Fourier Transform summary

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Version 3.5

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Fourier Transform

- **Periodic functions**
- Fourier Transform
- Fourier Transform Properties
- Fourier Transform and LTI systems

Periodic signals

A continuous-time signal $x(t)$ is ***periodic***, when there is a positive non-zero value T for which:

$$x(t + T) = x(t), \qquad \text{for all } t.$$

- T is referred to as the period of the signal.

- ***Frequency***: $F = \frac{1}{T}$. It is measured in Hertz (Hz).
- ***Angular frequency***: $\Omega = 2\pi F = \frac{2\pi}{T}$.

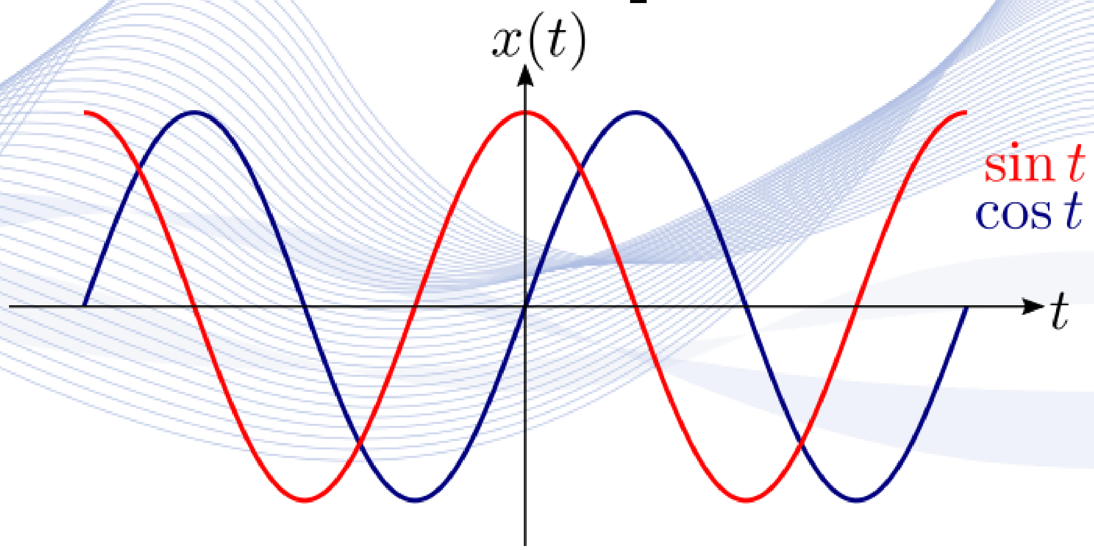
Periodic signals

Trigonometric signals (sine, cosine):

$$x(t) = A\cos(\Omega t + \varphi) = \frac{A}{2}\left[e^{i(\Omega t+\varphi)} + e^{-i(\Omega t+\varphi)}\right].$$

$$x(t) = A\sin(\Omega t + \varphi) = \frac{A}{2i}\left[e^{i(\Omega t+\varphi)} - e^{-i(\Omega t+\varphi)}\right].$$

- A: amplitude,
- Ω: angular frequency, φ: phase
- The period of the sinusoid is $T = \frac{2\pi}{\Omega}$.



Periodic signals

Complex exponential signal:

$$x(t) = \mathrm{e}^{st} = e^{(\sigma+i\Omega)t} = e^{\sigma t}\,(\cos\Omega t + i\sin\Omega t).$$

Special case:

$$x(t) = Ae^{i\Omega t} = A\cos\Omega t + i\,A\sin\Omega t.$$

Fourier Transform

- Periodic functions
- **Fourier Transform**
- Fourier Transform Properties
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Fourier Transform

Fourier transform (***FT***) of signal $x(t)$ is a function transformation $X(\Omega)$:

$$X(\Omega) \triangleq \mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t)e^{-i\Omega t}dt\,.$$

Inverse Fourier transform of $X(\Omega)$ is defined as:

$$x(t) \triangleq \mathcal{F}^{-1}\{X(\Omega)\} = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\Omega)e^{i\Omega t}d\Omega\,.$$

Fourier Transform

- Fourier transform pair is denoted by: $x(t) \leftrightarrow X(\Omega)$.

- Fourier transform essentially decomposes an 1D signal $x(t)$ into a sum of periodic complex exponential functions, resulting in signal ***spectrum*** (frequency content) $X(\Omega)$ at frequency Ω.

Fourier Transform

The Fourier transform $X(\Omega)$ is, in general, a complex one:

$$X(\Omega) = R(\Omega) + iI(\Omega) = |X(\Omega)|e^{i\varphi(\Omega)}.$$

- $X(\Omega)$ is referred as the (Fourier) ***spectrum*** of $x(t)$.
- $|X(\Omega)|$ is the ***spectrum magnitude***.
- $\varphi(\Omega)$ is the ***spectrum phase***.

Fourier Transform

- ***Band-limited signals*** (or ***low-pass signals***) are those satisfying:

$$|X(\Omega)| = 0, \qquad |\Omega| > \Omega_{max}.$$

- Ω_{max} is the band-limited signal ***bandwidth***.

Fourier Transform

FT $X(\Omega)$ is given by:

$$X(\Omega) = \int_{-\infty}^{\infty} x(t)\, e^{-i\Omega t} dt = \int_{-\infty}^{\infty} x(t)\big(\cos(\Omega t) - i\sin(\Omega t)\big)\, dt.$$

If $x(t)$ is a real-valued signal, then:

$$R(\Omega) = \int_{-\infty}^{\infty} x(t) \cos(\Omega t)\, dt\,.$$

$$I(\Omega) = -\int_{-\infty}^{\infty} x(t) \sin(\Omega t)\, dt.$$

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Fourier Transform

Consequently, if $x(t)$ is a real function:

- $R(\Omega), I(\Omega)$ are ***even/odd functions***, respectively:

$$R(-\Omega) = R(\Omega), \qquad I(-\Omega) = -I(\Omega).$$

- FT satisfies the ***complex conjugation*** property:

$$X(-\Omega) = X^*(\Omega),$$

- $*$ denotes complex conjugation.

Fourier Transform

Dirichlet conditions are sufficient for FT $X(\Omega)$ convergence:

1. $x(t)$ is absolutely integrable, as defined by:

$$\int_{-\infty}^{\infty} |x(t)| dt < \infty.$$

2. $x(t)$ has bounded variation within any finite interval $[a, b]$.

3. $x(t)$ is a continuous function or contains finite number of finite discontinuities in any given finite interval.

If conditions 2,3 are met, $x(t)$ is a ***piecewise smooth function***.

Fourier Transform

The ***square window function***:

$$x(t) = \begin{cases} 1, & |t| < T/2 \\ 0, & |t| > T/2 \end{cases}$$

satisfies Dirichlet conditions. Its FT is a real ***sync function***:

$$X(\Omega) = \int_{-\infty}^{\infty} x(t) e^{-i\Omega t} dt = \int_{-T/2}^{T/2} e^{-i\Omega t} dt = \frac{2}{\Omega} \sin\left(\frac{\Omega T}{2}\right)$$

$$= T \frac{\sin\left(\frac{\Omega T}{2}\right)}{\Omega T/2}.$$

Fourier and Laplace Transform

Fourier transform:

$$X(\Omega) = \int_{-\infty}^{\infty} x(t) e^{-i\Omega t} dt,$$

is a special case of bilateral ***Laplace transform*** (***LT***):

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt,$$

for $s = i\Omega$.

Fourier and Laplace Transform

If $s = \sigma + i\Omega$, Laplace transform becomes:

$$X(\sigma + i\Omega) = \int_{-\infty}^{\infty} x(t)e^{-(\sigma+i\Omega)t}dt = \int_{-\infty}^{\infty} |x(t)e^{-\sigma t}|e^{-i\Omega t}dt,$$

and:

$$X(\sigma + i\Omega) = \mathcal{F}\{x(t)e^{-\sigma t}\}.$$

- The bilateral Laplace transform of $x(t)$ can be interpreted as the Fourier transform of signal $x(t)e^{-\sigma t}$.
- This holds only if $x(t)$ is absolutely integrable.

Fourier and Laplace Transform

Delta function transforms:

- LT of delta function $\delta(t)$ is given by:

$$\mathcal{L}\{\delta(t)\} = 1, \qquad \text{for all } s.$$

- By definition, the FT of delta function is:

$$\mathcal{F}\{\delta(t)\} = \int_{-\infty}^{\infty} \delta(t) e^{-i\Omega t} dt = 1.$$

- Thus, LT and FT of $\delta(t)$ are the same.

Fourier and Laplace Transform

LT of right-sided exponential signal:

$$x(t) = e^{-at}u(t), a > 0,$$

is given by:

$$\mathcal{L}\{x(t)\} = X(s) = \frac{1}{s+a}, \qquad \mathrm{Re}(s) > -a.$$

By definition, the FT of the same signal is:

$$\mathcal{F}\{x(t)\} = X(\Omega) = \int_{-\infty}^{\infty} e^{-at}u(t)e^{-i\Omega t}dt = \int_{0}^{\infty} e^{-(a+i\Omega)t}dt = \frac{1}{i\Omega + a}.$$

Fourier and Laplace Transform

- Thus, LT and FT of $x(t)$ are the same for $s = i\Omega$:

$$X(\Omega) = X(s)\Big|_{s=i\Omega}.$$

- Note that $x(t)$ is absolutely integrable.

Fourier Transform pairs

Signal $x(t)$	Fourier Transform $X(\Omega)$
$\delta(t)$	1
$\delta(t-t_0)$	$e^{-i\Omega t_0}$
1	$2\pi\delta(\Omega)$
$e^{i\Omega_0 t}$	$2\pi\delta(\Omega-\Omega_0)$
$\cos(\Omega_0 t)$	$\pi[\delta(\Omega-\Omega_0)+\delta(\Omega+\Omega_0)]$
$\sin(\Omega_0 t)$	$-i\pi[\delta(\Omega-\Omega_0)-\delta(\Omega+\Omega_0)]$
$u(t)$	$\pi\delta(\Omega)+\frac{1}{i\Omega}$
$u(-t)$	$\pi\delta(\Omega)-\frac{1}{i\Omega}$

Fourier Transform pairs

Signal $x(t)$	Fourier Transform $X(\Omega)$
$e^{-at}u(t), \quad a > 0$	$\frac{1}{i\Omega + a}$
$e^{-a\lvert t\rvert}, \quad a > 0$	$\frac{2a}{a^2 + \Omega^2}$
$e^{-at^2}, \quad a > 0$	$\sqrt{\frac{\pi}{a}} e^{-\Omega^2/4a}$
$x(t) = \begin{cases} 1, & \lvert t\rvert < a \\ 0, & \lvert t\rvert > a \end{cases}$	$2a\frac{\sin(\Omega a)}{\Omega a}$
$\frac{\sin(at)}{\pi t}$	$X(\Omega) = \begin{cases} 1, & \lvert\Omega\rvert < a \\ 0, & \lvert\Omega\rvert > a \end{cases}$
$\mathrm{sgn}(t)$	$\frac{2}{i\Omega}$

Fourier Transform

- Periodic functions
- Fourier Transform
- **Fourier Transform Properties**
- Fourier Transform and LTI systems

Fourier Transform Properties

- Linearity:

$$a_1x_1(t) + a_2x_2(t) \leftrightarrow a_1X_1(\Omega) + a_2X_2\Omega.$$

- Time Shifting:

$$x(t - t_0) \leftrightarrow e^{-i\Omega t_0}X(\Omega).$$

  - A time shift results only in a FT phase change.

- Frequency Shifting:

$$e^{i\Omega_0 t}x(t) \leftrightarrow X(\Omega - \Omega_0).$$

Fourier Transform Properties

- Time Scaling:

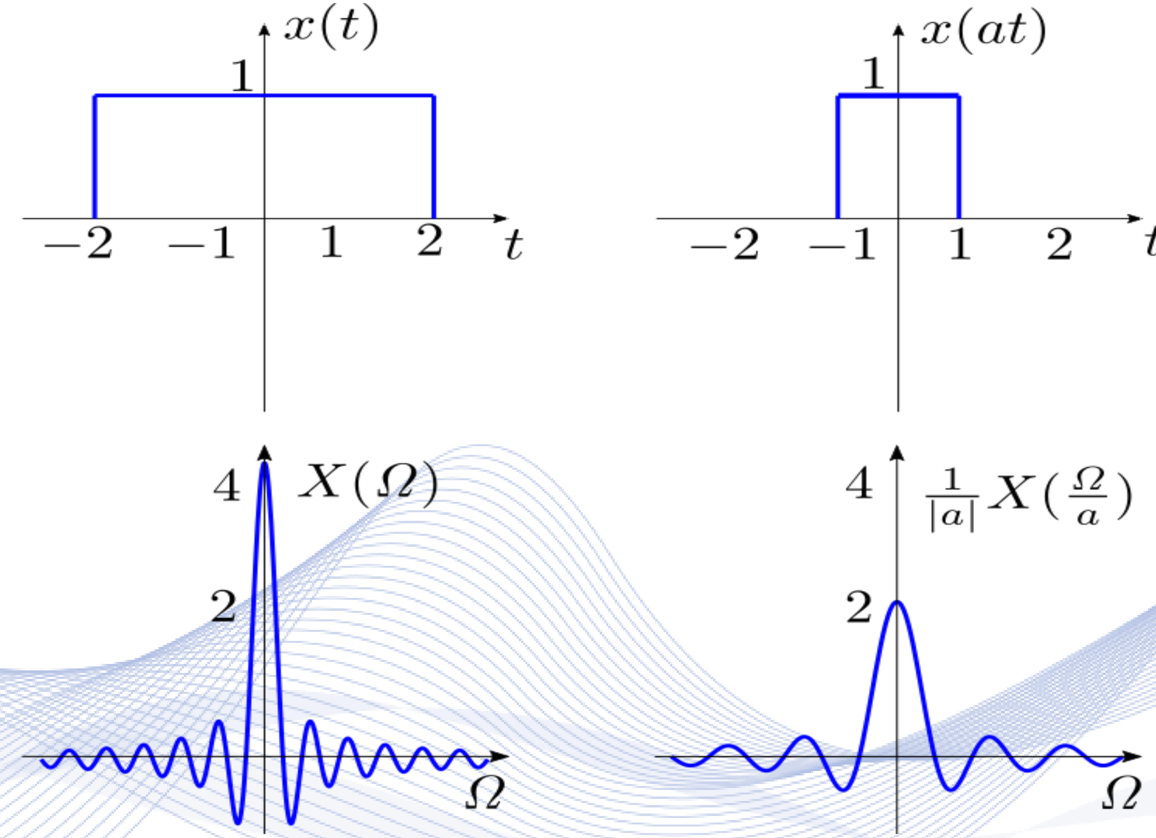
$$x(at) \leftrightarrow \frac{1}{|a|} X\left(\frac{\Omega}{a}\right).$$

- Effect of time scaling on a square wave signal:

$$x(t) = \begin{cases} 1, & |t| < 2 \\ 0, & |t| > 2 \end{cases}$$

for $a = 2$.

- Signal frequency range increases.



Effects of signal time scaling

Fourier Transform Properties

- Time Reversal (signal flipping about $t = 0$):

$$x(-t) \leftrightarrow X(-\Omega).$$

- Signal flipping is used in convolution definition.

- Duality (or Symmetry):

$$X(t) \leftrightarrow 2\pi x(-\Omega).$$

Fourier Transform Properties

- First order differentiation in the temporal domain:

$$\frac{dx(t)}{dt} \leftrightarrow i\Omega X(\Omega).$$

- Higher order differentiation in the temporal domain:

$$\frac{d^n x(t)}{dt^n} \leftrightarrow (i\Omega)^n X(\Omega).$$

- Multiplication by $(i\Omega)^n$ amounts to high-pass signal filtering.
- Therefore, differentiation is a ***high-pass system***.

Fourier Transform Properties

- Integration in the temporal domain:

$$\int_{-\infty}^{t} x(-\tau)d\tau \leftrightarrow \pi X(0)\delta(\Omega) + \frac{1}{i\Omega}X(\Omega).$$

- Division by $i\Omega$ amounts to low-pass signal filtering. Therefore, integration is a ***low-pass system***.

- Differentiation in the Frequency Domain:

$$(-it)x(t) \leftrightarrow \frac{dX(\Omega)}{d\Omega}.$$

Fourier Transform Properties

- ***Signal convolution***:

$$x(t) * h(t) \leftrightarrow X(\Omega)H(\Omega).$$

- As FT can be computed through Discrete Fourier Transform (DFT) for discrete signals, using the Fast Fourier Transform (FFT) algorithms, it leads to fast convolution calculation.
- Signal multiplication:

$$x_1(t)x_2(t) \leftrightarrow \frac{1}{2\pi} X_1(\Omega) * X_2(\Omega).$$

Fourier Transform Properties

- ***Parseval's theorem***:

$$\int_{-\infty}^{\infty} x_1(t)x_2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_1(\Omega)X_2(-\Omega)d\Omega\,.$$

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\Omega)|^2 d\Omega\,.$$

- Essentially, FT preserves energy after transformation.

Fourier Transform Properties

- If $x(t)$ is real:

$$X(-\Omega) = X^*(\Omega),$$

- If $x(t) = x_e(t) + x_o(t)$, where $x_e(t) = x_e(-t)$ and $x_o(t) = -x_o(-t)$ are the even and odd components of $x(t)$ and:

$$x_e(t) \leftrightarrow A(\Omega),$$

$$x_o(t) \leftrightarrow iB(\Omega).$$

then:

$$x(t) \leftrightarrow X(\Omega) = A(\Omega) + iB(\Omega).$$

Fourier Transform

- Periodic functions
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- Fourier Transform Properties
- **Fourier Transform and LTI systems**

Frequency response of LTI systems

- The output $y(t)$ of a continuous-time LTI system is given by the convolution:

$$y(t) = x(t) * h(t).$$

- Applying the convolution property, we get:

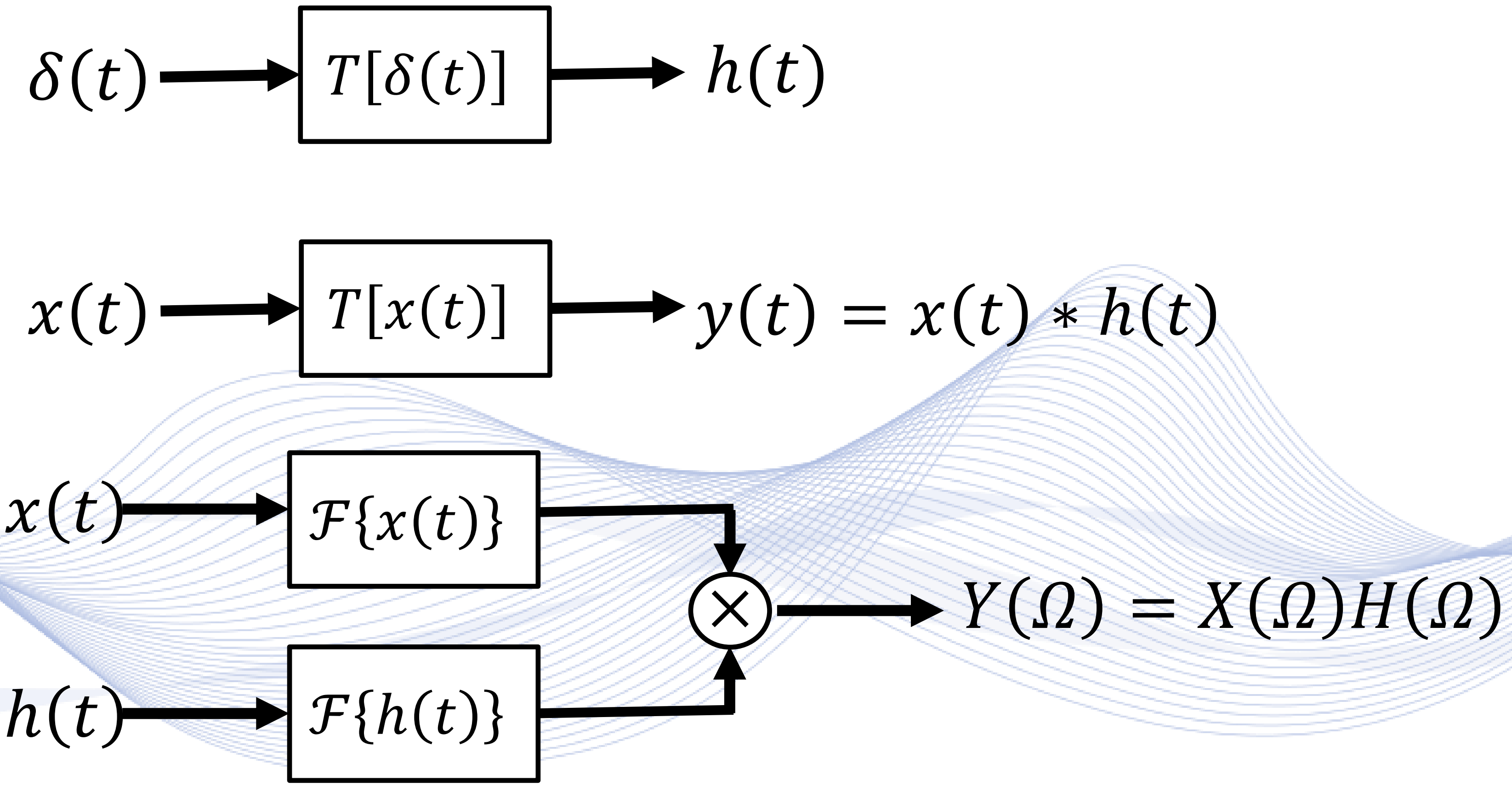
$$Y(\Omega) = X(\Omega)H(\Omega).$$

- LTI system ***frequency response*** $H(\Omega)$:

$$H(\Omega) = \frac{Y(\Omega)}{X(\Omega)}$$

is the FT of its impulse response $h(t)$.

Frequency response of LTI systems



Frequency response of LTI systems

Polar representation of system frequency response $H(\Omega)$:

$$H(\Omega) = |H(\Omega)|e^{i\theta_H(\Omega)}.$$

- $|H(\Omega)|$ is the system ***frequency response magnitude***.
- $\theta_H(\Omega)$ is the system ***frequency response phase***.

Frequency response of LTI systems

The behavior of the LTI system in the frequency domain is completely characterized by its frequency response $H(\Omega)$:

- If $|H(\Omega)| < 1$, the respective input signal frequency is attenuated.
- If $|H(\Omega)| = 0$, the respective frequency is cutoff.
- If $|H(\Omega)| = 1$, the respective frequency passes through the system.
- If $|H(\Omega)| > 1$, the respective frequency is amplified.

Frequency response of LTI systems

An input signal $x(t) = e^{i\Omega_0 t}$ with Fourier Transform:

$$X(\Omega) = 2\pi\delta(\Omega - \Omega_0),$$

results in system output:

$$Y(\Omega) = 2\pi H(\Omega_0)\delta(\Omega - \Omega_0).$$

- Taking the inverse FT of $Y(\Omega)$, we get:

$$y(t) = H(\Omega_0)e^{i\Omega_0 t}.$$

- An LTI does ***not*** change the frequency Ω_0.
- $e^{i\Omega t}$ is an LTI system ***eigenfunction*** with eigenvalue $H(\Omega_0)$.

Frequency response of LTI systems

- Let:

$$X(\Omega) = |X(\Omega)|e^{i\theta_X(\Omega)}, \quad H(\Omega) = |H(\Omega)|e^{i\theta_H(\Omega)},$$
$$Y(\Omega) = |Y(\Omega)|e^{i\theta_Y(\Omega)}.$$

- Then from $Y(\Omega) = X(\Omega)H(\Omega)$ we get:

$$|Y(\Omega)| = |X(\Omega)||H(\Omega)|,$$
$$\theta_Y(\Omega) = \theta_X(\Omega) + \theta_H(\Omega).$$

- System frequency response magnitude $|H(\Omega)|$ is referred as the gain of the system.

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Frequency response of LTI systems

- Both signal differentiation and integration can be considered as linear systems having frequency response $H(\Omega)$.
- In signal differentiation:

$$\frac{d^n x(t)}{dt^n} \leftrightarrow Y(\Omega) = (i\Omega)^n X(\Omega).$$

multiplication in the frequency domain by $(i\Omega)^n$ results in a ***high-pass system***:

$$H(\Omega) = (i\Omega)^n.$$

Frequency response of LTI systems

- The higher the differentiation order is, the more profound its high-pass characteristics are.
- Differentiation typically enhances high-frequency signal noise.
- Therefore, low-pass signal filtering must be performed before signal differentiation.
- Second-order differentiation and beyond are typically avoided, due to their noise sensitivity.

Frequency response of LTI systems

- In integration:

$$\int_{-\infty}^{t} x(-\tau)d\tau \leftrightarrow \frac{1}{i\Omega}X(\Omega), \qquad \text{if } X(0) = 0.$$

division by $i\Omega$ the frequency domain amounts to low-pass signal filtering:

$$H(\Omega) = \frac{1}{i\Omega}.$$

- Therefore, integration is a ***low-pass system***.

Frequency response of LTI systems

A LTI system can be described by a linear differential equation with constant coefficients:

$$a_n \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \cdots + a_0 y(t)$$
$$= b_m \frac{d^m x(t)}{dt^m} + b_{m-1} \frac{d^{m-1} x(t)}{dt^{m-1}} + \cdots + b_0 x(t),$$

- $a_i, b_j \in \mathbb{R}$.

Frequency response of LTI systems

Then, by applying the LT in both parts of the previous formula, we have:

$$H(i\Omega) = \frac{Y(i\Omega)}{X(i\Omega)} = \frac{b_m\ (i\Omega)^m + b_{m-1}(i\Omega)^{m-1} + \cdots + b_0}{a_n(i\Omega)^n + a_{n-1}(i\Omega)^{n-1} + \cdots + a_0}.$$

- Frequency response has rational form.
- Under certain conditions, its magnitude $|H(i\Omega)|$ can become:
  - 0, thus completely attenuating the respective frequencies;
  - Very large (towards infinity), thus greatly amplifying the respective frequencies.

1D Filters

Filtering is the process where the amplitude/phase of the frequency components of a signal are modified or even reduced to zero.

- A "filter" is an LTI system, whose frequency response shows this selective frequency modification behavior.

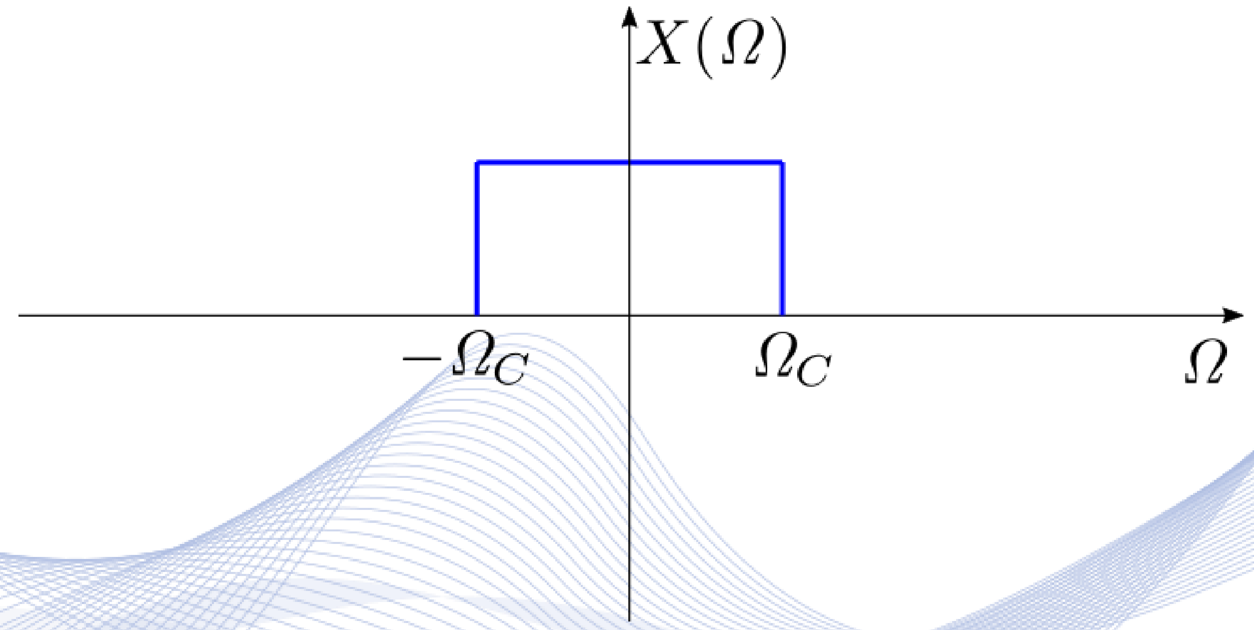
1D Filters

- An ***ideal filter*** allows some selected signal frequencies to pass, while completely attenuating the rest.
- ***Pass-band*** is the range of frequencies passed by the filter.
- ***Stop-band*** is the range of frequencies rejected by the filter.

1D Filters

Ideal ***Low-Pass*** (***LP***) Filter:

$$|H(\Omega)| = \begin{cases} 1, & |\Omega| < \Omega_{max} \\ 0, & |\Omega| > \Omega_{max} \end{cases}.$$



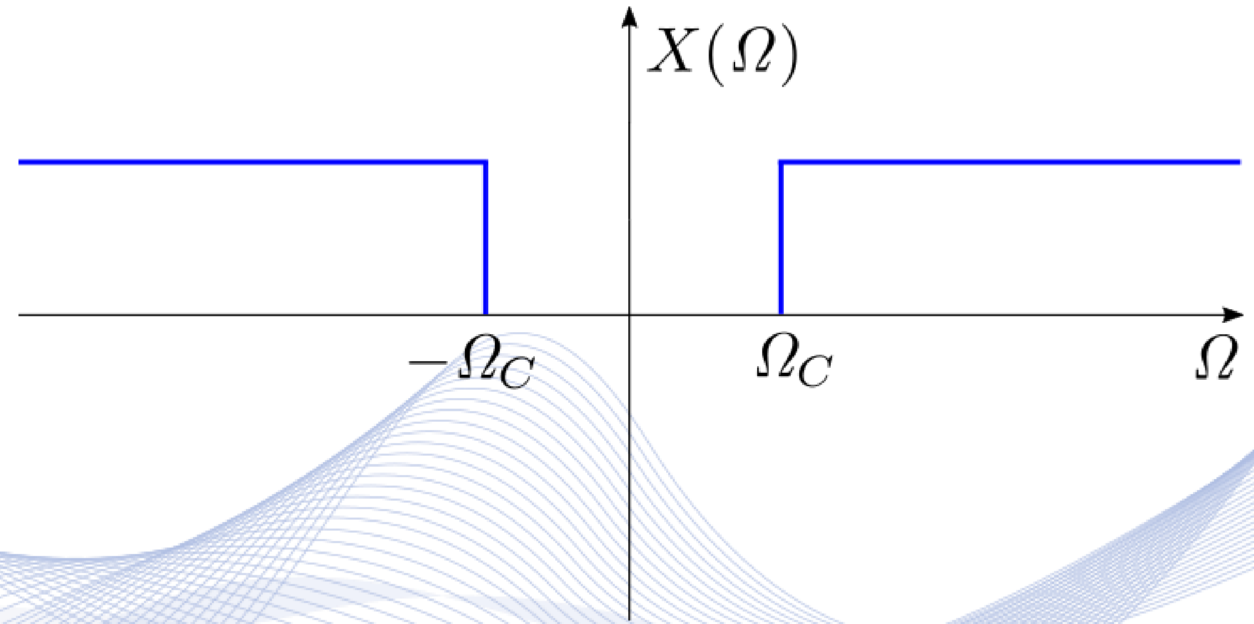
- Ω_{max} : ***LP cut-off frequency***.

1D Filters

Ideal ***High-Pass*** (***LP***) Filter:

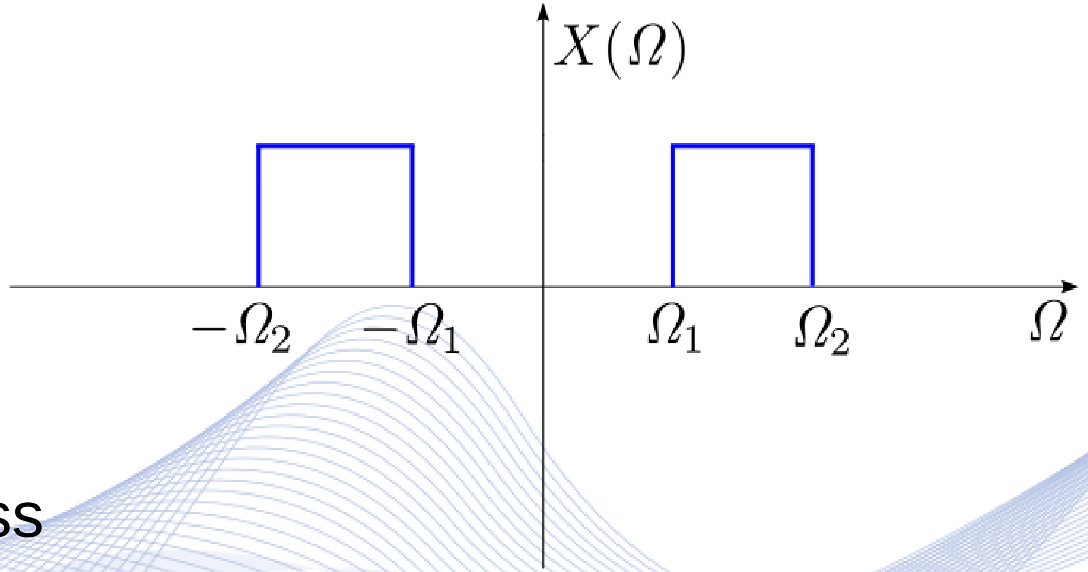
$$|H(\Omega)| = \begin{cases} 1, & |\Omega| > \Omega_{min} \\ 0, & |\Omega| < \Omega_{min} \end{cases}.$$

Ω_{min} : ***HP cut-off frequency***.



1D Filters

$$|H(\Omega)| = \begin{cases} 1, & \Omega_1 < |\Omega| < \Omega_2 \\ 0, & \text{eslewhere.} \end{cases}$$

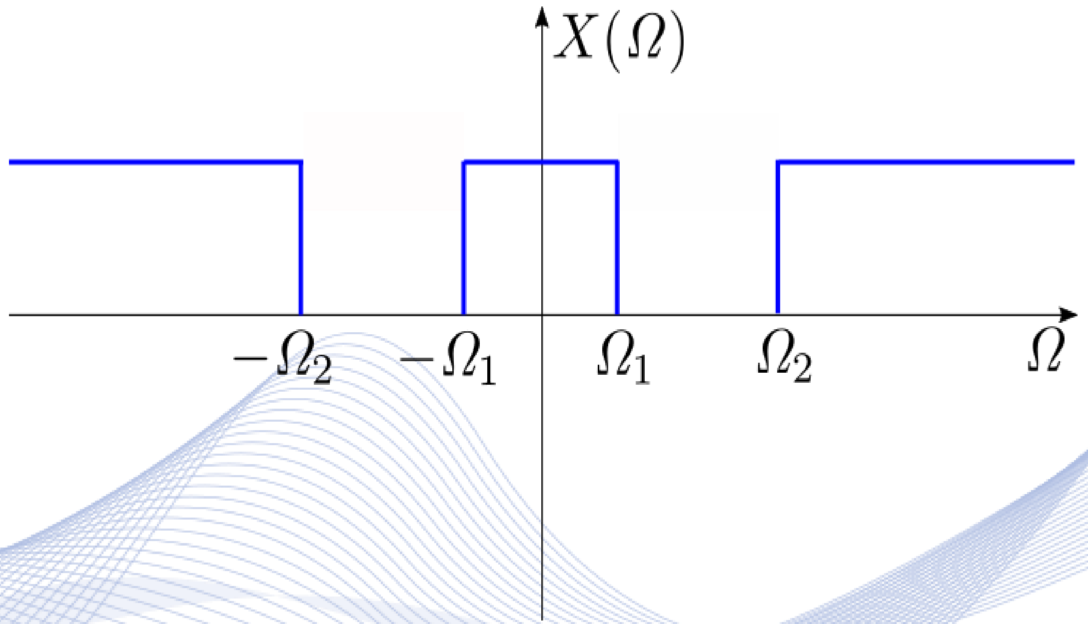


- Ω_1, Ω_2 : ***BP cut-off frequencies***.
- ***Bandpass resonator*** is a band-pass filter having very narrow passpand band, typically around resonation frequency Ω_p: $\Omega_1, \simeq \Omega_2 \simeq \Omega_s$.
- Radio transmitter/receiver ***oscillators*** and ***resonators***.

1D Filters

Ideal ***Bandstop*** (***BS***) Filter:

$$|H(\Omega)| = \begin{cases} 0, & \Omega_1 < |\Omega| < \Omega_2 \\ 1, & \text{otherwise.} \end{cases}$$



- Ω_1, Ω_2 : ***BS cut-off frequencies***.
- **notch filter** is a band-stop filter having very narrow stop band, typically around frequency Ω_s: $\Omega_1, \simeq \Omega_2 \simeq \Omega_s$.
- 50/60 Hz rejection filters.

1D Filters

- To avoid phase distortion, a filter phase response $\theta_H(\Omega)$ is typically linear over the frequency range of interest:

$$\theta_H(\Omega) = a\Omega.$$

- Ideal frequency-selective filters are noncausal systems.

1D Filters

- ***Filter bandwidth*** is the difference between filter cut-off frequencies, if its pass-band is not infinite:
- For an ideal low-pass filter, its bandwidth is equal to its cutoff frequency: $W_B = \Omega_c$.
- For an ideal bandpass filter, its bandwidth is the difference between its two cuttoff frequencies: $W_B = \Omega_2 - \Omega_1$.
- Typically a ***transition band*** exists between passpands and stop bands.

1D Filters

- ***Half-Power Bandwidth*** $W_{3\,dB}$ is defined by the frequency at which the frequency response amplitude $|H(\Omega)|$ drops to a value equal to $|H(\Omega_m)|/\sqrt{2}$.
- $H(\Omega_m)$ is the maximal frequency response amplitude.
- It shows the point at which the output power has dropped to half of its peak value.
- At this frequency, we have $3\ dB$ ***attenuation***:

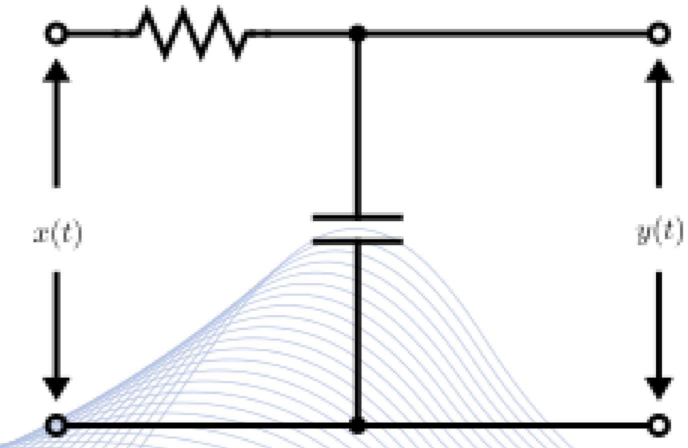
$$L = 10\log_{10}(|H(\Omega)|^2/|H(\Omega_m)|^2) = 10\log_{10}(1/2) \approx 3\ dB.$$

1D Filters

Analog ***electric filters***:

- They are electric networks consisting of ***resistances***, ***capacitors***, ***inductors***.
- The input-output relation of an RC filter is given by:

$$RC\frac{dy(t)}{dt} + y(t) = x(t).$$



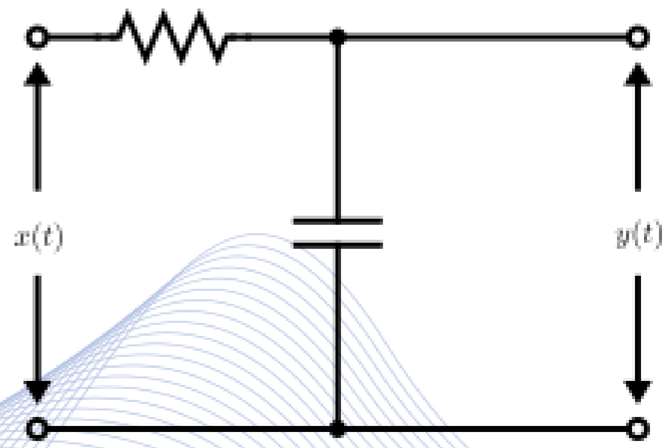
1D Filters

The frequency response $H(\Omega)$ of the RC filter can be found by taking the FT of both sides:

$$H(\Omega) = \frac{Y(\Omega)}{X(\Omega)} = \frac{1}{1 + i\Omega RC}$$

$$= \frac{1}{1 + \frac{i\Omega}{\Omega_0}},$$

where $\Omega_0 = {}^{1}/_{RC}$.

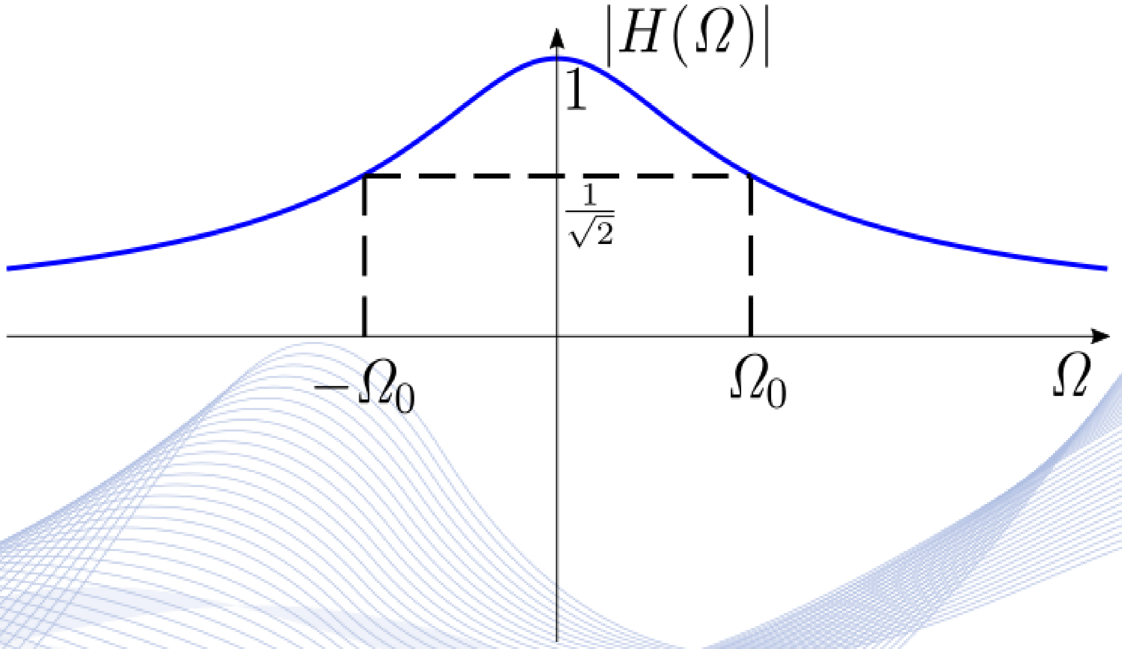


1D Filters

- The frequency response amplitude $|H(\Omega)|$ is:

$$|H(\Omega)| = \frac{1}{\left|1 + \left(\frac{\Omega}{\Omega_0}\right)^2\right|^{1/2}}.$$

- The RC filter is a low-pass one.
- High frequencies are attenuated.



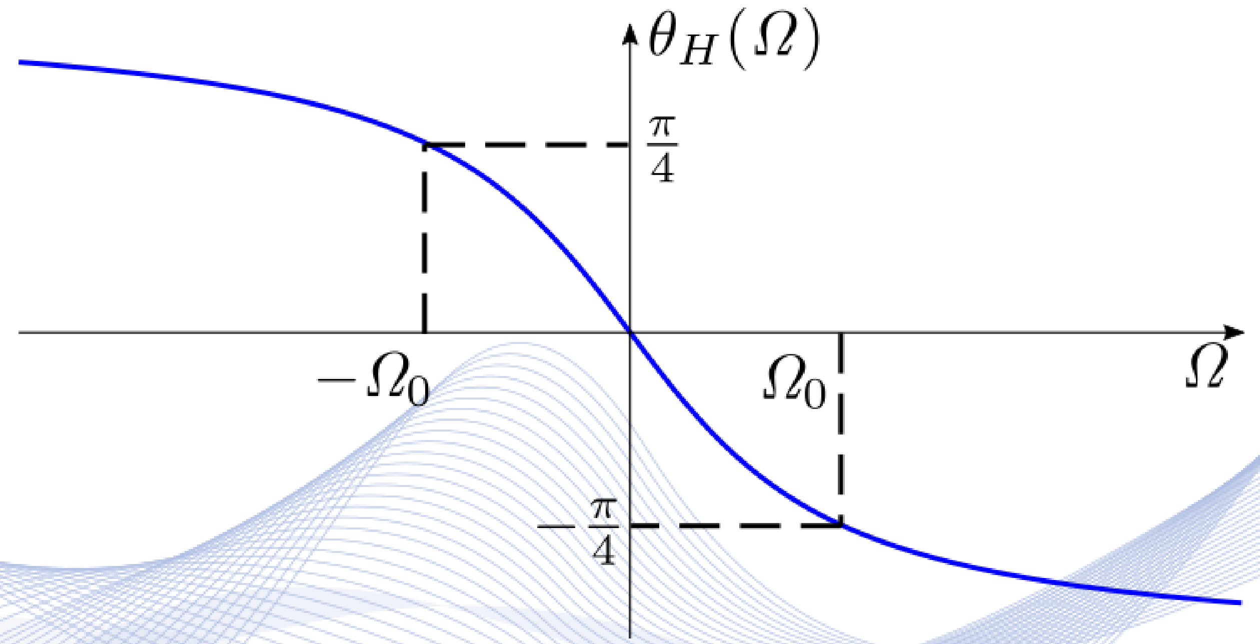
1D Filters

- Its frequency response phase $\theta_H(\Omega)$ is given by:

$$\theta_H(\Omega) = -\tan^{-1}\left(\frac{\Omega}{\Omega_0}\right).$$

- It is a nonlinear function of Ω.
- It can become almost linear for small Ω:

$$\theta_H(\Omega) \approx -\frac{\Omega}{\Omega_0}.$$



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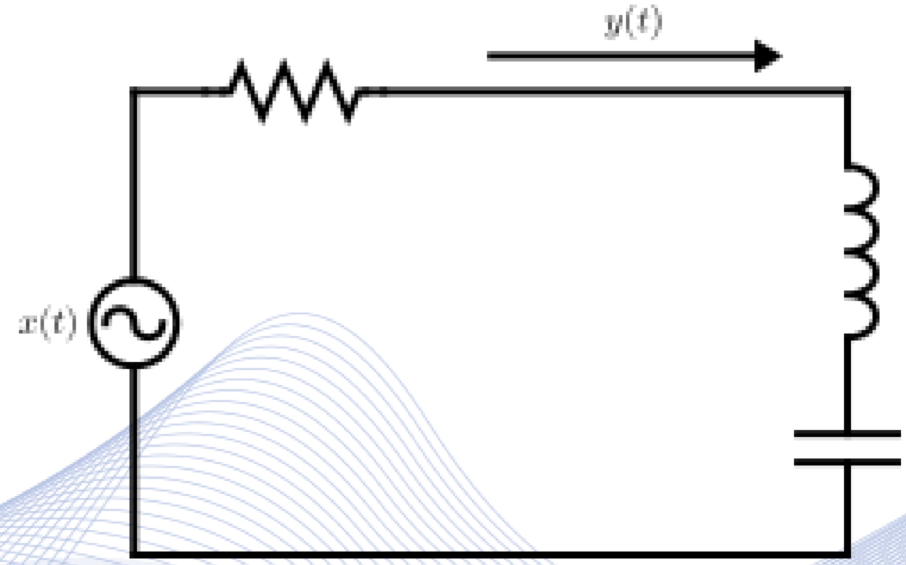
1D Filters

RLC filter input-output relation is given by:

$$L\frac{dx(t)}{dt} + Rx(t) + \frac{1}{C}\int_{-\infty}^{1} x(t)dt = y(t).$$

- $x(t)$: input voltage,
- $y(t)$: output current.



1D Filters

Its differentiation results in a second-order differential equation:

$$L\frac{d^2x(t)}{dt^2} + R\frac{dx(t)}{dt} + \frac{1}{C}x(t) = \frac{dy(t)}{dt}$$

and frequency response:

$$H(\Omega) = \frac{Y(\Omega)}{X(\Omega)} = \frac{i\Omega}{-L\Omega^2 + iR\Omega + \frac{1}{C}}.$$

1D Filters

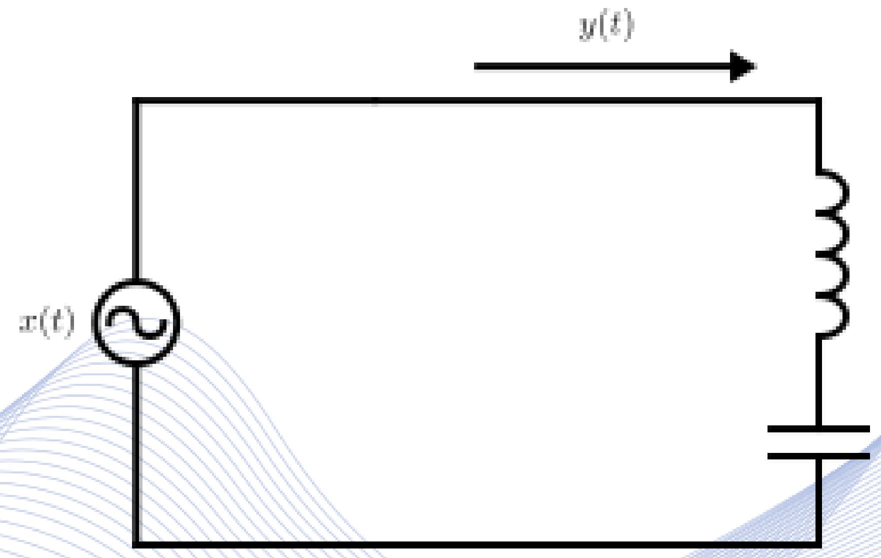
- If $R = 0$, the frequency response becomes:

$$H(\Omega) = \frac{i\Omega}{-L\Omega^2 + \frac{1}{C}}.$$

- It is an ***electric oscillator*** typically resonating at frequency:

$$\Omega = 1/\sqrt{LC}.$$

- It has been extensively used in radio transmitters.

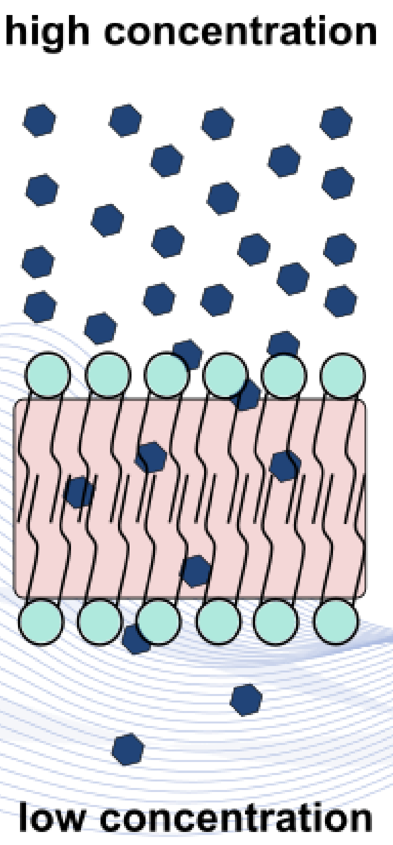


1D Systems

Temporal diffusion system:

$$\frac{dy(t)}{dt} = c\big(x(t) - y(t)\big).$$

- c: diffusion coefficient.



1D Systems

Its frequency response $H(\Omega)$ can be found by taking the FT of both sides:

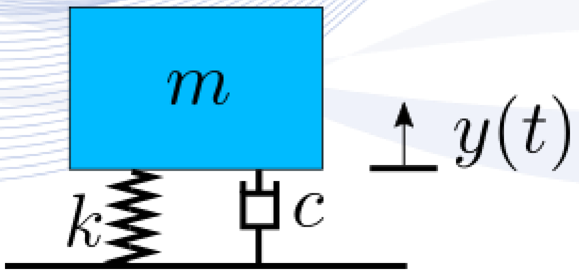
$$H(\Omega) = \frac{Y(\Omega)}{X(\Omega)} = \frac{c}{c + i\Omega}.$$

- It is a low-pass system.
- It can model many phenomena, e.g., pharmacokinetics
- It can be extended to 1D, 2D and 3D spatiotemporal diffusion and to information diffusion over graphs.

1D Systems

Mass-spring-damper mechanical system. If force $f(t)$ is exercised on a mass m that is attached to a spring having constant k that follows Hooke's law and to a damper having damping constant β, the displacement $y(t)$ is given by the exercised forces:

$$m\frac{d^2y(t)}{dt^2}+\beta\frac{dy(t)}{dt}+ky(t)=f(t).$$



- It models car suspension systems.

1D Systems

Fourier Transform of this system is:

$$-m\Omega^2 Y(\Omega) + \beta i\Omega Y(\Omega) + kY(\Omega) = F(\Omega).$$

Therefore, its transfer function is given by:

$$H(\Omega) = \frac{Y(\Omega)}{F(\Omega)} = \frac{1}{-m\Omega^2 + \beta i\Omega + k}.$$

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Q & A

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