FIR Filter Design summary

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Version 1.3.1

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FIR Filter Design

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FIR Filter Design

In signal processing, a ***Finite Impulse Response*** (**FIR**) filter is a filter whose impulse response (or response to any finite length input) is of finite duration, because it settles to zero in finite time.

This is in contrast to ***Infinite Impulse Response*** (**IIR**) filters, which may have internal feedback and may continue to respond indefinitely (usually decaying).

FIR Filter Design

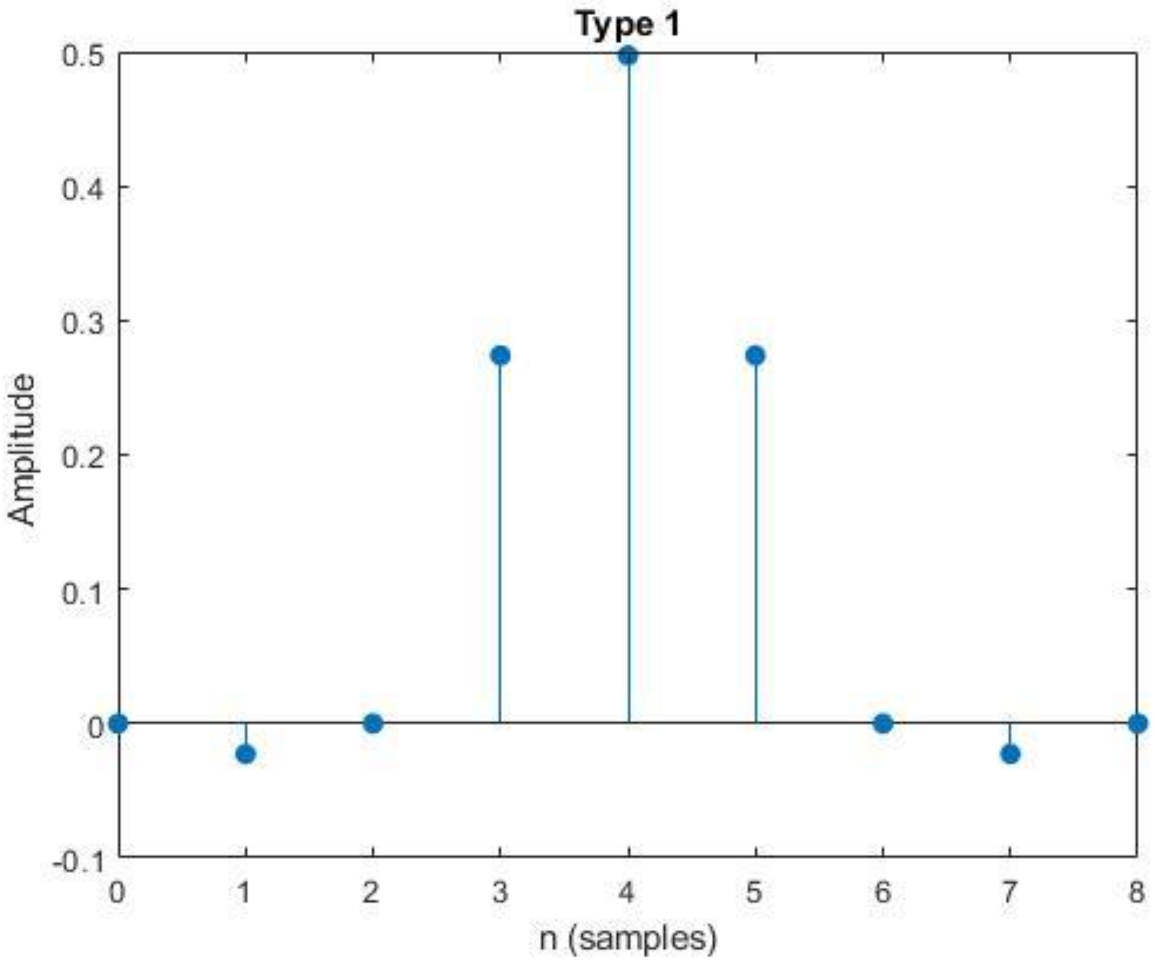
A ***FIR filter*** of order N, is a filter of the form:

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-1} z^{-n}.$$

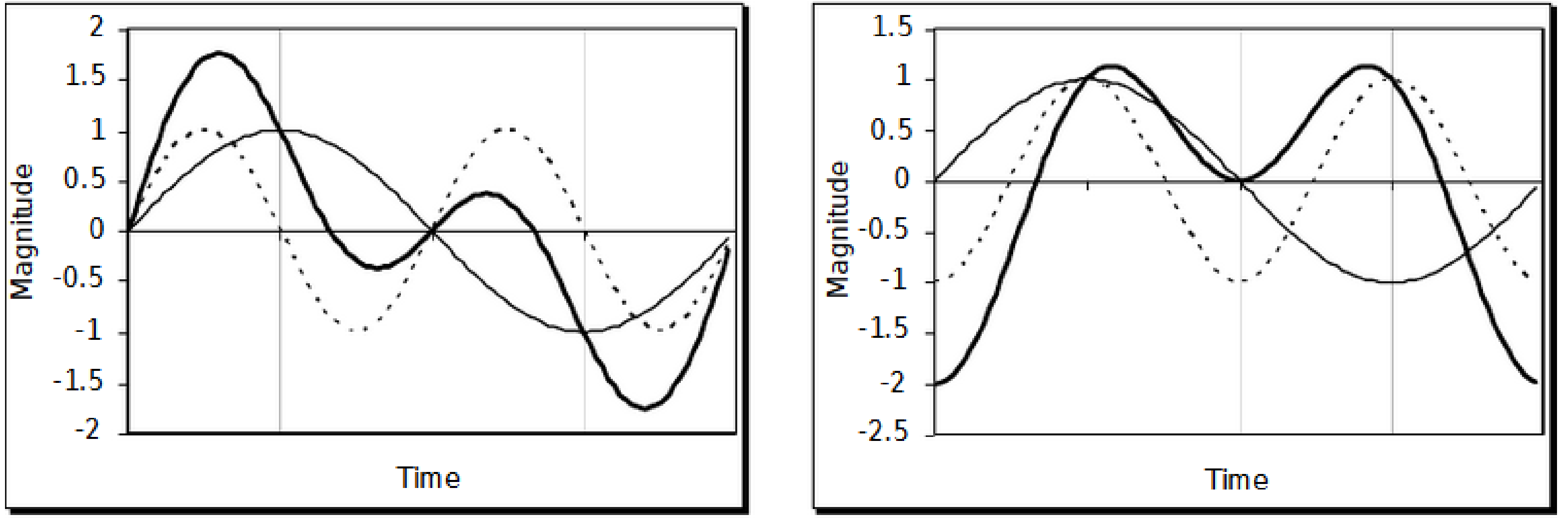
Its ***Frequency Response*** is given by:

$$H\left(e^{iW}\right) = \sum_{n=0}^{N-1} h(n)\, e^{-iWn}.$$

FIR - Linear Phase



FIR - Linear Phase



(a) (b)

Figure 8.5. Effects of linear and non-linear phase on signals. In both parts (a) and (b), the bold trace denotes the sum of a 1 Hz and a 2Hz sine wave. However, in part (b), the 2 Hz sine wave has also been phase delayed by $\pi/2$, ie. 90°.

Window Method

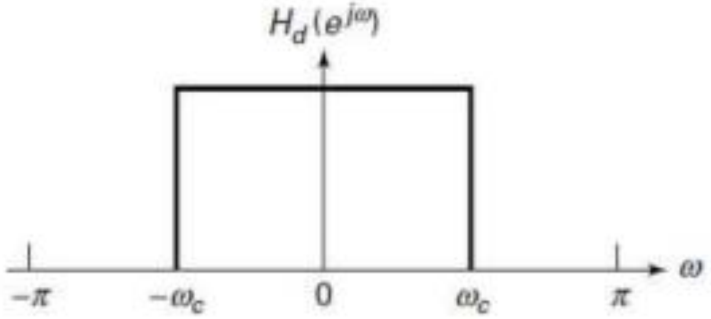
If the *Impulse Response* $h_d(n)$, $-\infty < n < \infty$ needed is known, a **FIR** filter of length N can be designed as:

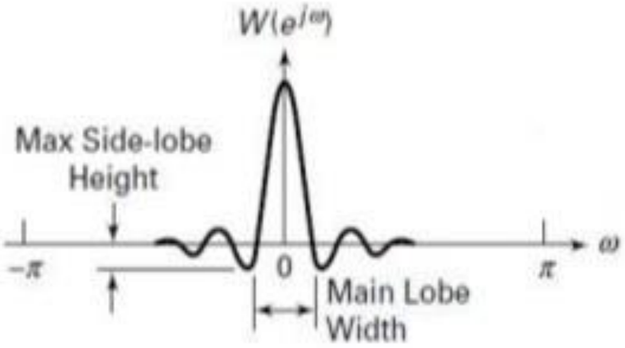
$$h(n) = h_d(n)w(n)$$

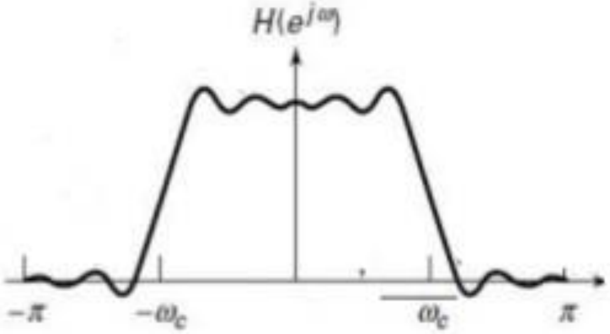
The following function $w(n)$ is called "**Window**":

$$w(n) = \begin{cases} \neq 0, 0 \leq n \leq N-1 \\ 0, elsewhere \end{cases}$$

Window Method







Types of Windows

- ***Rectangular Window***:

$$w(n) = 1, 0 \leq n \leq N - 1$$

- ***Bartlett Window***:

$$w(n) = \begin{cases} \frac{2n}{N-1}, 0 \leq n \leq \frac{N-1}{2} \\ 2 - \frac{2n}{N-1}, \frac{N-1}{2} \leq n \leq N-1 \end{cases}$$

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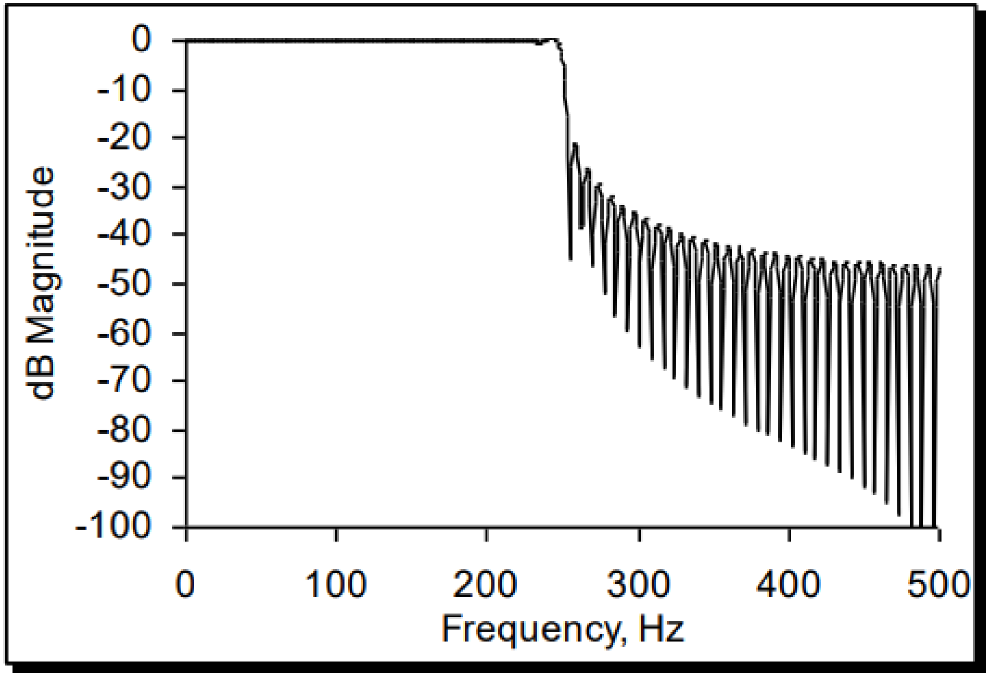
Window Method

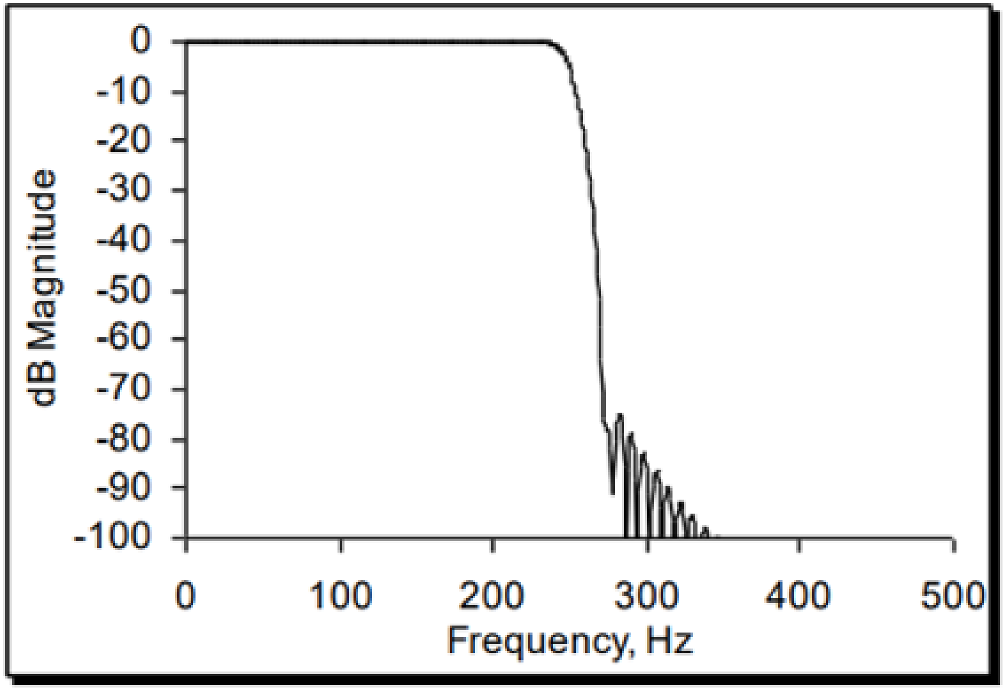
Magnitude *Frequency Responses*, shown in dBs, of a *low-pass* **FIR** filter with a cut-off of 250 Hz and a sample rate of 1 kHz using: (a) a ***Rectangular***



(a)

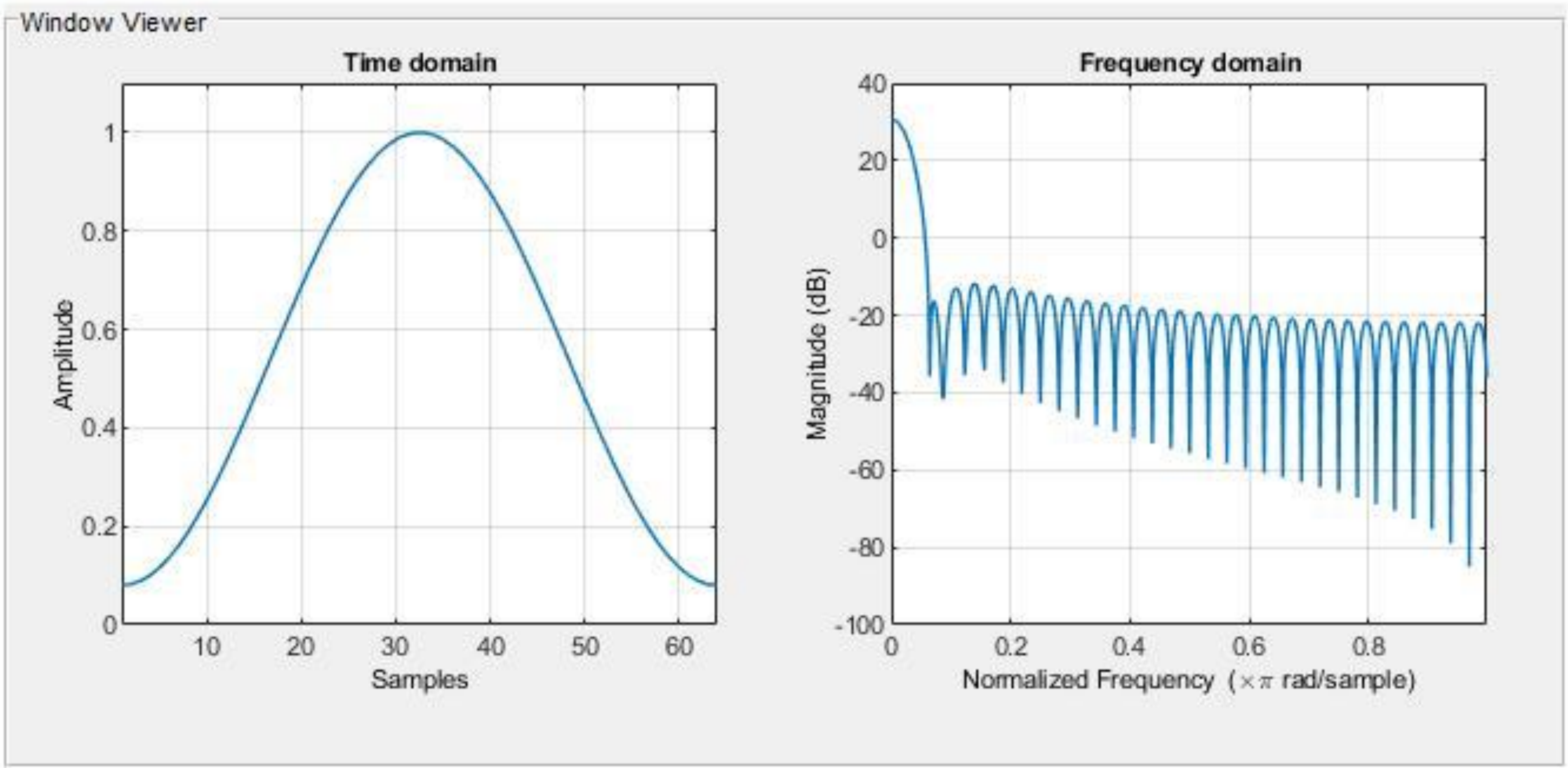
Window Method

Magnitude *Frequency Responses*, shown in dBs, of a *low-pass* **FIR** filter with a cut-off of 250 Hz and a sample rate of 1 kHz using: and (d) a ***Blackman window***.



(d)

Window Method



Optimisation Method

An effective design method where the coefficients $h(n)$, $0 \leq n \leq N-1$ in a way to minimize other criteria, such as L_2 or the L_p of the *approximation error*:

$$E_2 = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|H_d\left(e^{iW}\right) - H\left(e^{iW}\right)\right|^2 dW$$

$$E_p = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|H_d\left(e^{iW}\right) - H\left(e^{iW}\right)\right|^p dW$$

Frequency Sampling Method

A very simple design method is by choosing:

$$\tilde{H}(k) = H_d\left(e^{i\frac{2\pi k}{N}}\right), k = 0,1,\dots,N-1$$

We can optimize this by choosing the values of $\tilde{\mathrm{H}}(\mathrm{k})$ so that the filter response is optimized in the *passband* and the *stopband*.

Frequency Sampling Method

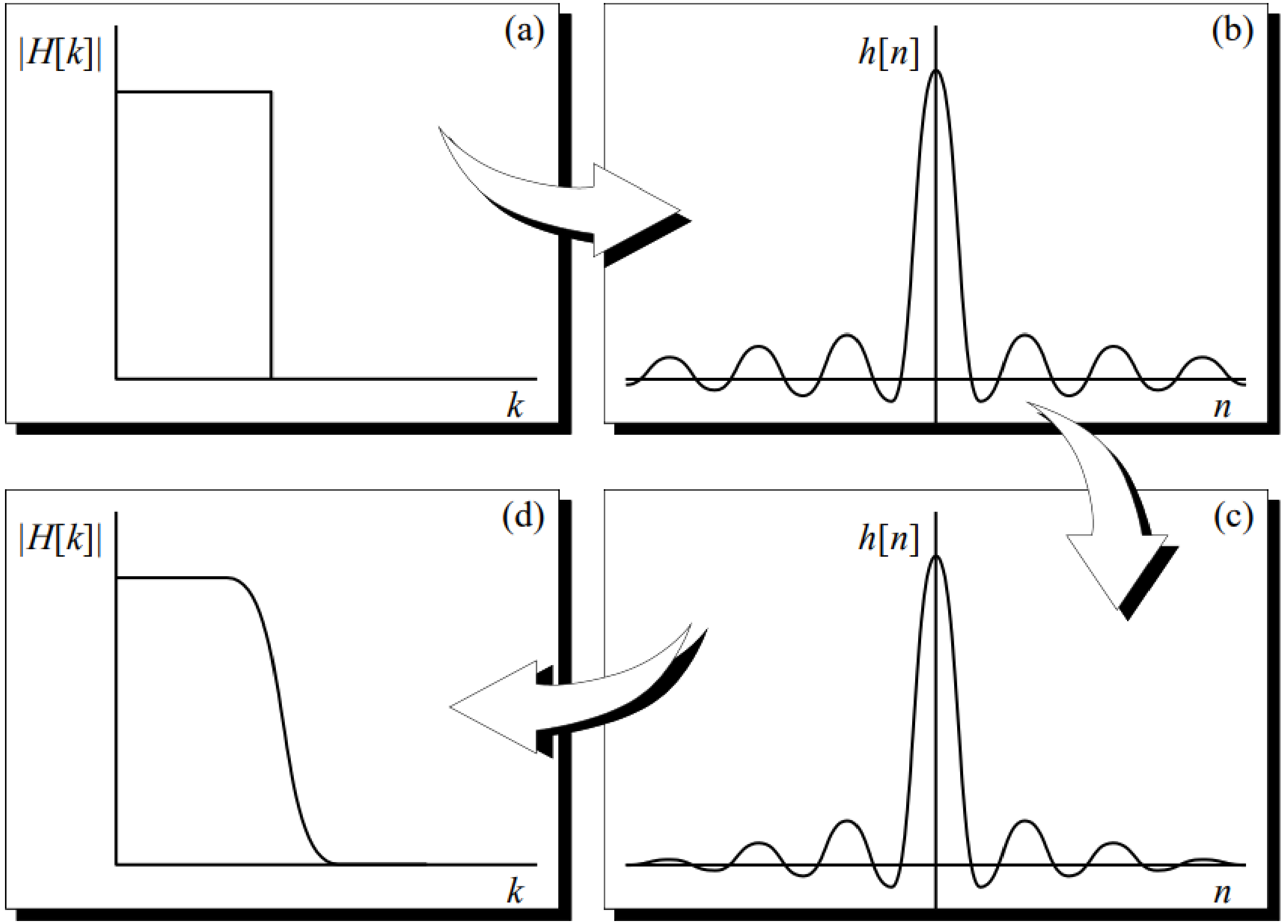


Figure 8.6. Design steps using the frequency sampling method.

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Equiripple FIR Filter Design

Design of *Liner Phase* **FIR** Filters

$$H(e^{iW}) = \sum_{n=-M}^{M} h(n)e^{-iWn}, h(n) = h(-n)$$

In this case the *Frequency Response* is *real*:

$$H(e^{iW}) = h(0) + \sum_{n=1}^{M} 2h(n)\cos(Wn)$$

Equiripple FIR Filter Design

And the unknown constants are *M+1*.

Usually, $M, \delta 1, \delta 2$ are *constants* and $W_{p,} W_s$ are *variables*, or the opposite.

Given the $M, W_{p,} W_{s,}$ we're trying to minimize the error:

$$E_{max} = max|E(W)|, 0 \leq W \leq W_{p,} W_8 \leq W \leq \pi$$

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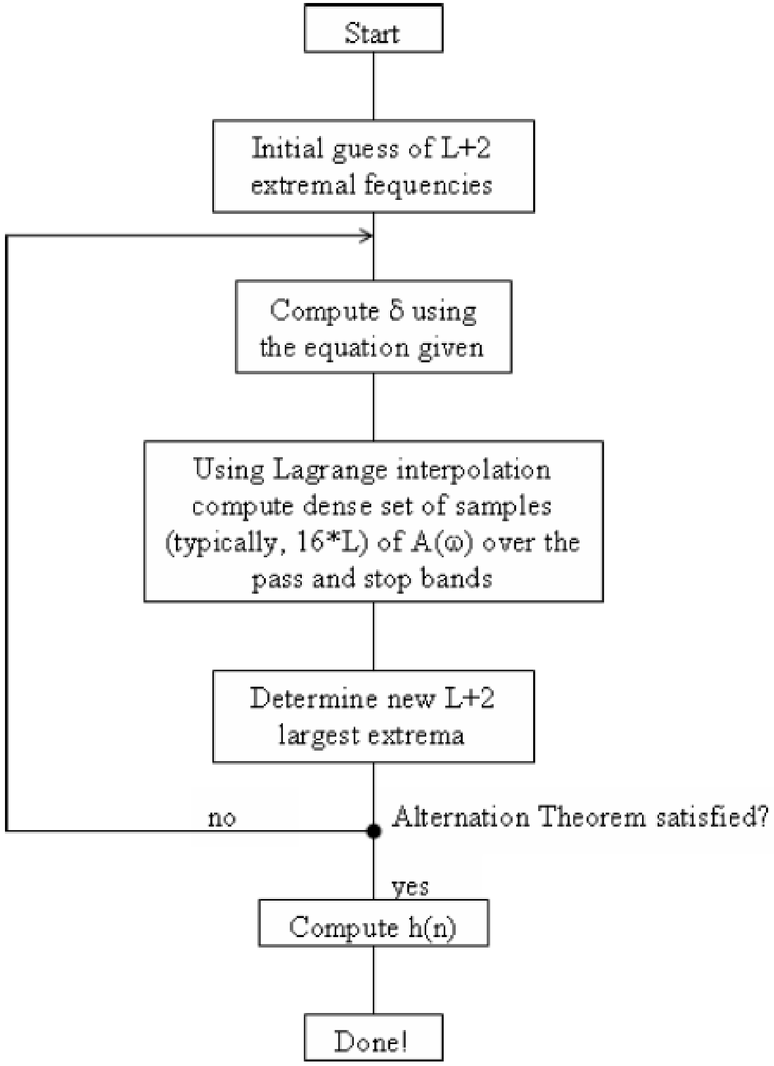
REMEZ Algorithm



FIR vs IIR

- The main advantage of **IIR** Filters is that they demand a less amount of operations than the **FIR** Filters.

- The main advantage of **FIR** Filters is their *Phase Linearity*.

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Q & A

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