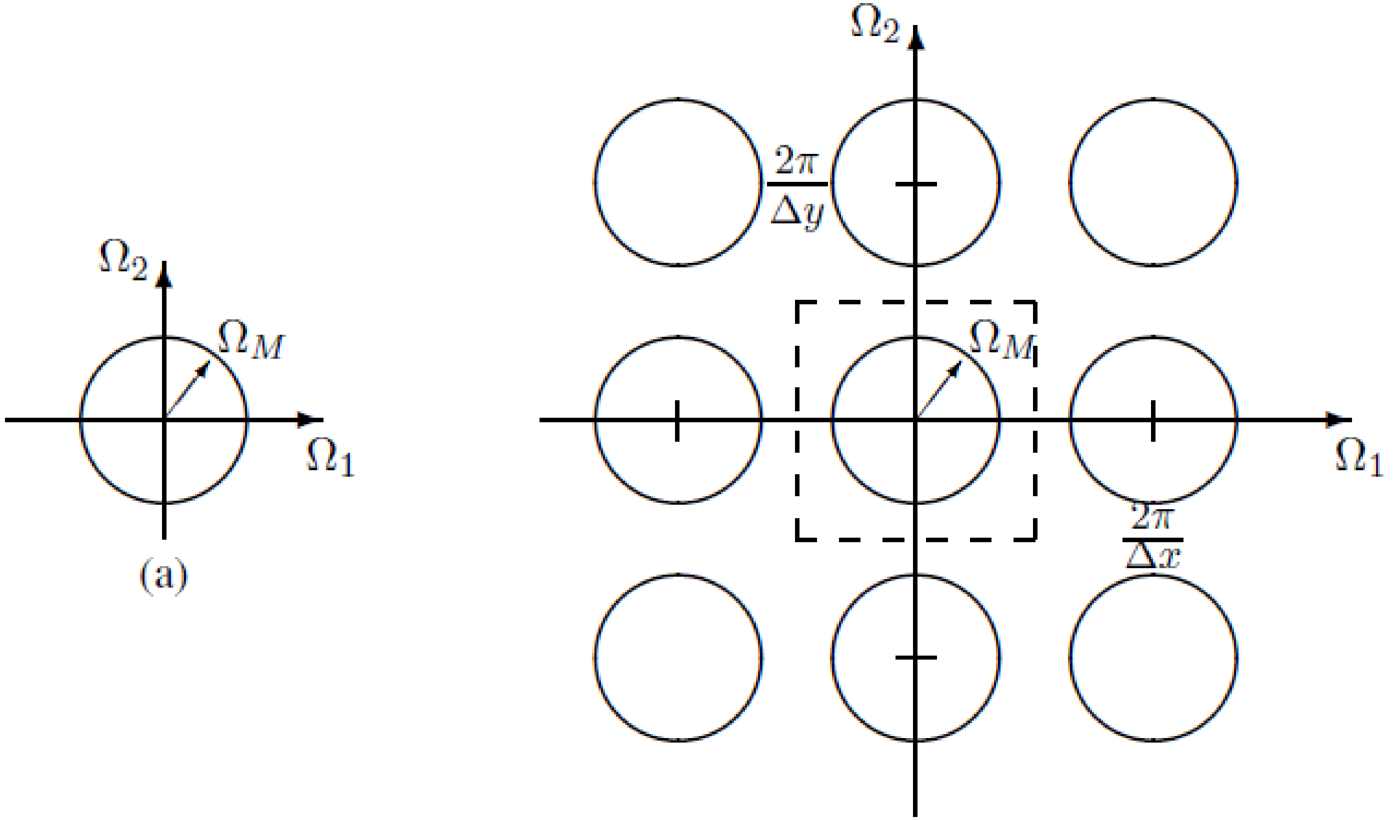
$$X_a(\Omega_1, \Omega_2) = 0, \qquad |\Omega_1| \geq \frac{\pi}{T_1}, \qquad |\Omega_2| \geq \frac{\pi}{T_2},$$

the discrete 2D image spectrum is given by:

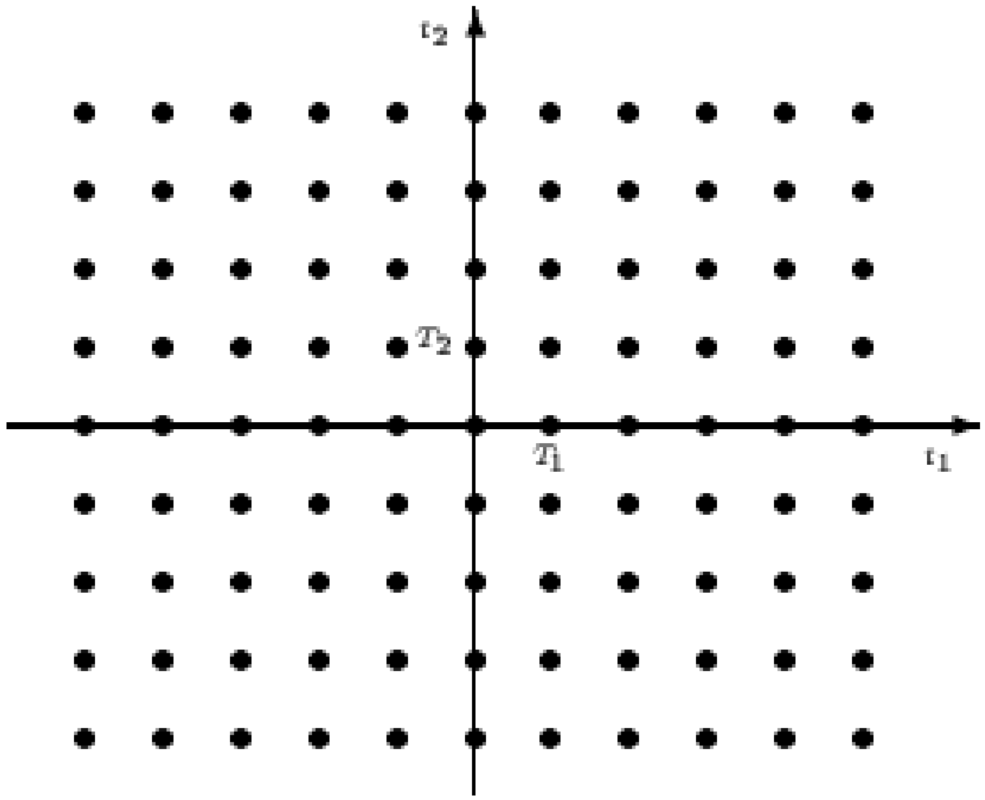
$$X(\Omega_1 T_1, \Omega_2 T_2) = \frac{1}{T_1 T_2} X_a(\Omega_1, \Omega_2), \qquad |\Omega_1| \leq \frac{\pi}{T_1}, \qquad |\Omega_2| \leq \frac{\pi}{T_2}.$$

2D signal reconstruction

Therefore, a square low-pass filter $H(\omega_1, \omega_2)$ can be used to recover $X_a(\Omega_1, \Omega_2)$ from the discrete 2D image spectrum $X(\omega_1, \omega_2)$ by retaining only its basic period.



2D signal sampling



Rectangular lattice (sampling grid).

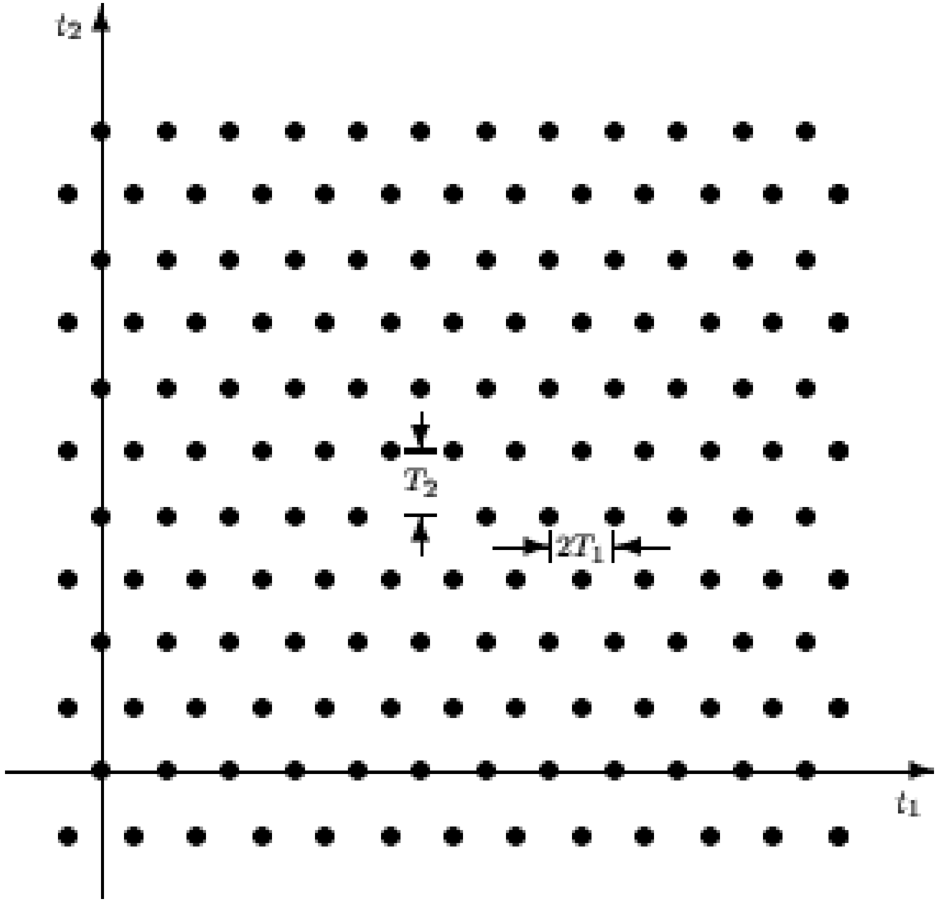
2D signal sampling

Hexagonal lattice:

- special 2D sampling lattice.
- All lattice vertices have equal distance from each other.
  - Uniform sampling of 2D curves on a hexagonal grid.
- Sampling matrix $\mathbf{V}$:

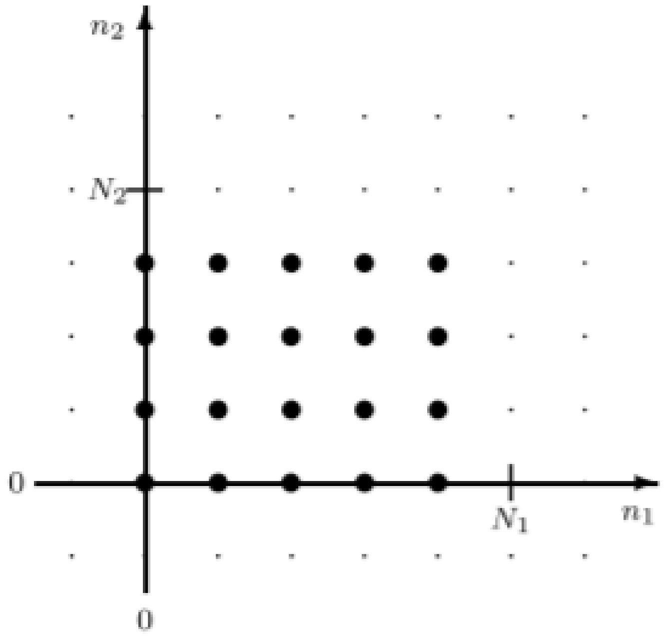
$$\mathbf{V} = \begin{bmatrix} T_1 & T_1 \\ T_2 & -T_2 \end{bmatrix}.$$

2D signal sampling



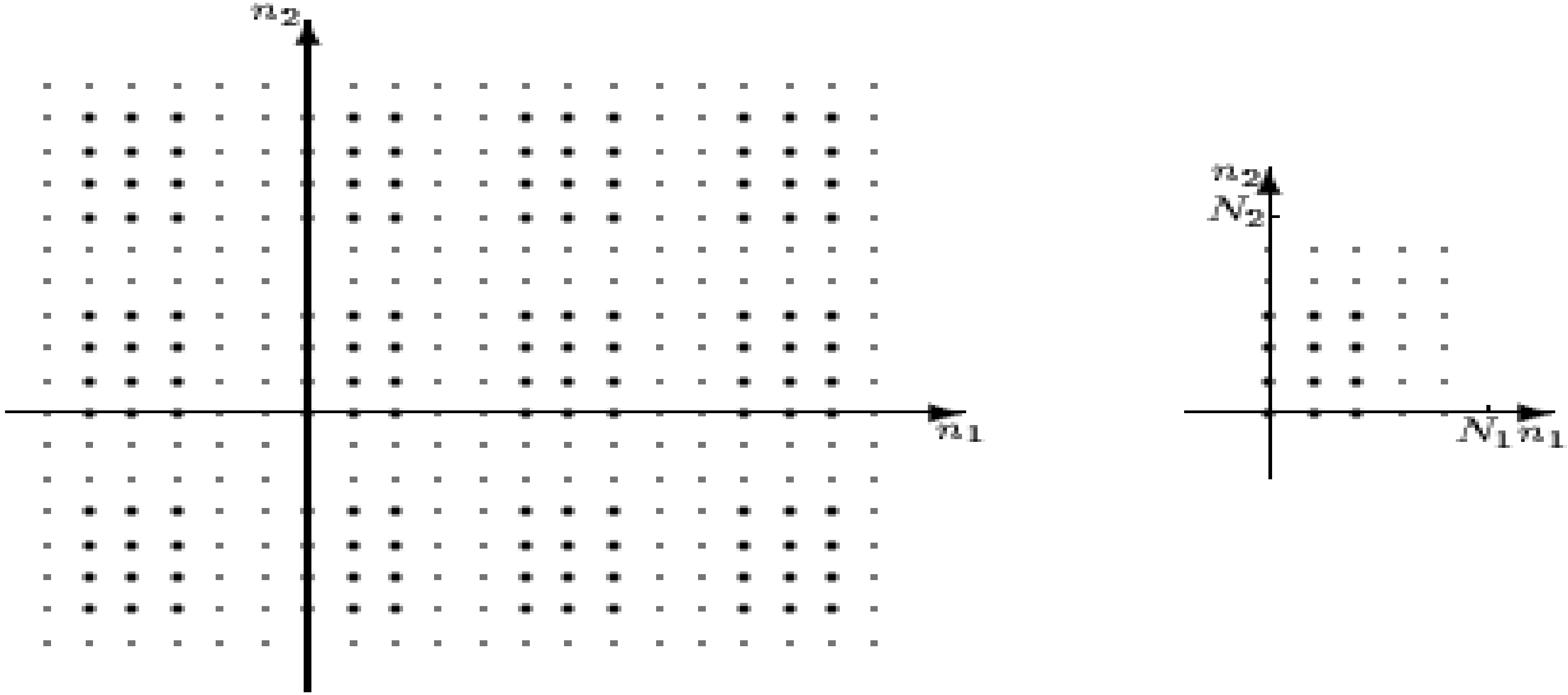
Hexagonal lattice.

2D Discrete Signals



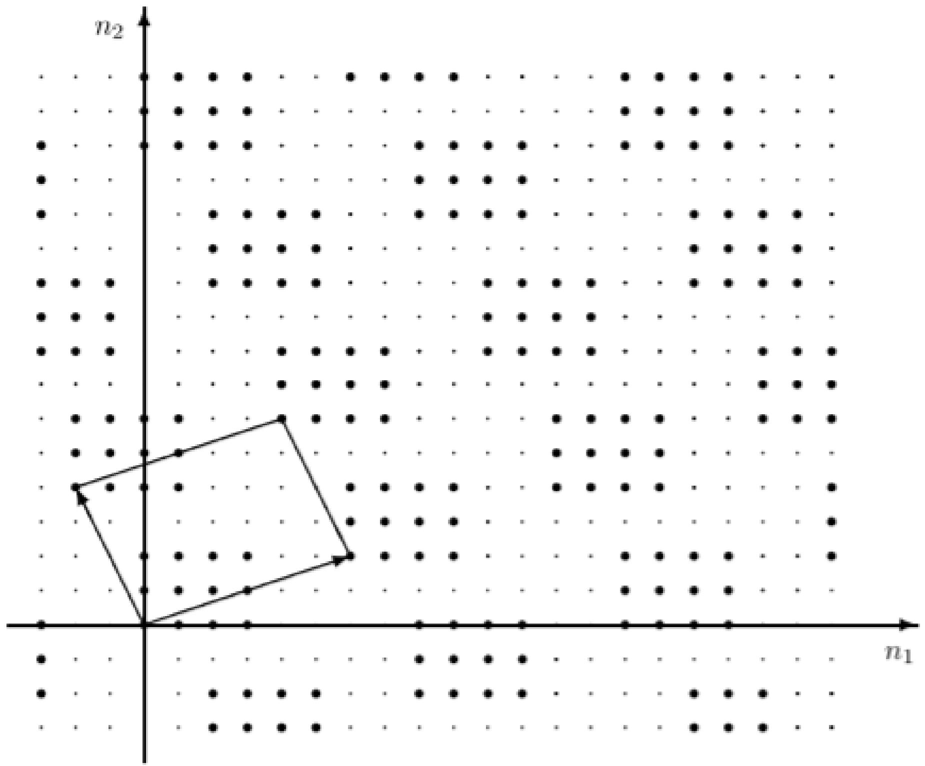
Support region of a 2D finite signal.

2D Discrete Signals



Rectangularly periodic sequence and its fundamental period $N_1 = 5, N_2 = 6$.

2D Discrete Signals



2D periodic sequence with periodicity vectors $\mathbf{N}_1 = [6,2]^T, \mathbf{N}_2 = [-2,5]^T$.

Bibliography

[PIT2021] I. Pitas, “Computer vision”, Createspace/Amazon, in press.
[PIT2017] I. Pitas, “Digital video processing and analysis ” , China Machine Press, 2017 (in Chinese).
[PIT2013] I. Pitas, “Digital Video and Television ” , Createspace/Amazon, 2013.
[PIT2000] I. Pitas, Digital Image Processing Algorithms and Applications, J. Wiley, 2000.
[NIK2000] N. Nikolaidis and I. Pitas, 3D Image Processing Algorithms, J. Wiley, 2000.
[CIR] https://www.circuitstoday.com/working-of-scanner

Q & A

Thank you very much for your attention!

More material in
http://icarus.csd.auth.gr/cvml-web-lecture-series/

Contact: Prof. I. Pitas
pitas@csd.auth.gr