

# Digital Image Formation summary

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# Digital Image Formation

- **Image formation**
- Optical sensors and cameras
- Image digitization
- Image noise

# Digital Image Formation

**An image is the optical representation of an object illuminated by a radiating source.**

Image formation involves:

- Object;
- Radiating/illumination source;
- Image formation system (camera).

Primary image formation model:

- visible light reflected on an object.
- Other modes: X-ray, ultrasound, seismic sources.

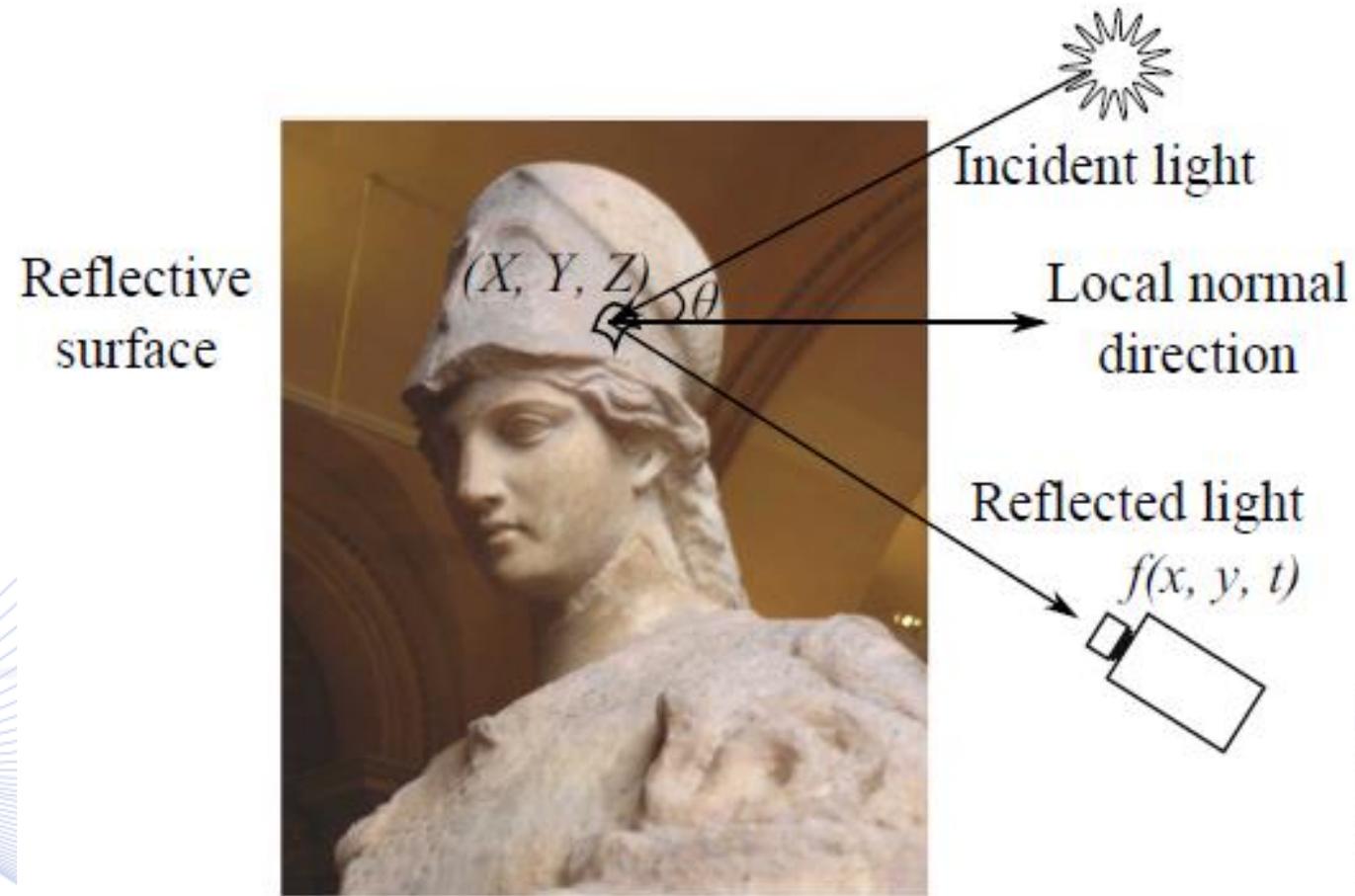
# Light reflection

- Objects reflect or emit light.
- **Reflection** can be decomposed in two components:
  - **Diffuse reflection** (distributes light energy equally along any spatial direction, allows perceiving object color).
  - **Specular reflection** (strongest along the direction of the incident light, incident light color is perceived).
- **Lambertian surfaces** perform only diffuse reflection, thus being dull and matte (e.g., cement surface).

# Light reflection

- ***Ambient illumination*** sources emit the same light energy in all directions (e.g., a cloudy sky).
- ***Point illumination*** sources emit light energy isotropically or anisotropically (e.g., ordinary light bulbs) along various directions.
  - If point illumination sources are far away (e.g., sun), their rays are considered to be parallel.

# Light reflection



Reflection geometry.

# Light reflection

Reflected irradiance when object surface produces diffuse reflectance and incident light source comes from:

- Ambient illumination:

$$f_r(X, Y, Z, t, \lambda) = r(X, Y, Z, t, \lambda)E_\alpha(t, \lambda).$$

- Point light source:

$$f_r(X, Y, Z, t, \lambda) = r(X, Y, Z, t, \lambda)E_p(t, \lambda) \cos \theta.$$

- Distant point source and ambient illumination:

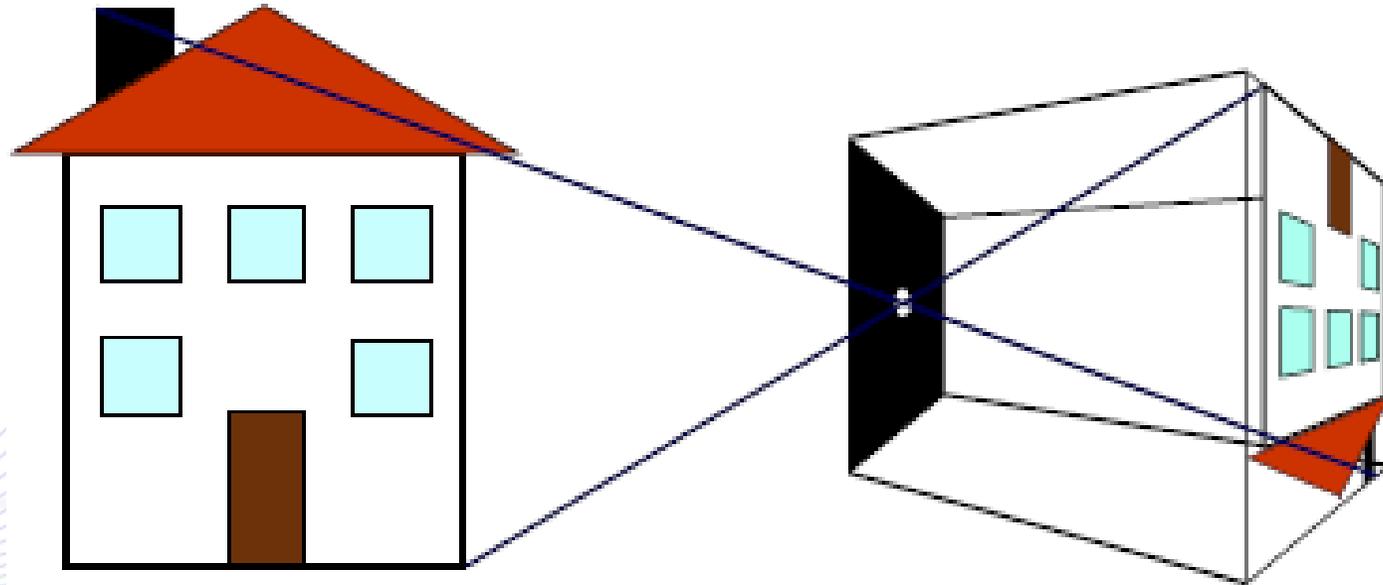
$$E(t, \lambda) = E_\alpha(t, \lambda) + E_p(t, \lambda) \cos \theta.$$

- This is a special case of ***Phong reflection model***.

# Digital Image Formation

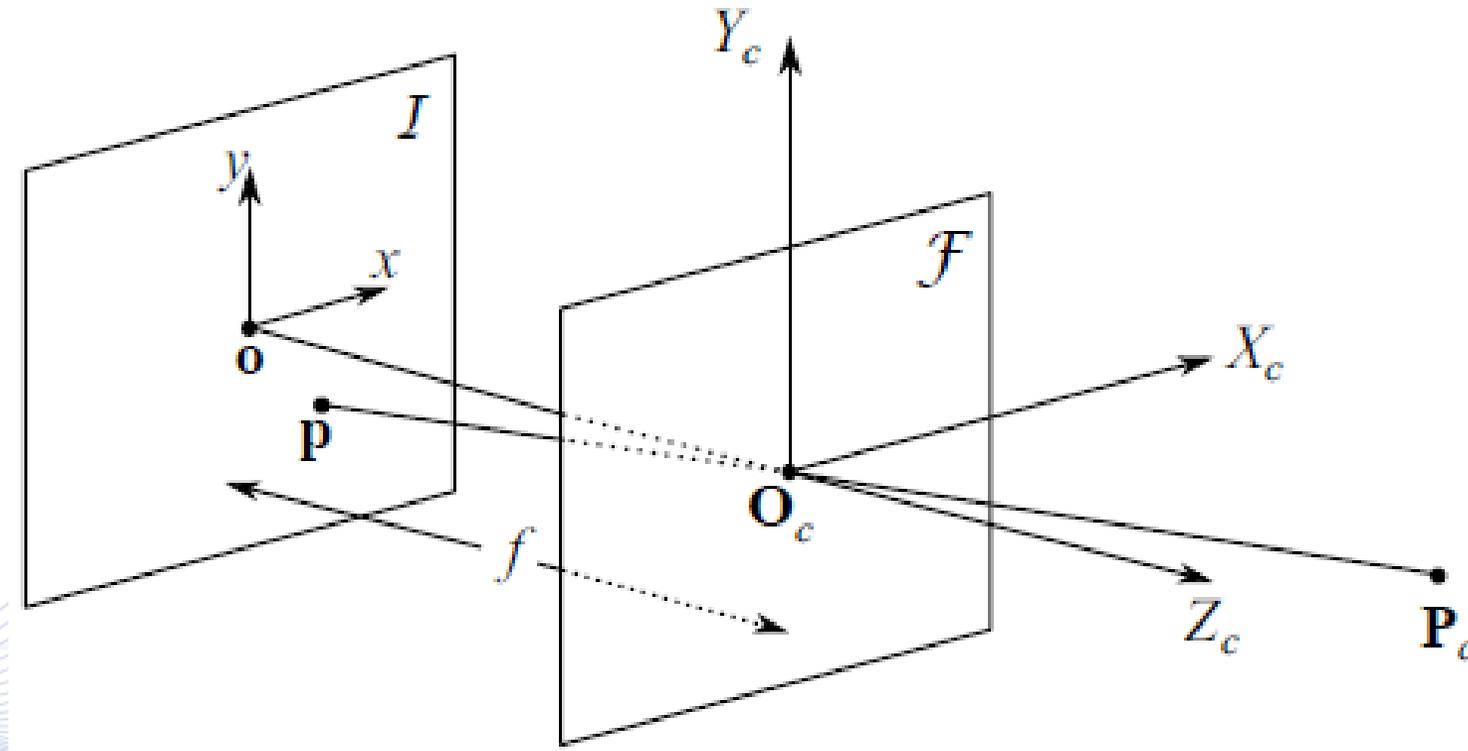
- Image formation
- **Optical sensors and cameras**
- Image digitization
- Image noise

# Pinhole Camera



Pinhole camera geometry.

# Pinhole Camera



Depiction of focal length  $f$ .

# Camera Structure

- There are two kinds of lenses:
  - Fixed focal length (e.g., ***prime lens***) and
  - Variable focal length (e.g., ***zoom lens***).

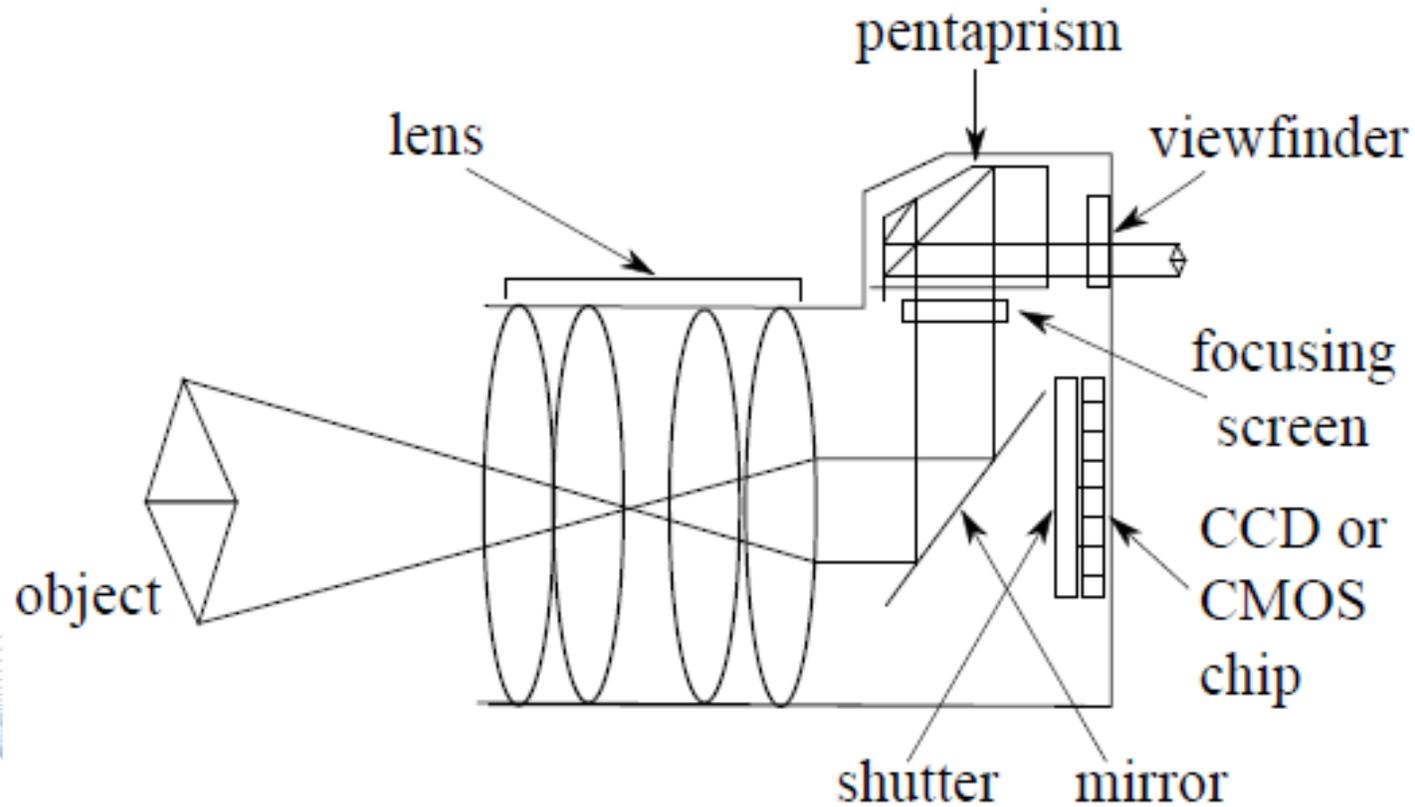
# Camera Structure

- Based on their focal length, lenses are categorized in wide-angle, normal and telephoto lenses:
  - **Wide-angle lens** has smaller focal length than normal, thus capturing wider parts of the scene and exaggerating differences in the relative distance and size between foreground and background objects.
  - **Telephoto lens** has larger focal length and can take photos from a distance.

# Camera Structure

- Aperture size is usually expressed in ***f-numbers***. The bigger the f-number the smaller the aperture size.
- It controls the ***Depth of Field (DOF)***, the distance between the nearest and farthest focused objects in the image.
- The smaller the aperture size is, the longer the depth of field, since less light rays are captured on the image for each visible 3D scene point.

# Camera Structure



Camera structure.

# Camera Structure



**Color temperature** of a light source is the temperature of a black body (expressed in Kelvin) that emits a light of the same hue to that of the light source.

- Light sources range from warm ones (~1000 K) to cool ones (up to 10000 K).

Temperature In Kelvins	Light Source
1000 – 2000 K	Candle Light
2500 – 3500 K	Tungsten Bulb
3000 – 4000 K	Sunrise/Sunset (clear sky)
4000 – 5000 K	Fluorescent lamps
5000 – 5500 K	Electronic Flash
5000 – 6500 K	Daylight with clear sky
6500 – 8000 K	Moderately overcast sky
9000 – 10000 K	Shade or heavily overcast sky

# Camera Structure



Image before white balance.

Image after white balance.

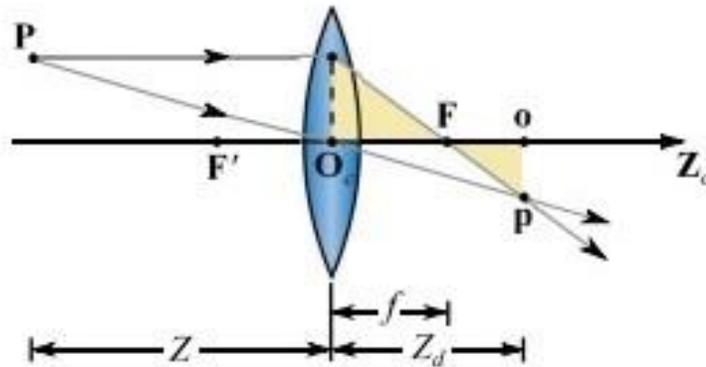
# Camera Model



Image acquisition model.

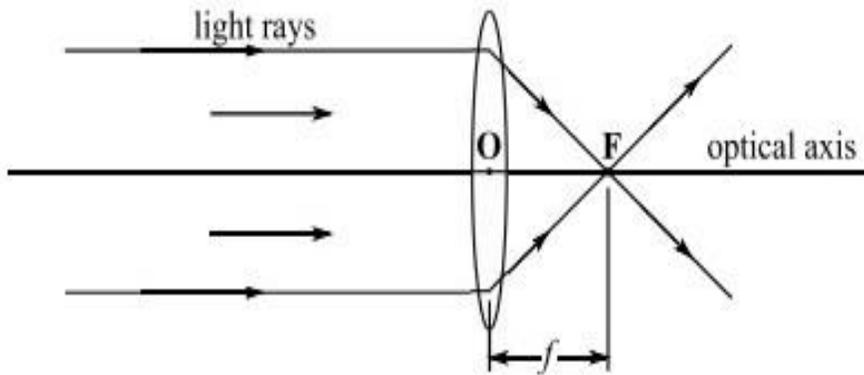
# Optical Lens Models

- The simplest optical systems use **thin lenses**. There are two dominant types of thin lenses, the *converging* and the *diverging* ones.

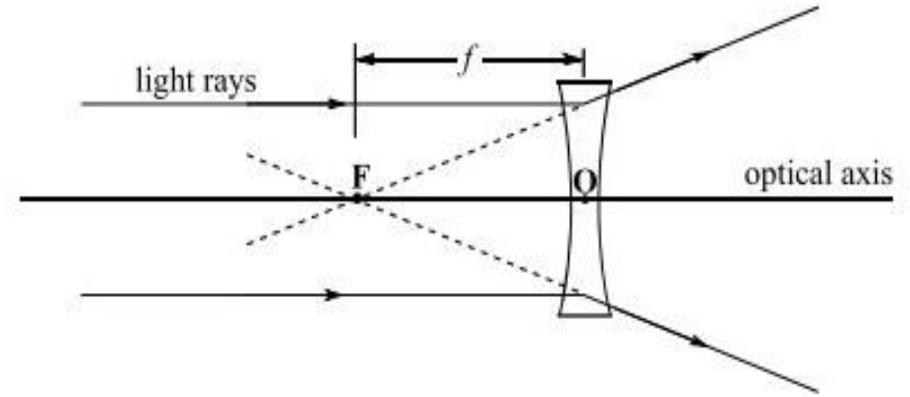


Graphical model of a thin lens.

# Optical Lens Models



Converging thin lens.



Diverging thin lens.

# Optical Lens Models



## *Fundamental Equation of Thin Lenses:*

$$\frac{1}{Z} + \frac{1}{Z_d} = \frac{1}{f},$$

- $Z$  is the distance of a scene point  $\mathbf{P}$  from the lens along the optical axis.
- $Z_d$  is the distance of its corresponding focused image point  $\mathbf{p}$  from the lens along the optical axis.
- $f$  is the lens focal length.

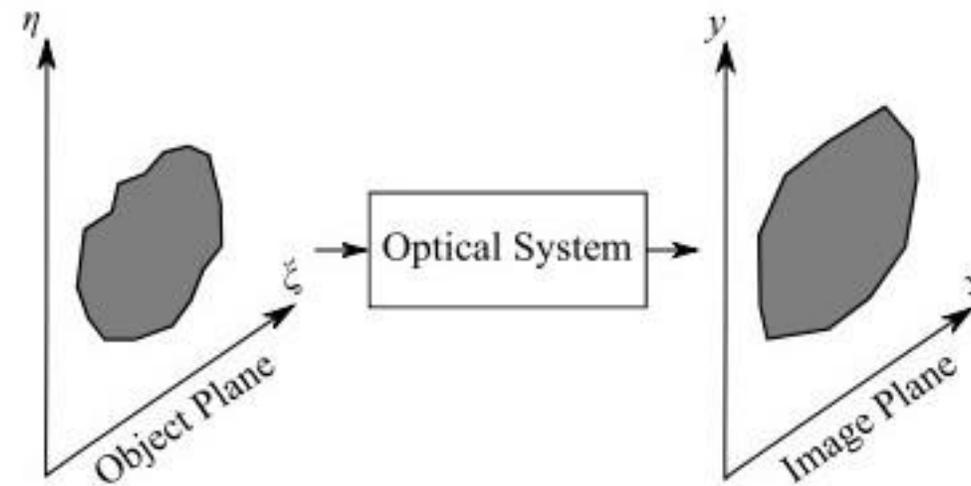
# Optical Lens Models

- For a thin double-convex lens, having a small diameter in comparison to its focal length,  $f$  is given by the famous ***Lensmaker's equation***.

$$\frac{1}{f} = (n - 1) \left( \frac{1}{R_1} + \frac{1}{R_2} \right),$$

- $n$  is the lens material refractive index.
- $R_1, R_2$  are the radii of the front and rear spherical lens surfaces.

# Image Formation Models



Optical system input and output images.

# Image Formation Models

Both signals  $f(\xi, n)$ ,  $b(x, y)$  represent optical intensities:

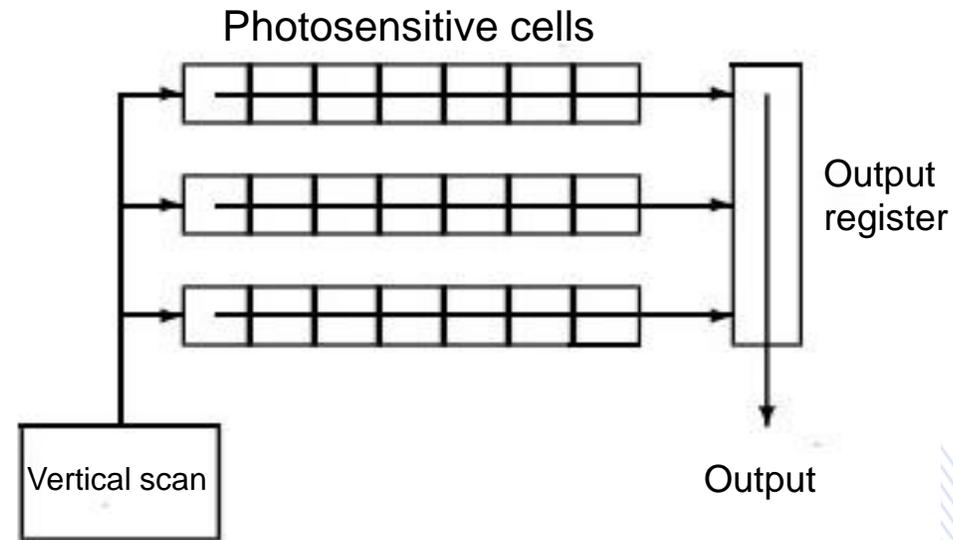
- They must take non-negative values, therefore:

$$f(\xi, n) \geq 0, \quad b(x, y) \geq 0.$$

The input-output relation of the optical subsystem is given by a **2D convolution**:

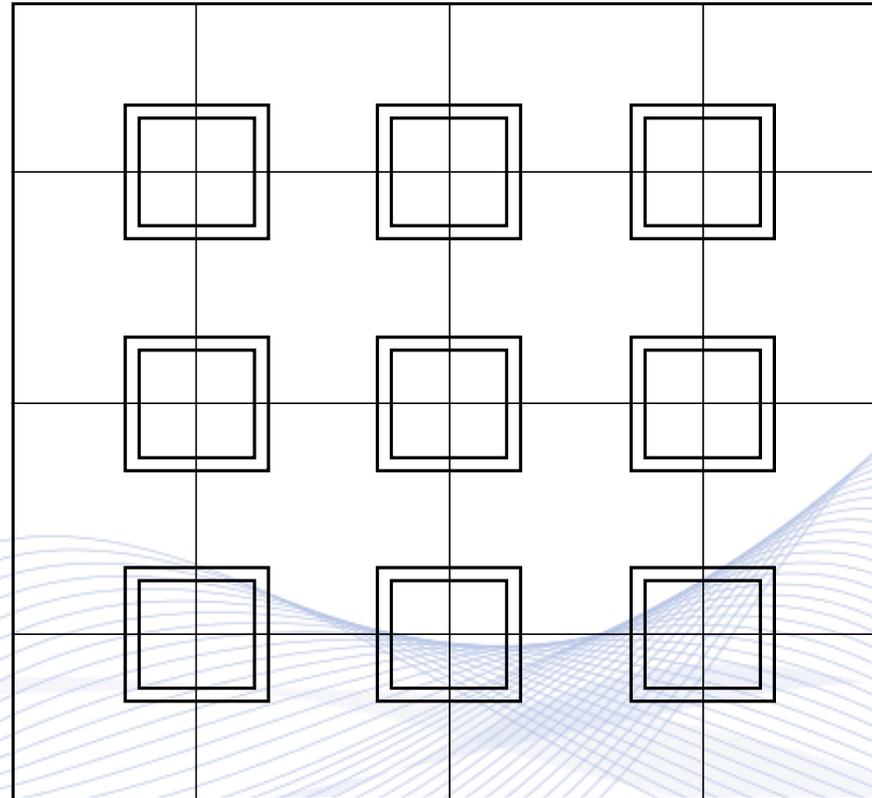
$$b(x, y) = \iint_{-\infty}^{\infty} f(\xi, n) h(x - \xi, y - n) d\xi dn.$$

# Optical sensors



CCD Camera Structure.

# Optical sensors



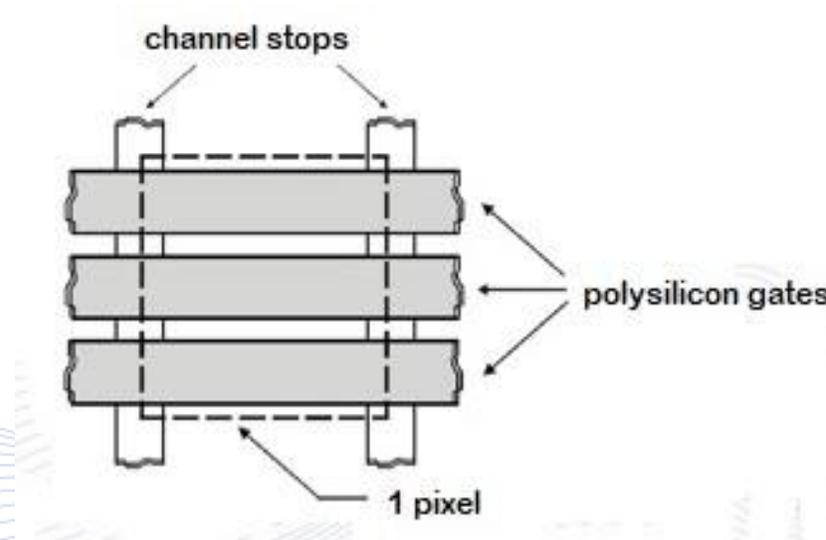
Square CCD cell grid.

# Optical sensors



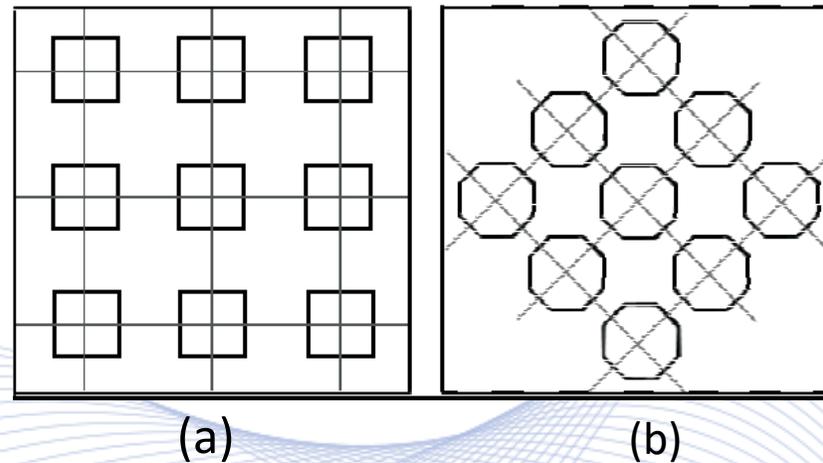
- **Charge-Coupled Device (CCD)** is the most popular optical sensor technology.
- A three-phase CCD pixel consists of three polysilicon gates vertically oriented towards two channel stops.
- There are some CCD structures which use one, two or four polysilicon gates to define a pixel.
- CCD grid topology can be square or orthorhombic.

# Optical sensors



Three-phase CCD cell.

# Optical sensors



CCD cell grid topologies: (a) square; (b) orthorhombic.

# Optical sensors



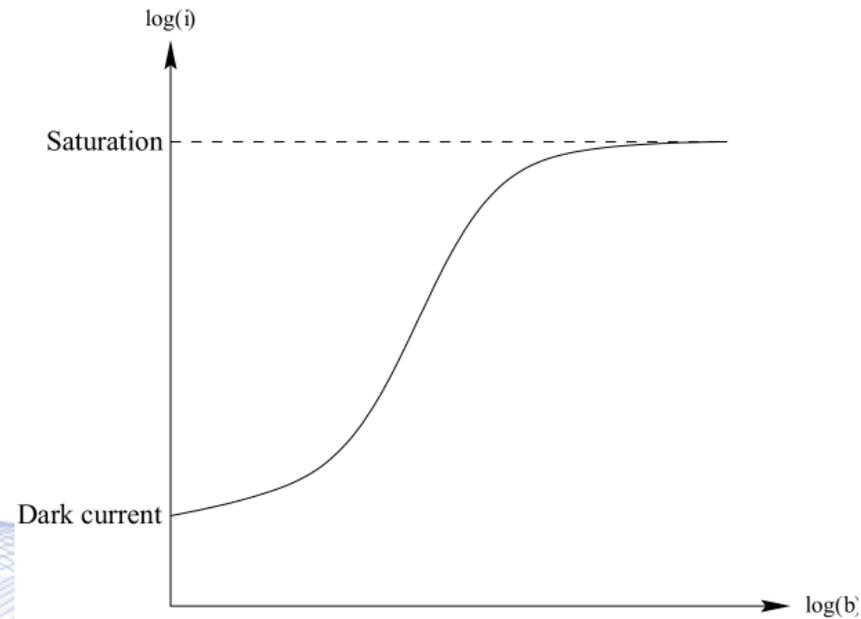
- A simplified image recording model with a CCD sensor follows the form:

$$i = g^\gamma (b + n)^\gamma,$$

- $b, i$ : input and output (recorded) image brightness.
- $g$ : **sensor gain** (can be set automatically).
- $\gamma$ : sensor  **$\gamma$  coefficient** determining sensor nonlinearity. It can be evaluated for each sensor. In many cameras, it is in the range  $[0.55, 1.00]$ .
- $n$ : CCD noise.

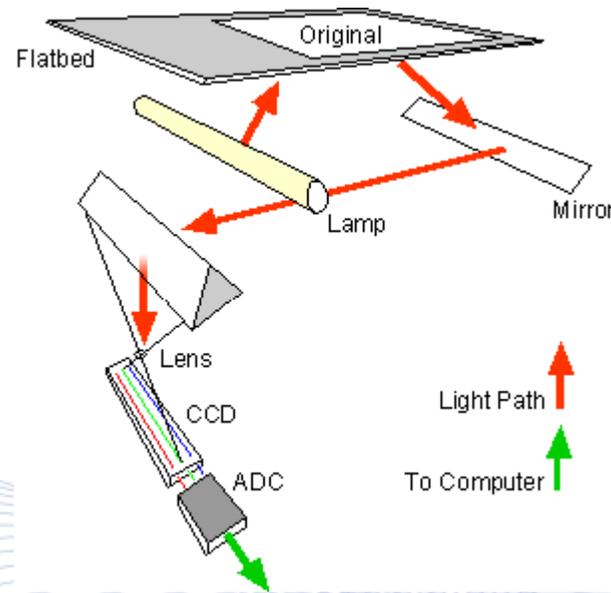
# Optical sensors

- $b$ : input luminance.
- $i$ : output current.



Characteristic curve of the light sensor.

# Image scanners



Scanner structure [CIR].

# $\gamma$ Correction



Before gamma correction.



After gamma correction.

# Digital Image Formation

- Image formation
- Optical sensors and cameras
- **Image digitization**
- Image noise

# Digital Image Formation

## ***Sampling and digitization:***

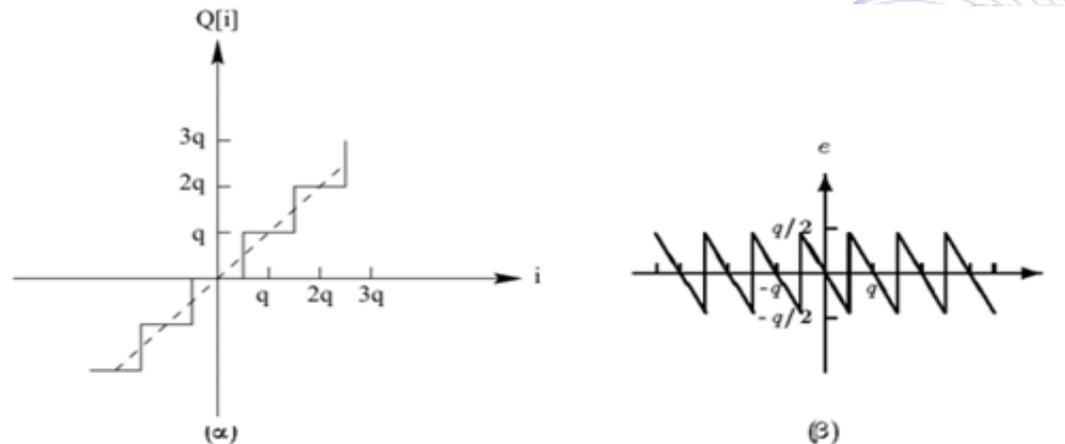
- They are performed by an  $A/D$  converter (in a frame grabber).
- It transforms the analog image  $i(x, y)$  to a digital image:

$$i(n_1, n_2) = i(n_1T_1, n_2T_2), \quad n_1 = 1, \dots, N, \quad n_2 = 1, \dots, M.$$

# Image digitization

## Quantization:

- It is performed by the  $A/D$  converter;
- $q$ : quantization step  $q = \frac{1}{2^b}$ ;
- quantized image illumination levels  $kq, k = 0, 1, 2, \dots, 2^b - 1$ .



a) Input-output curve of quantizer; b) Quantization error.

# Image digitization

- **Grayscale** images are quantized at 256 levels:
  - 1 byte (8 bits) for pixel representation.
- **Binary** images have only two quantization levels: {0,1}.
  - They are represented with 1 bit per pixel.

# Image digitization



(a)



(b)



(c)



(d)

(a) Original image at 256 grayscale levels; (b) 64 levels;  
(c) 8 levels; (d) 2 levels.

# Probabilistic Image Description

- In many cases, it is useful to estimate a probabilistic characterization of an image. In this case we assume that each pixel is a random variable.
- The image vector has a probability distribution of the form:

$$p(\mathbf{i}) = p\{i(1,1), \dots, i(N, M)\}.$$

- If the probability distribution is Gaussian, it is of the form:

$$p(\mathbf{i}) = (2\pi)^{-\frac{NM}{2}} |C_i|^{-\frac{1}{2}} e^{\{-\frac{1}{2}(\mathbf{i}-\mathbf{m}_i)^T C_i^{-1}(\mathbf{i}-\mathbf{m}_i)\}}.$$

# Digital Image Formation

- Image formation
- Optical sensors and cameras
- Image digitization
- **Image noise**

# Image generation noise

- Digital image corruption by noise:
  - during a) image acquisition or b) image transmission.
- Image acquisition noise:
  - photoelectronic noise;
  - film-grain noise (signal-dependent).
- Image formation model:

$$g(x, y) = c(f(x, y))^{\gamma} n(x, y).$$

# Image generation noise

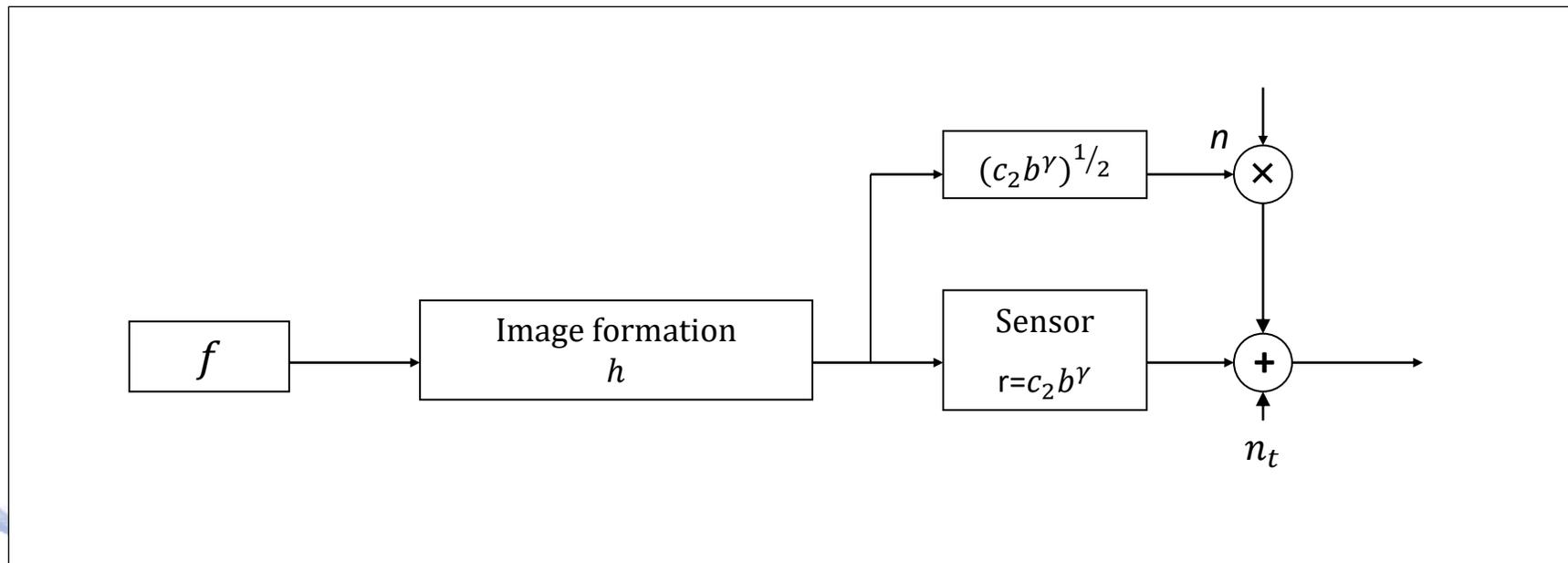


Image formation model.

# Image noise generators

Artificial noise generation is needed for:

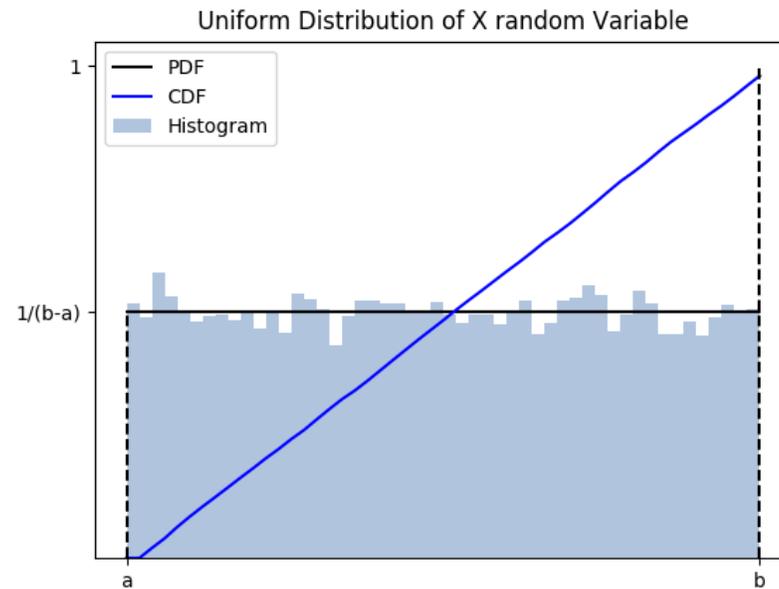
- Simulations
- Certain artistic applications (e.g. for the simulation of film-grain noise).
- Noise  $n(i, j)$  generation:
  - Random number generators: uniform noise in  $[0,1]$ .
  - Noise transformation to a different pdf, e.g., Gaussian or Laplacian.

# Image noise generators



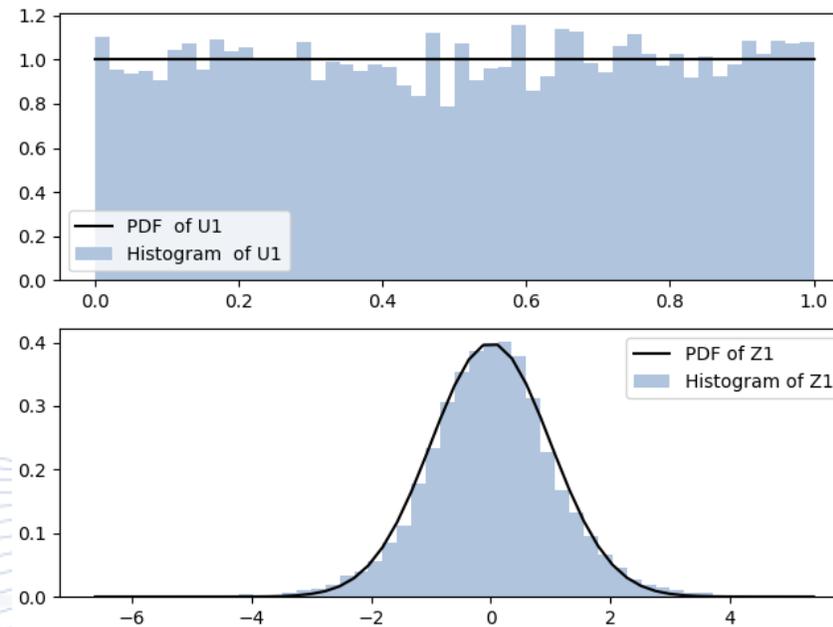
(a) Original image; (b) Image corrupted by additive Gaussian noise; (c) Image corrupted by multiplicative Gaussian noise; (d) image corrupted by salt-pepper noise.

# Image noise generators



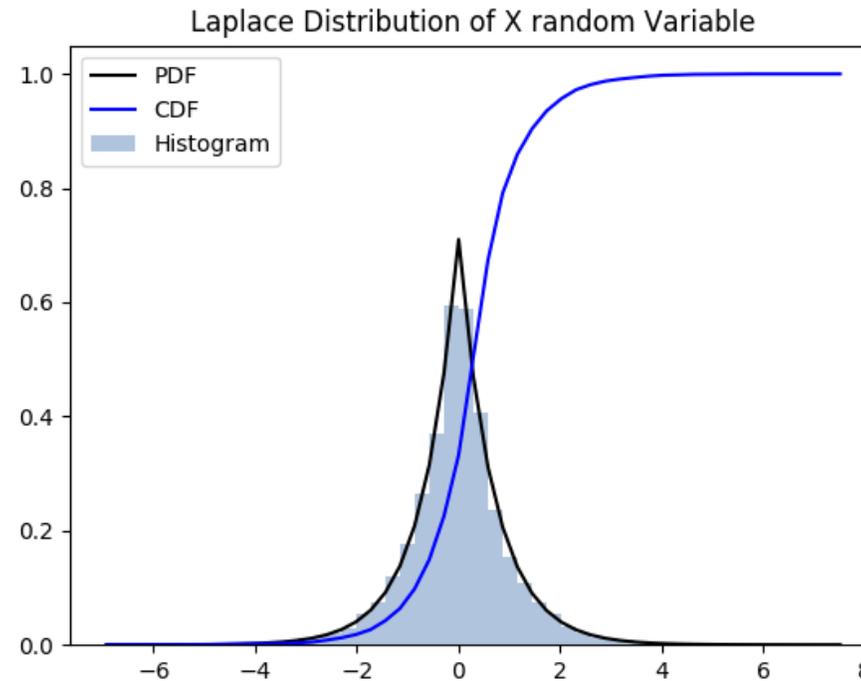
Uniform random number generation in  $[0,1]$ .

# Image noise generators



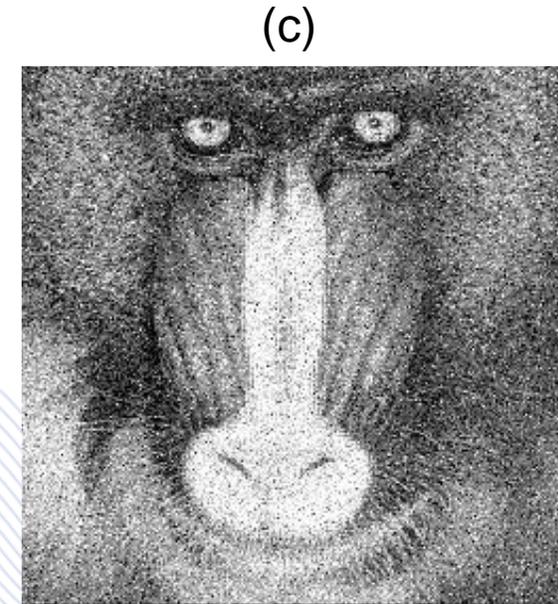
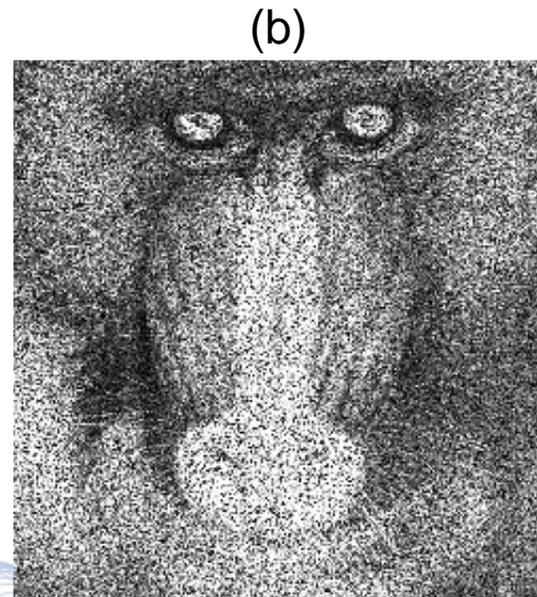
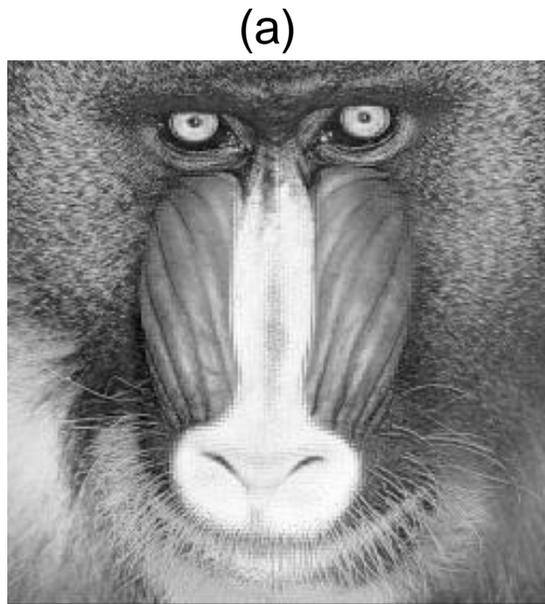
Gaussian random number generation using uniform distribution.

# Random Number Generation



Laplacian random number generator histogram.

# Image noise generators



(a) Original image; (b) Image corrupted by multiplicative Gaussian noise; (c) image corrupted by additive Laplacian noise.

# Bibliography

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# Q & A

**Thank you very much for your attention!**

**More material in  
<http://icarus.csd.auth.gr/cvml-web-lecture-series/>**

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