Digital Image Filtering summary

C. Georgiadis, Prof. Ioannis Pitas
Aristotle University of Thessaloniki
pitas@csd.auth.gr
www.aiia.csd.auth.gr

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Artificial Intelligence & Information Analysis Lab

Digital Image Filtering

- **Image noise**
- 2D FIR filters
- Moving average filters
- Spatial filters
- Median filters
- Digital filters based on order statistics
- Adaptive order statistic filters
- Anisotropic Diffusion
- Image interpolation

Image noise

- ***White additive noise***:

$$x(i,j) = s(i,j) + n(i,j),$$

- ***White multiplicative noise***:

$$x(i,j) = s(i,j)n(i,j),$$

- ***White signal-dependent noise***:

$$x(i,j) = s^{\gamma}(i,j)n(i,j),$$

- Noise can have various distributions: Gaussian, uniform, Laplacian.

Image noise

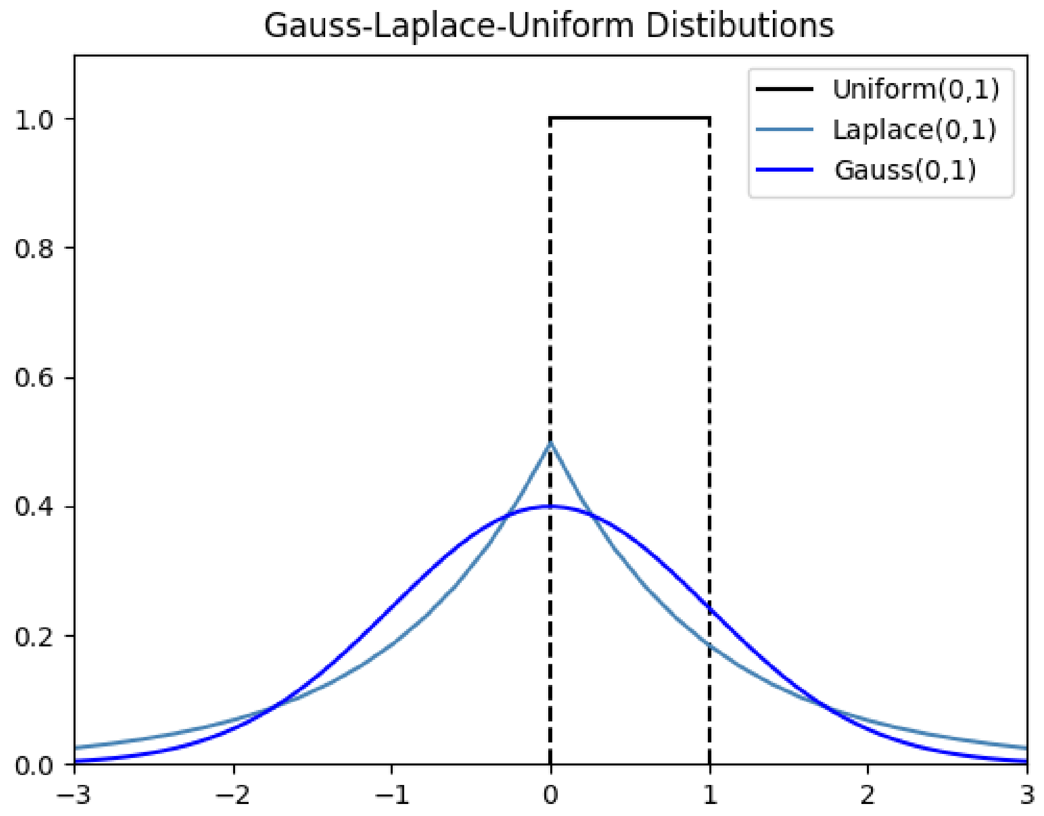
- ***Salt-pepper noise*** consists of black and/or white image impulses:

$$g(i,j) = \begin{cases} z(i,j), & \text{with probability} \quad p. \\ f(i,j), & \text{with probability} \quad 1-p. \end{cases}$$

Image noise

- Uniform noise has a ***short-tailed*** probability distribution.
- Laplacian noise has a ***long-tailed*** probability distribution.

- Gaussian noise is at the borderline between long- and short tailed probability distributions.



from [PIT2000].

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2D FIR Digital Filters

The output of a 2D FIR filter is given by a ***linear convolution***:

$$y(n_1, n_2) = \sum_{k_1=0}^{M_1-1} \sum_{k_2=0}^{M_2-1} h(k_1, k_2) x(n_1 - k_1, n_2 - k_2).$$

for a ***filter window*** (region of support) $[0, M_1 - 1] \times [0, M_2 - 1]$.

- For centered filter window $[-v_1, v_1] \times [-v_2, v_2]$, $M_i = 2v_i + 1$, $i = 1, 2$:

$$y(n_1, n_2) = \sum_{k_1=-v_1}^{v_1} \sum_{k_2=-v_2}^{v_2} h(k_1, k_2) x(n_1 - k_1, n_2 - k_2).$$

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2D FIR Digital Filters

Moving Average filter:

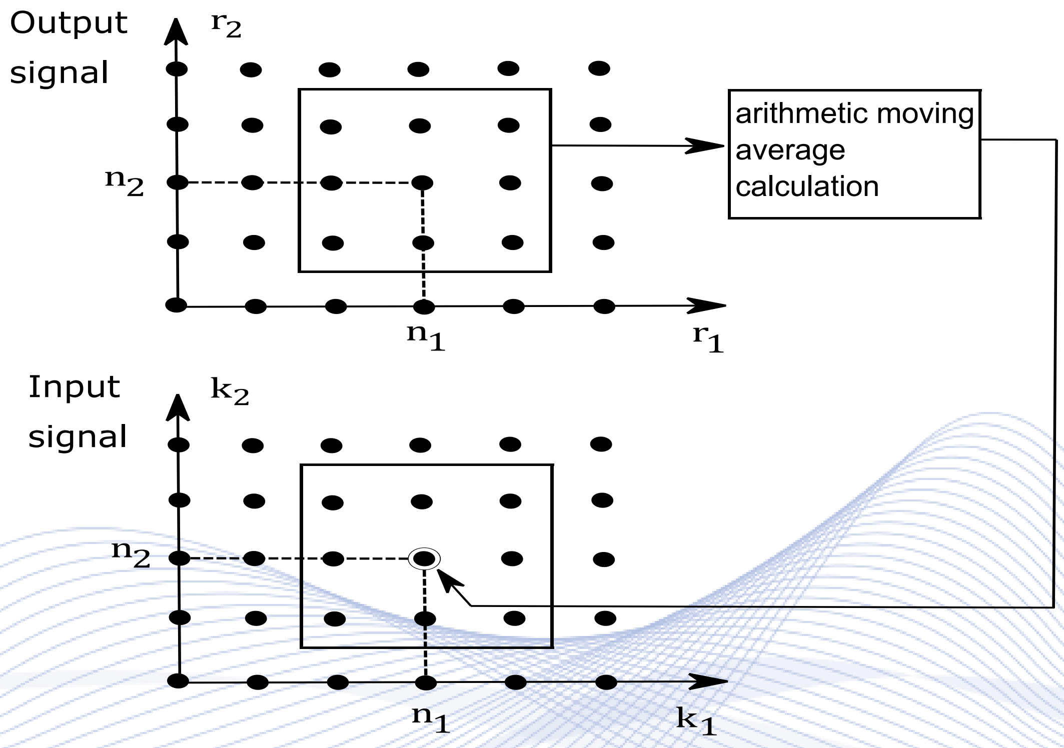
$$y(n_1, n_2) = \left(\frac{1}{M_1 M_2}\right) \sum_{k_1=-\nu_1}^{\nu_1} \sum_{k_2=-\nu_2}^{\nu_2} x(n_1 - k_1, n_2 - k_2),$$

where $M_i = 2\nu_i + 1,\ i = 1, 2$.

Properties:

- It is a linear FIR ***low-pass filter***.
- It tends to blur edges and image details (e.g., lines, fine texture).
- It degrades image quality, particularly for large filter windows.

Moving Average Filter

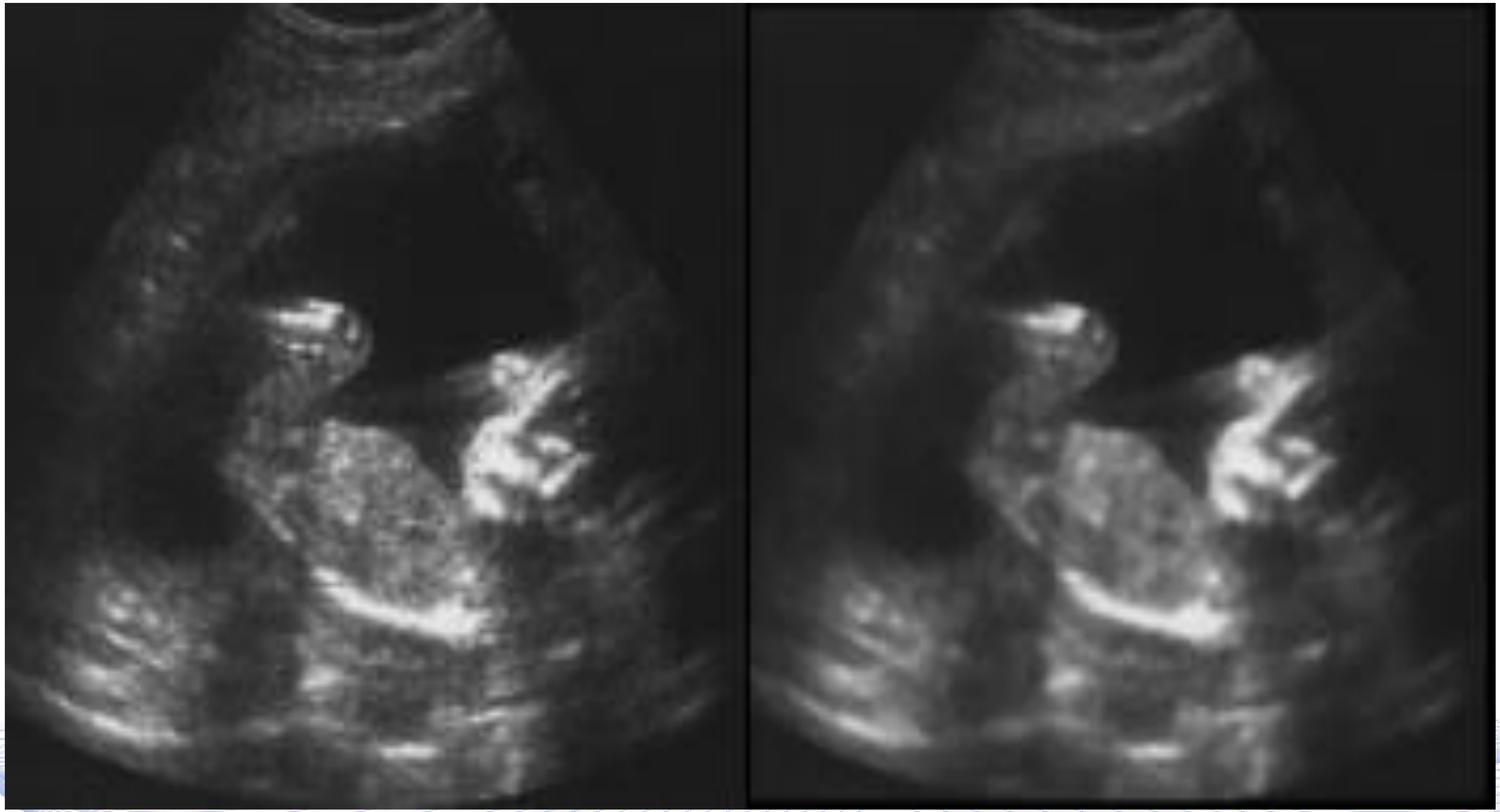


3×3 arithmetic moving average filter structure.

Moving Average Filter

5×5 moving average image filtering [PIT2000].

L_p Mean Filter



a) Ultrasound image; b) Output of an L_2 filter [PIT2000].

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Spatial Filters

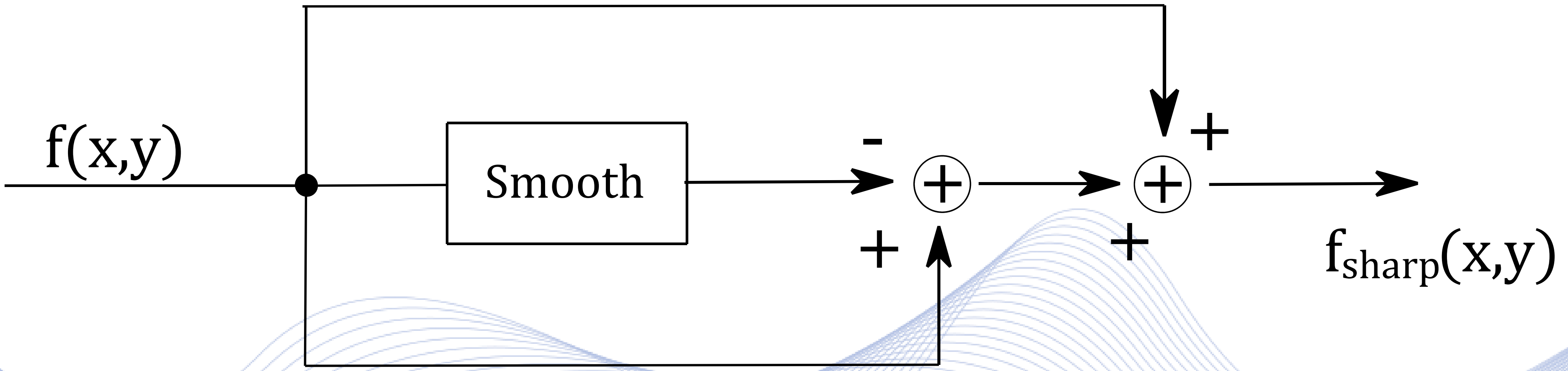
$$\frac{1}{273}$$

1	4	7	4	1
4	16	26	16	4
7	26	41	26	7
4	16	26	16	4
1	4	7	4	1

5×5 discrete approximation of a Gaussian kernel for $\sigma = 1$.

Spatial Filters

Unsharp Filter



Block diagram of the unsharp filter.

Spatial Filters

Conservative smoothing assumes that noise has a high spatial frequency.

- It can be attenuated by a local operation which ensures pixel intensity consistency in local image neighborhoods.
- It ensures that pixel intensities are bounded within the intensity ***range*** of its neighbors, defined by their ***minimum*** and ***maximum*** intensity values.
- If the central pixel intensity lies within the intensity range of its neighbors, it remains unchanged.
- If it is greater/smaller than the maximum/minmum value, it is set equal to the maximum/minimum value, respectively.

Spatial Filters

Conservative smoothing

- The central pixel intensity is 150, so it will be replaced with the maximum intensity value (127) of its 8 nearest neighbors.

123	125	126	130	140
122	124	126	127	135
118	120	150	125	134
119	115	119	123	133
111	116	110	120	130

Conservative smoothing in a local pixel neighborhood.

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Median Filters

2D median filter:

$$y(i,j)=\text{med}\{x(i+r,j+s),(r,s)\in A,\ (i,j)\in Z^2\}.$$

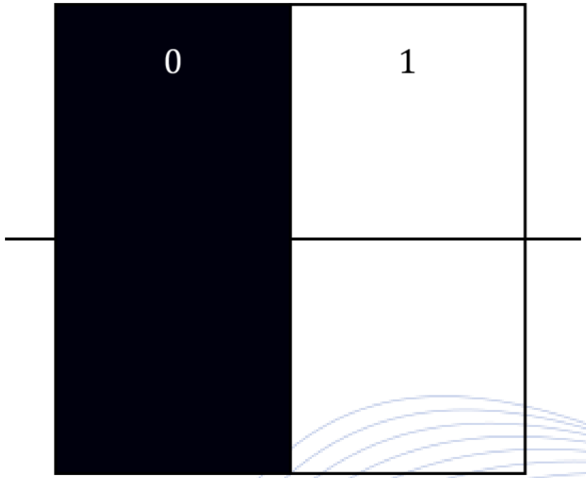
Median filter properties:

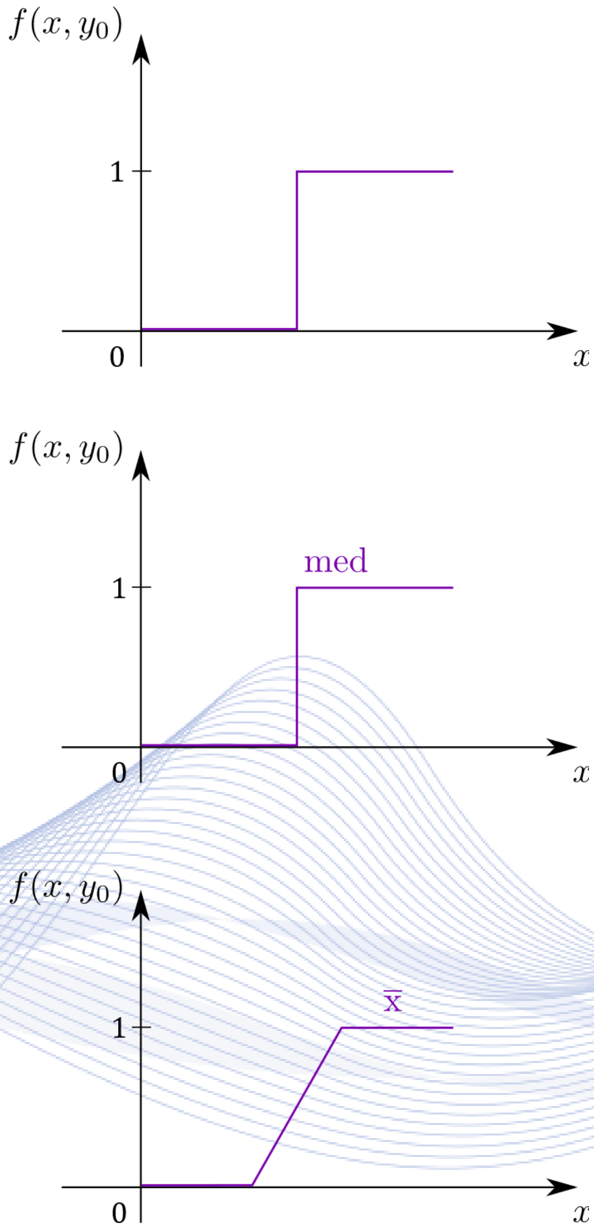
• They have low-pass characteristics and remove additive white noise.

• They are very efficient in the removal of:

- impulsive noise,
- noise with long-tailed distribution (e.g., having Laplacian distribution).

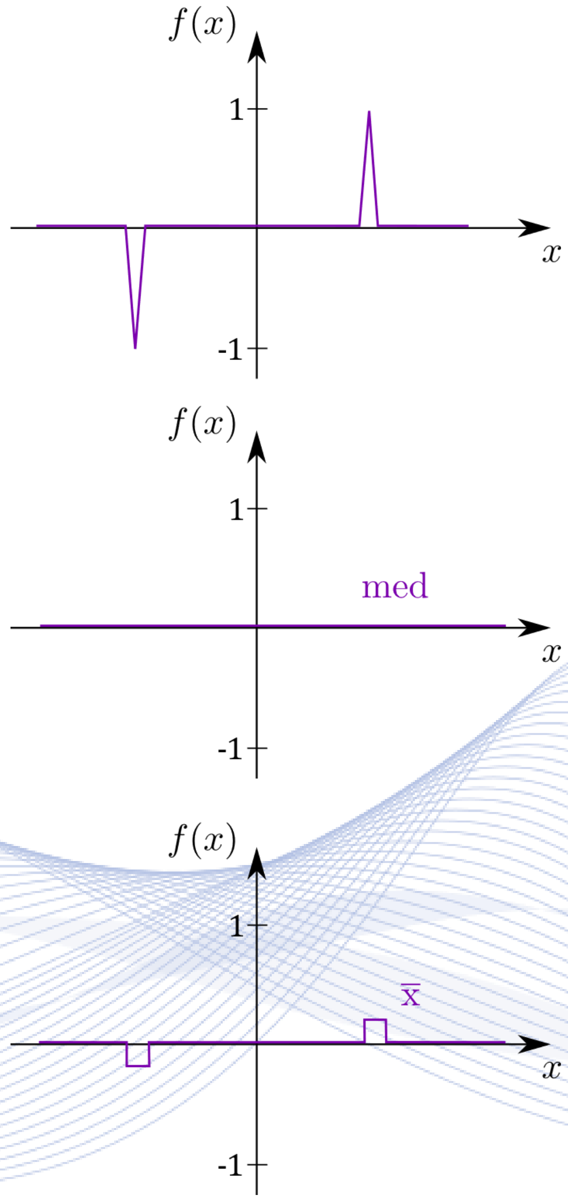
Median Filters





Edge filtering

Median Filters



Impulsive noise filtering

Median Filters

a) Baboon image corrupted by mixed impulsive noise;
b) 7×7 median filter output; c) 7×7 moving average filter output [PIT2000].

Median Filters

Weighted median is the estimator T that minimizes the weighted L_1 norm:

$$\sum_{i=1}^{n} w_i |x_i - T| \rightarrow \min.$$

It is described by:

$$y_i = \mathrm{med}\{w_{-v} \diamond x_{i-v}, \ldots, w_v \diamond x_{i+v}\},$$

where $w \diamond x$ denotes duplication of x, w times to $\{x, \ldots, x\}$.

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Order Statistics Filters

Ranked order filters:

An $r\text{–th}$ ranked filter y_i output is the $r\text{– th}$ order statistic of signal x_i samples $\{x_{i-v}, \ldots, x_i, \ldots, x_{i+v}\}$, $n = 2v + 1$ that exist in a ***running filter*** window.

• It introduces a strong bias in the estimation of the mean, when the rank is small or large (tending to ***min*** or ***max filters***).

• The bias is even stronger when the input data have a long-tailed distribution.

Order Statistics Filters

Max/min filters:

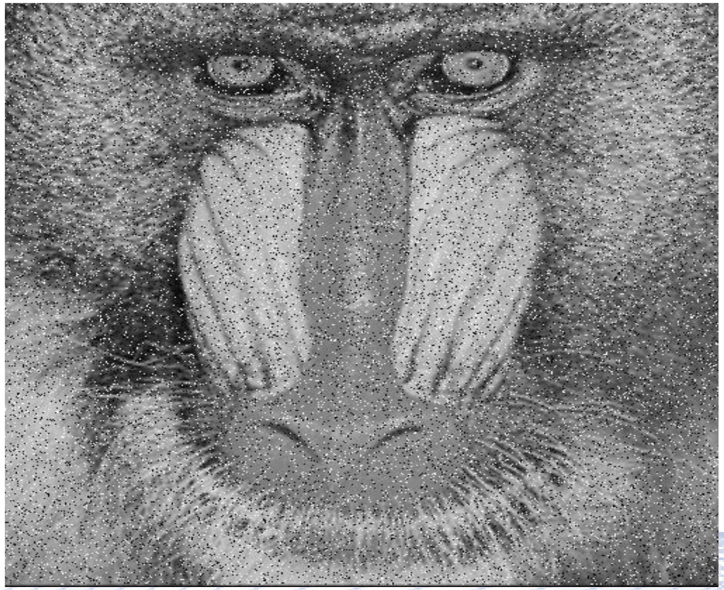
Running maximum $x_{(n)}$ and ***minimum*** $x_{(1)}$ are the two extremes of the ranked-order filters.

- Maximum filter effectively removes negative impulses in an image.
- Minimum filter removes positive impulses.

- Both filters fail in the removal of mixed impulse noise.
- Both filters have good edge preservation properties (but shift edges).
- Max/min filters tend to enhance bright and dark image regions, respectively.

Order Statistics Filters

Max/min filters



a) Baboon image corrupted by mixed impulsive noise;
b) The output of a cascade of a min and max filter [PIT2000].

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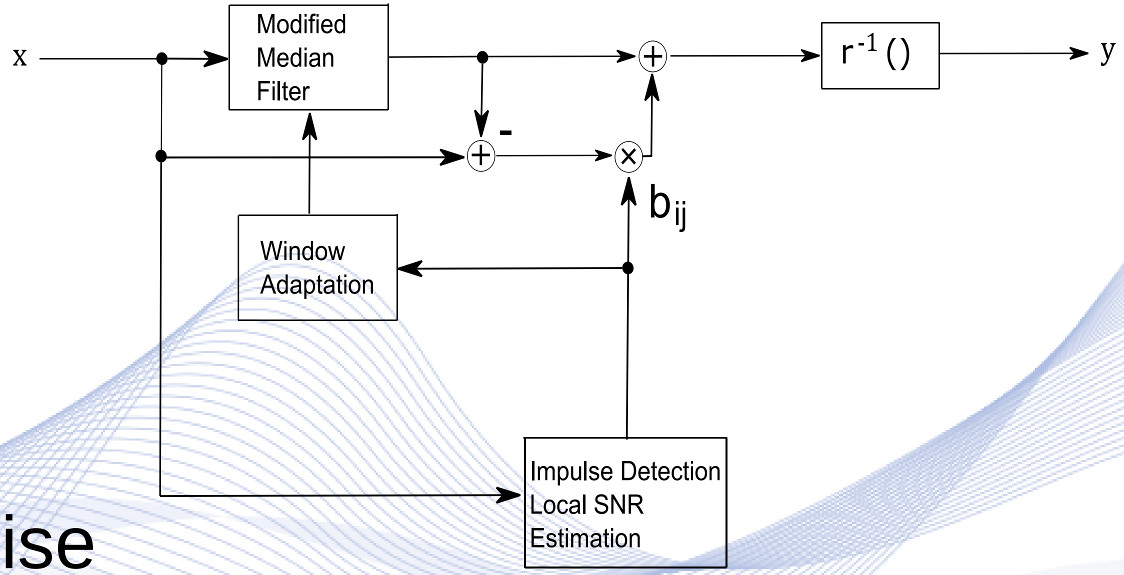
Adaptive Order Statistic Filters

Signal-adaptive median (SAM) filter:

- It is an adaptive filter based on the two-component image model:

$$y_{1ij} = \widehat{m}_x + b_{ij}(x_{ij} - \widehat{m}_x).$$

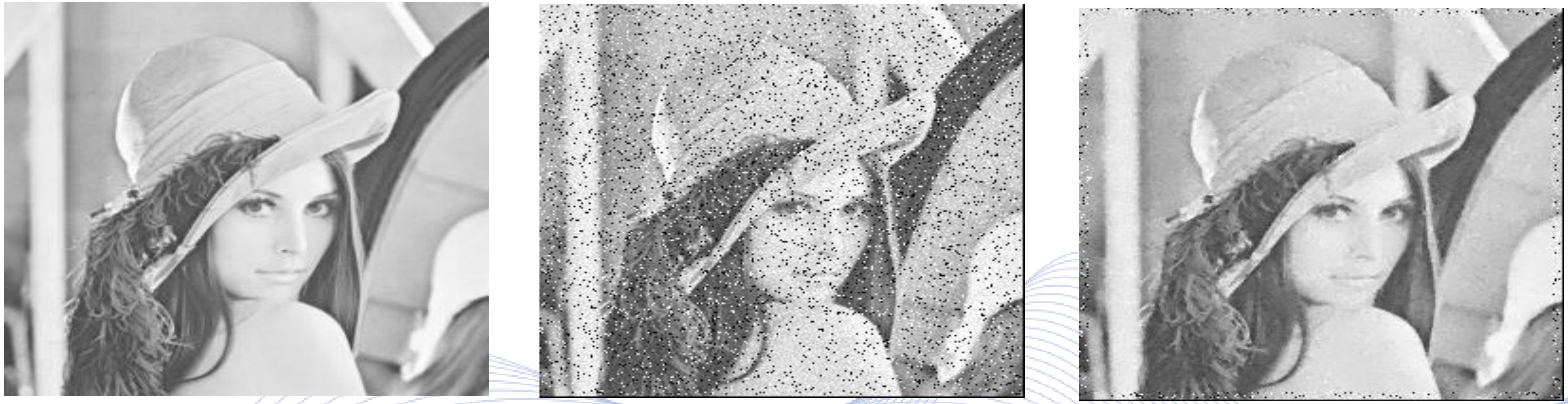
$$y_{ij} = r^{-1}(y_{1ij}).$$



- It has excellent performance in noise filtering, edge and image detail preservation.

Adaptive Order Statistic Filters

Two-component model filters



a) Original image;
b) Image corrupted by Gaussian noise (variance=100) and mixed impulsive noise; c) SAM filter output [PIT2000].

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Anisotropic Diffusion

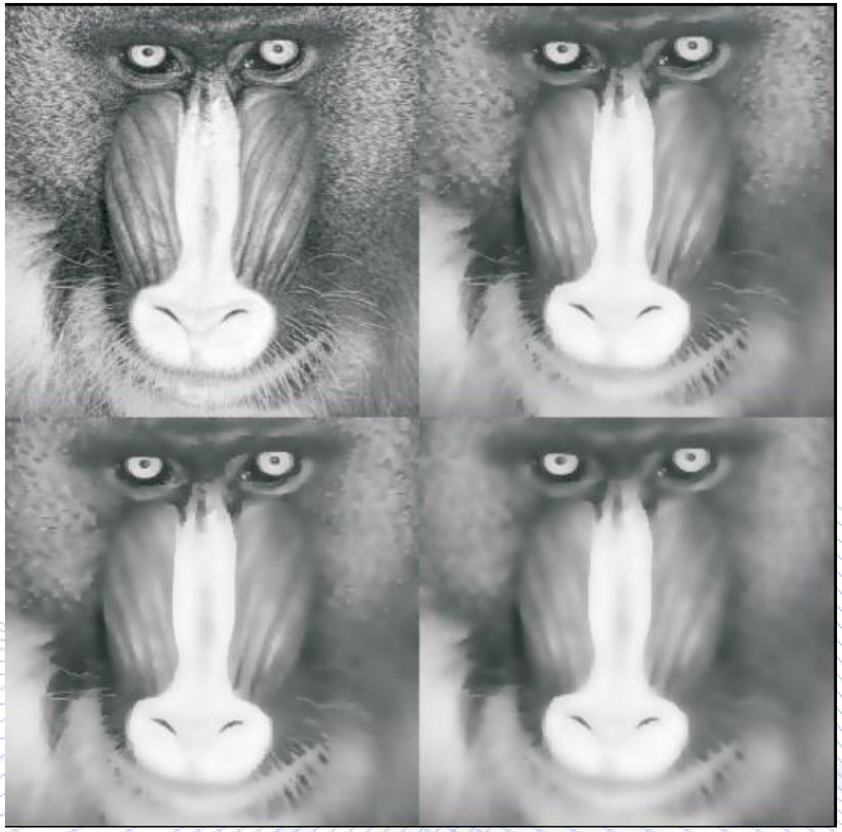
Image intensity $f(i,j)$ can be considered as ***pixel temperature*** that can be diffused over the entire image domain, in an iterative process described by $f(i,j,t)$ over various steps t.

Isotropic diffusion filtering can perform image smoothing:

$$\frac{\partial f}{\partial t} = c\,\mathrm{div}(\nabla f) = c\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right).$$

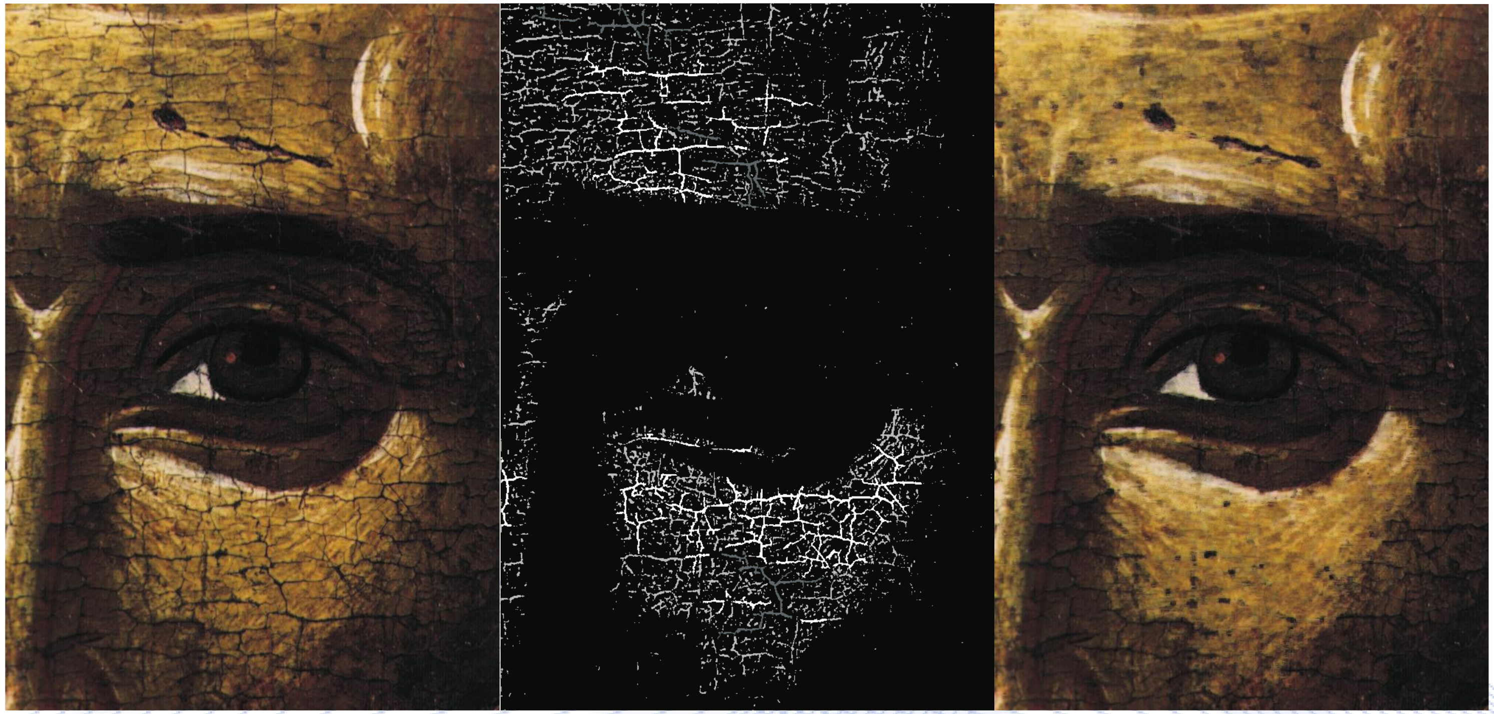
- c: diffusion coefficient.
- Diffusion is also used for image segmentation.

Anisotropic Diffusion



a) Original image; b-d) Various anisotropic diffusion iterations.

Anisotropic Diffusion



a) Original Byzantine painting with cracks.
b) Localized cracks.
c) Filled cracks using anisotropic diffusion.

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Image Interpolation

Image interpolation is an important operation with many applications:

- Image zooming (e.g., for video games)
- Image upsampling (e.g., in neural autoencoders or in neural semantic region segmentation.
- Image magnification/upsampling.
- Video format conversion.

Image Interpolation

Zero-order (hold) interpolation: pixel (x, y) is assigned the value of the geometrically closest pixel in the image array:

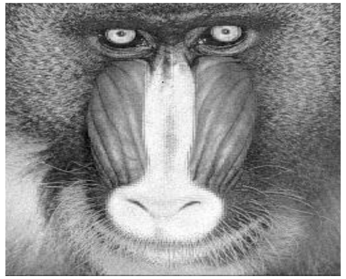
$$f_i(n_1, n_2) = f([n_1/2], [\ n_2/2]).$$

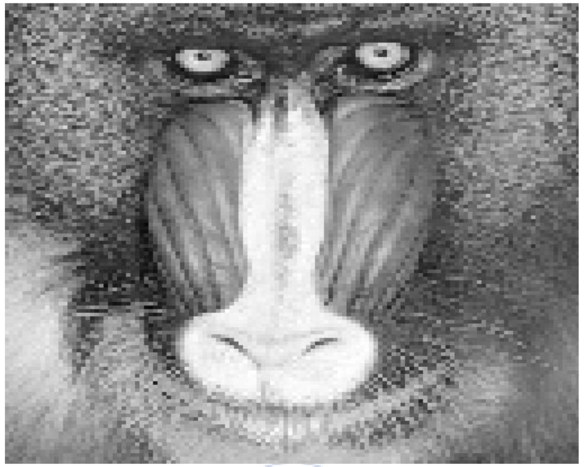
- Repeated application: zooming by a factor of $2^n \times 2^n$.
  - For large n, regions of constant intensity (image blobs) are visible.

- It is sometimes used in video effect creation.

Image Interpolation

BABOON Image.





Zero-order interpolation.

Linear Interpolation.

Cubic spline Interpolation.

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Q & A

Thank you very much for your attention!

More material in
http://icarus.csd.auth.gr/cvml-web-lecture-series/

Contact: Prof. I. Pitas
pitas@csd.auth.gr