Digital Image Compression summary

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Version 4.4

Artificial Intelligence & Information Analysis Lab

Digital Image Compression

- Introduction
- Huffman coding
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Introduction

Digital image compression:

- Techniques and algorithms reducing the memory needed to represent and store digital images.

Compression factors:

- Storage and transmission of digital images.
- Bit-rate reduction during transmission.

Compression ratio is the ratio of the sizes of the compressed and the original image.

Lossless Compression

Lossless compression:

- No loss of information.
- Reduction of number of bits required for the original image representation, by eliminating ***statistical image redundancy***.

It is used when:

- raw image data are difficult to obtain or
- images contain vital information that may be destroyed by compression, e.g., in forensics or medical diagnostic imaging.

Lossless Compression

- ***Pros***: The decompressed image is numerically exactly the same as the original image.

- ***Cons***: Compression ratio is not very big (e.g., up to 1:4).

Lossy Compression

Lossy compression

- Inflicts ***distortion*** on the original image, up to an ***allowable*** level.
- Distortion level determines the compression ratio.

It is used when:

- raw image data can be easily produced or
- information loss can be tolerated at the receiver site, e.g., in Digital Television, Teleconferencing.

Lossy Compression

- ***Pros***: It can offer a very good compression ratio (e.g., 1:100), by adjusting the compressed image distortion level appropriately.

- ***Cons***: The compressed image distortion is sometimes perceivable.

Lossy Compression

Distortion $D(x, \hat{x})$ between the original image $x(n_1, n_2)$ and the reconstructed image $\hat{x}(n_1, n_2)$ can be measured in various ways.

- The default image ***distortion measure*** is the ***Mean Squared Error*** (***MSE***) which is defined as :

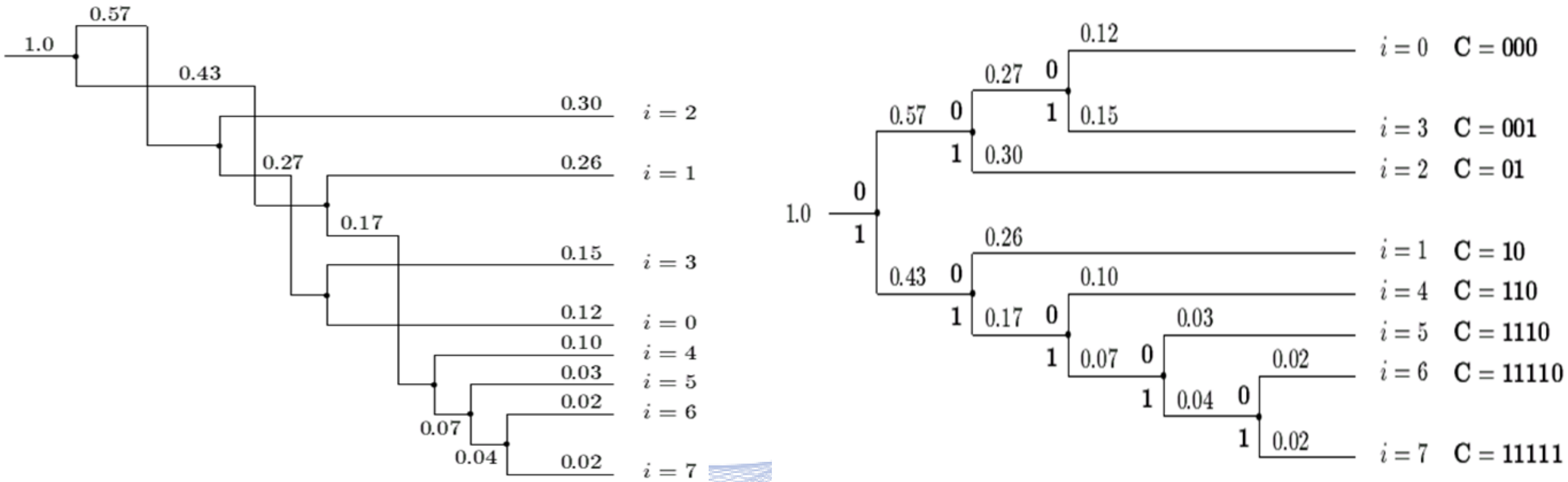
$$\text{MSE} \triangleq \frac{1}{N_1 N_2} \sum_{n_1=0}^{N_1-1} \sum_{n_2=0}^{N_2-1} \left(x(n_1, n_2) - \hat{x}(n_1, n_2)\right)^2 .$$

Huffman coding

Pulse Coding Modulation (***PCM***):

- 2^B codewords having B bits/pixel.
- The ***probability density function*** (***pdf***) $p(i)$ of image i can be estimated by calculating its histogram.
- The average number of bits per pixel can be reduced, by assigning binary codes of different bit length to the various image intensities.
- Short codewords are assigned to image intensities having a high probability of occurrence.
- Larger codewords to are assigned to less frequent image intensity levels.

Huffman coding



a) Construction of Huffman code tree; b) Huffman tree rearrangement.

Run-length coding

Each image line can be represented as follows:

$$(x_1, \ldots, x_M) \rightarrow (g_1, l_1), (g_2, l_2), \ldots, (g_k, l_k)$$

$$g_1 = x_1, \qquad g_k = x_M,$$

$$\sum_{i=1}^{k} l_i = M.$$

Each couple (g_i, l_i) is called gray-level run.

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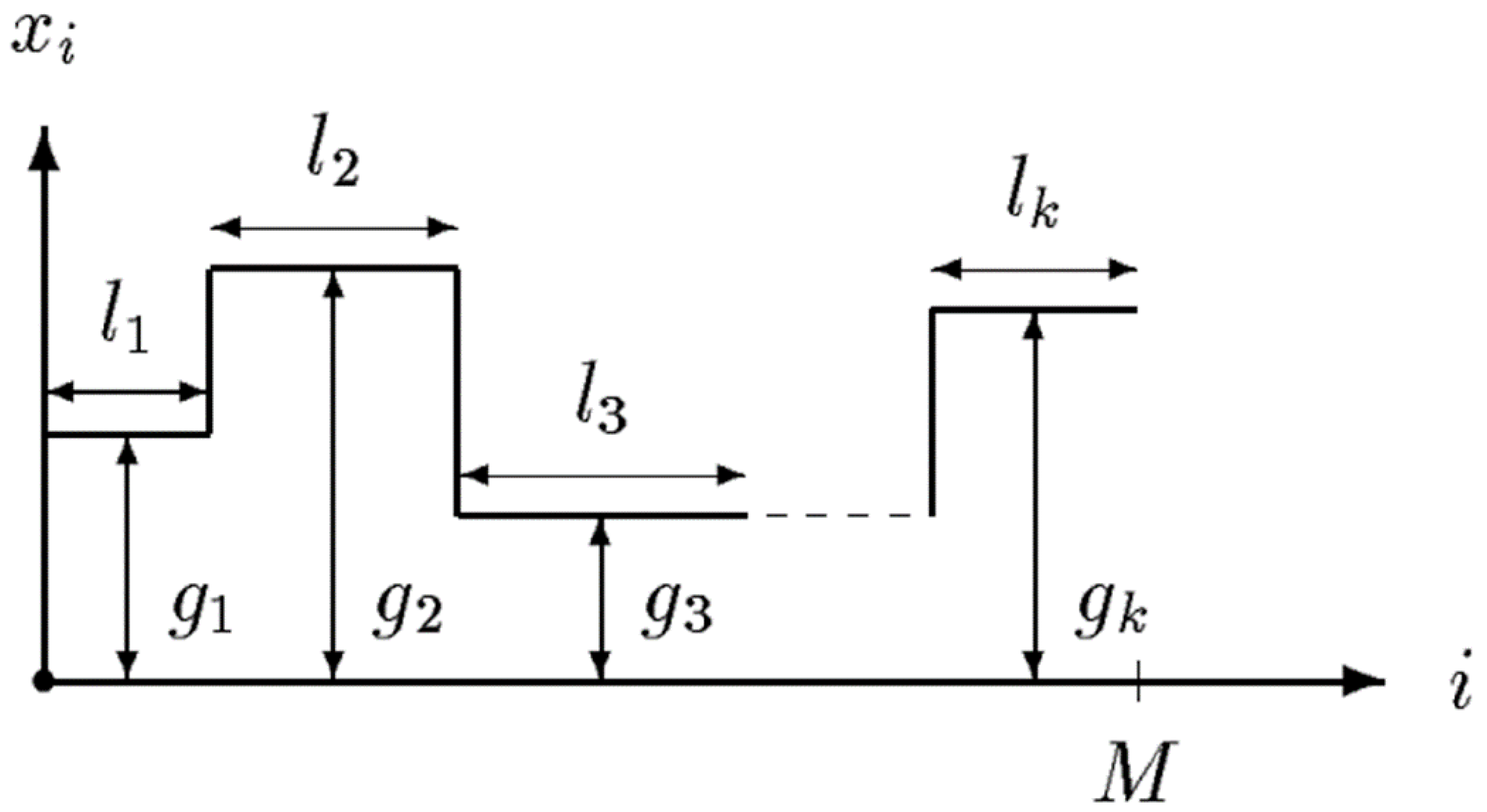
Run-length coding



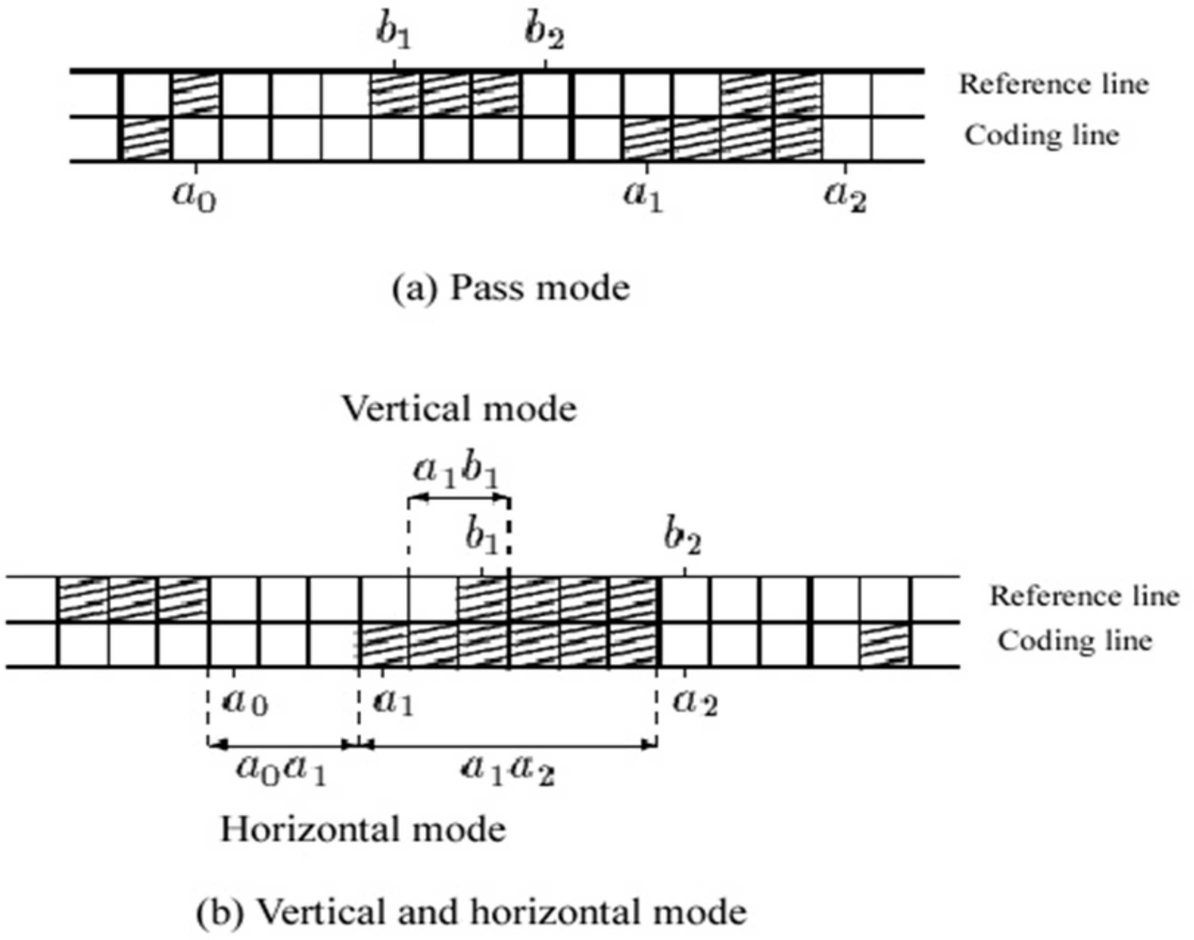
Graphical representation of an image line.

Modified READ coding

- Run-length coding is a one-dimensional scheme that cannot take into account vertical correlations among run transitions in consecutive image lines.

- ***Modified READ*** (***Relative Element Address Designate***) ***coding*** is a two-dimensional coding scheme that codes a binary image line with reference to the previous line.

Modified READ coding



Transition elements in modified READ coding.

LZW compression

- General-purpose compression scheme proposed by Lempel-Ziv and Welch.
- It can be used for the compression of any binary data file.
- It is incorporated in several de facto image storage standards (e.g., TIFF, GIF).

LZW compression

- It is a lossless, fast and effective algorithm and can operate on images of any bit depth.
- LZW compression is based on the construction of a code table that maps ***frequently encountered bit strings*** to output codewords.
- The digital image as well as the coded one is treated as a one-dimensional bit string.

Predictive coding

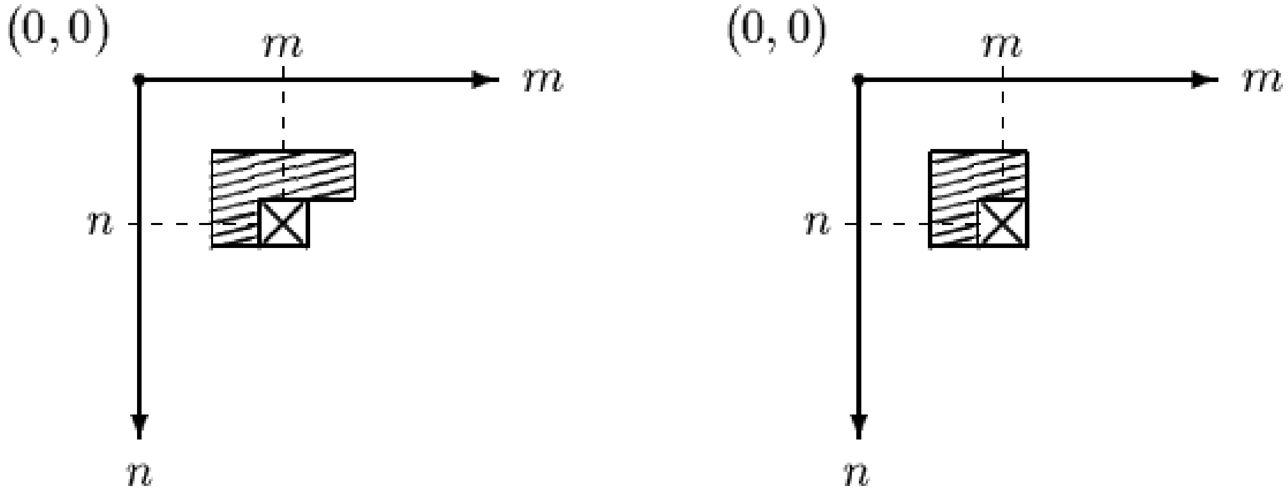
- One way to describe information redundancy in digital images is to use ***local image neighborhood predictability*** .
- Pixel intensity $f(n,m)$ can be predicted from the pixel intensities in its ***local neighborhood*** $\mathcal{A}$:

$$\hat{f}(n,m) = L\left[f(n-i,m-j), \quad (i,j) \in \mathcal{A}, \quad (i,j) \neq (0,0)\right].$$

- ***Causal prediction*** is used, which is based on already reconstructed past pixel values:

$$\hat{f}(n,m) = L\left[f_r(n-i,m-j), \quad (i,j) \in \mathcal{A}\right].$$

Predictive coding

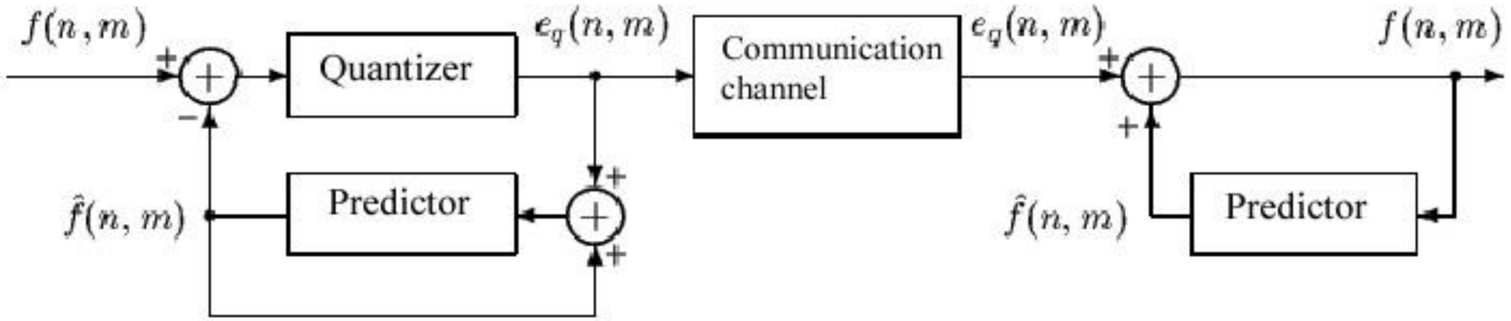


Causal windows used in image prediction.

Predictive coding

Predictive Differential Pulse Code Modulation (***DPCM***) is extensively used in telecommunications.

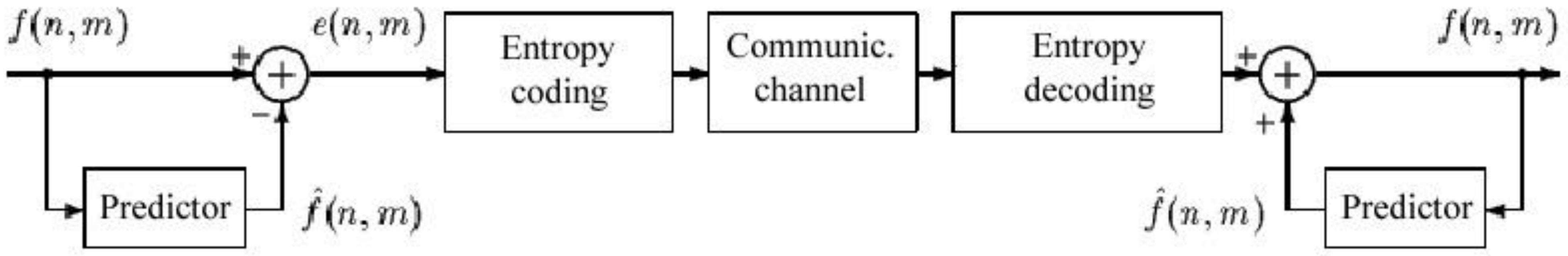
- It is a lossy coding scheme.
- Error signal quantization always creates an irrecoverable amount of distortion.



Predictive differential pulse code modulation (DPCM).

Predictive coding

- DPCM performance greatly depends on the ***predictor*** used and on the choice of its coefficients.
- Differential Pulse Code Modulation (DPCM) with ***entropy coding*** is a lossless coding scheme.



DPCM with entropy coding.

Predictive coding

- Let us suppose that image line $f(m)$ can be modeled as a stationary ***autoregressive*** (***AR***) ***process***:

$$f(m) = \sum_{k=1}^{p} a(k) f(m-k) + \varepsilon(m), \qquad E[\varepsilon^2(m)] = \sigma^2,$$

- $\varepsilon(m)$ is a white additive Gaussian noise that is uncorrelated to $f(m)$.

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Predictive coding

- The prediction coefficients can be estimated, by solving the system of ***normal equations***:

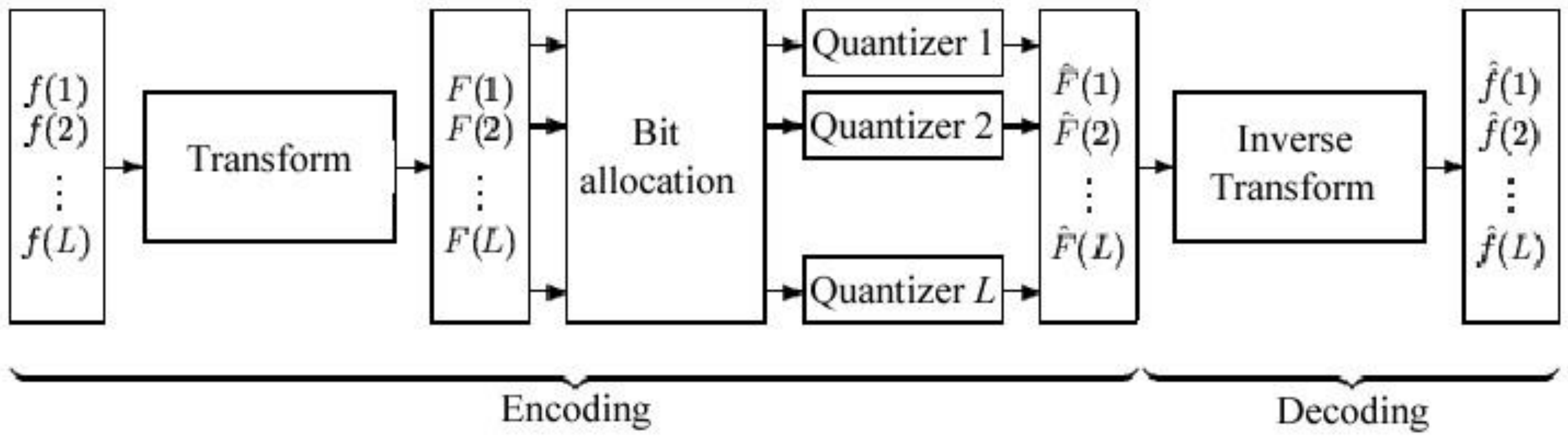
$$\begin{bmatrix} R(0) & R(1) & \cdots & R(p-1) \\ R(1) & R(0) & \cdots & R(p-2) \\ \vdots & \vdots & \vdots & \vdots \\ R(p-1) & R(p-2) & \cdots & R(0) \end{bmatrix} \begin{bmatrix} a(1) \\ a(2) \\ \vdots \\ a(p) \end{bmatrix} = \begin{bmatrix} R(1) \\ R(2) \\ \vdots \\ R(p) \end{bmatrix}.$$

- The matrix is called ***circulant*** or ***Toeplitz***.
- $R(k)$ is the image row ***autocorrelation function***.

Transform image coding

Digital image transforms concentrate image energy in a few transform coefficients.

- Heavy quantization or deletion of most transform coefficients leads to big lossy compression.



Transform encoding/decoding.

Transform image coding

- Let $\mathbf{f}$ be a vector representing an image of size $L = N \times M$ pixels.
- The transform coefficient vector $\mathbf{F}$ is given by:

$$\mathbf{F} = \mathbf{A}\mathbf{f}.$$

- $\mathbf{A}$ is the ***transform matrix***.
- The inverse transform is defined as follows:

$$\mathbf{f} = \mathbf{A}^{-1}\mathbf{F}.$$

Transform image coding

a) Image LENNA; b) Energy concentration in low DFT frequencies.

2D Discrete Cosine Transform

2D $N_1 \times N_2$ DCT is defined as:

$$C(k_1,k_2)=\sum_{n_1=0}^{N_1-1}\sum_{n_2=0}^{N_2-1} 4x(n_1,n_2)\cos\frac{(2n_1+1)k_1\pi}{2N_1}\cos\frac{(2n_2+1)k_2\pi}{2N_2},$$

for $0 \le k_1 \le N_1 - 1, 0 \le k_2 \le N_2 - 1$.

$$x(n_1,n_2)=\frac{1}{N_1N_2}\sum_{k_1=0}^{N_1-1}\sum_{k_2=0}^{N_2-1} w_1(k_1)w_2(k_2)C(k_1,k_2)\cos\frac{(2n_1+1)k_1\pi}{2N_1}\cos\frac{(2n_2+1)k_2\pi}{2N_2},$$

where:

$$w_1(k_1)=\begin{cases}1/2 & k_1=0\\ 1 & 1\le k_1\le N_1-1\end{cases} \qquad w_2(k_2)=\begin{cases}1/2 & k_2=0\\ 1 & 1\le k_2\le N_2-1\end{cases}.$$

Transform image coding

a) Image LENNA; b) Energy concentration in low DCT frequencies.

Transform image coding

8	7	6	5	3	3	2	2	2	1	1	1	1	1	0	0
7	6	5	4	3	3	2	2	1	1	1	1	1	0	0	0
6	5	4	3	3	2	2	2	1	1	1	1	1	0	0	0
5	4	3	3	3	2	2	2	1	1	1	1	1	0	0	0
3	3	3	3	2	2	2	1	1	1	1	1	0	0	0	0
3	3	2	2	2	2	2	1	1	1	1	1	0	0	0	0
2	2	2	2	2	2	1	1	1	1	1	0	0	0	0	0
2	2	2	2	1	1	1	1	1	1	1	0	0	0	0	0
2	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Bit allocation in a 16×16 block of DCT coefficients.

Transform image coding

a) Original image; b) JPEG compressed image.

Transform image coding

a) Original image; b) 90% JPEG compression quality;
c) 5% JPEG compression quality; d) 1% JPEG compression quality.

JPEG2000 compression

- JPEG2000 standard was introduced in order to overcome weaknesses of other compression methods (e.g., JPEG).
- Designed to be used by many image types (Greyscale, color, 2D, 3D) for various applications (scientific, medical etc.).
- Its goal was to achieve better ***rate-distortion*** characteristics and ***subjective quality*** of the compressed image.
- ***Rate-Distortion Optimization*** (***RDO***) was employed.
- It is the only standard compression scheme that provides both lossless and lossy compression.

JPEG2000 compression

- JPEG2000 performs extremely well in some applications, where other compression methods are frailling.
- It is ideal for large images, or images having low-contrast edges (e.g., medical images).

- Losseless and lossy compression can be provided within a single compressed bit-stream by using ***Discrete Wavelet Transform*** (***DWT***). It is the only compression standard that offers that solution.

JPEG2000 compression

JPEG2000 supports progressive compression:

Progressive transmission By Resolution (***PBR***).

- Image size increases by loading new bits, until the original image size is reached.

Progressive transmission By pixel Accuracy (***PBA***).

- Image quality is improved by loading new bits, until original pixel quality is reached.

JPEG2000 compression

Region-Of-Interest* (*ROI*) *coding.

- Favored image regions are compressed at top resolution.
- Other image regions can be encoded at smaller resolution.
- In some cases, irrelevant image regions can be entirely masked out.

- The user can randomly access and modify image regions that are not heavily distorted.

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Q & A

Thank you very much for your attention!

More material in
http://icarus.csd.auth.gr/cvml-web-lecture-series/

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