

Computational Geometry summary

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Version 3.0

Date: 06/06/2021

Computational Geometry

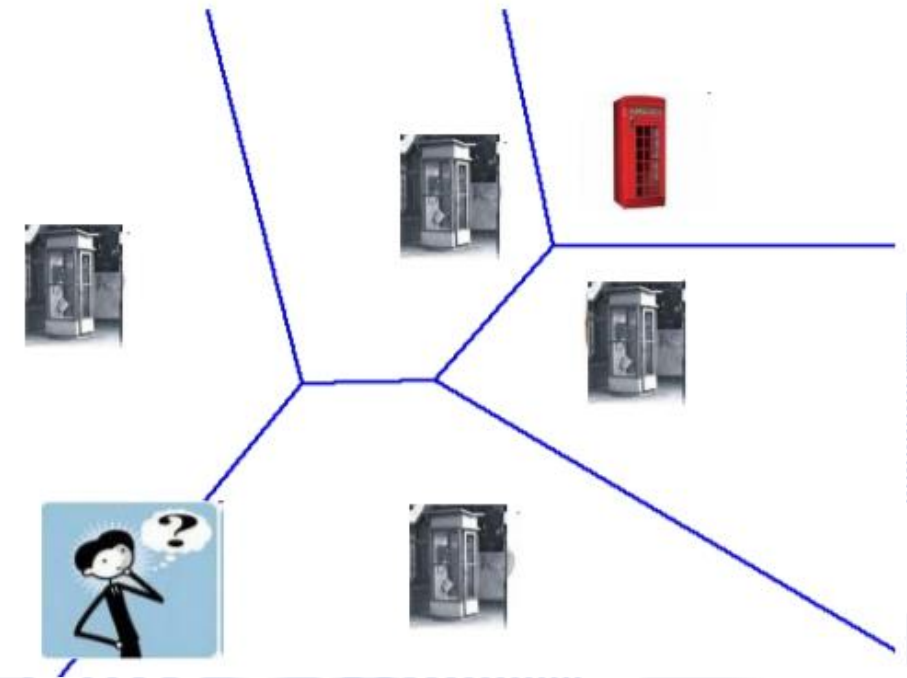


- **Introduction**
- Computational Geometry Processing
 - Convex Hull
 - Voronoi Diagrams
 - Delaunay Triangulation
 - Line Segment Intersection
 - Point Location
- Application Domains
 - Computer Graphics
 - Robotics
 - Geographic Information Systems
 - CAD / CAM
- Conclusion: *“Useful geometric tools”*

Introduction

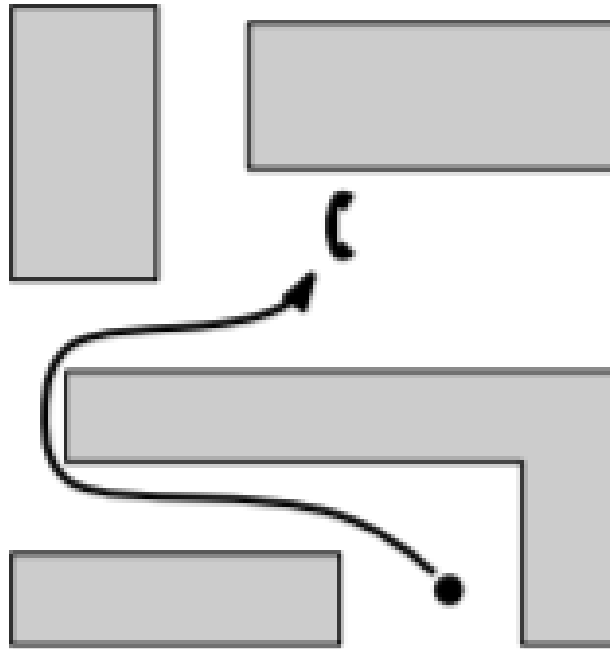
What is Computational Geometry

- Copes with geometric problems requiring smart geometric algorithms [CVU].
- What is the location of the nearest phone?



[CVU]

Introduction



[CVU]

- What is the best way to locate the nearest phone?
- Assemble information from various maps.

Introduction

- ***Mathematical approach***

The use and study of theorems, explanations, and mathematical proofs.

- ***Practical approach***

Deal with a single description in computational geometry and demonstrate how to model an engineering problem and arrive at an exact solution [MIT1995].

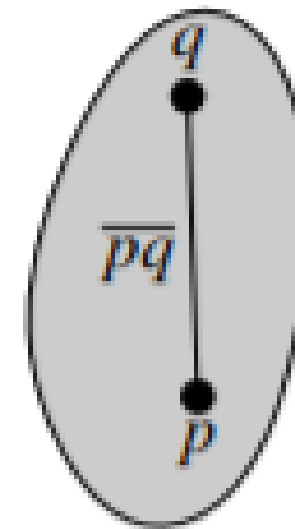
Computational Geometry

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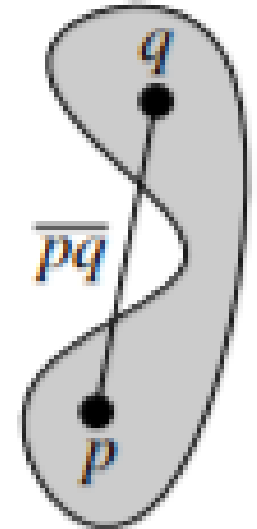
Convex Hull

A subset S of the plane is called **convex** if and only if for any pair of points $p, q \in S$ the line segment \overline{pq} is completely contained in S . The convex hull $C\mathcal{H}(S)$ of a set S is the smallest convex set that includes S .

To be more precise, it is the intersection of all convex sets that contain S [KREV2000].



convex



not convex

[KREV2000]

Convex Hull

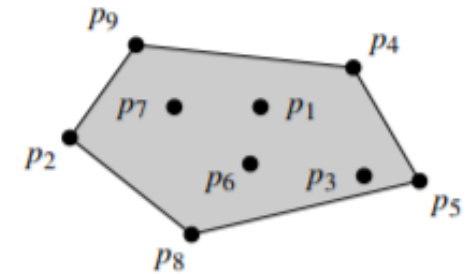
- The convex hull of a finite set P of n points in the plane is a difficult problem.
- Given a set $P = \{p_1, p_2, \dots, p_n\}$ of points in the plane, calculate an index that contains those points from P that are the vertices of $C\mathcal{H}(P)$, listed rightward.
- The observation that $C\mathcal{H}(P)$ is a convex polygon is more applicable.

input = set of points:

$p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9$

output = representation of the convex hull:

p_4, p_5, p_8, p_2, p_9



[KREV2000]

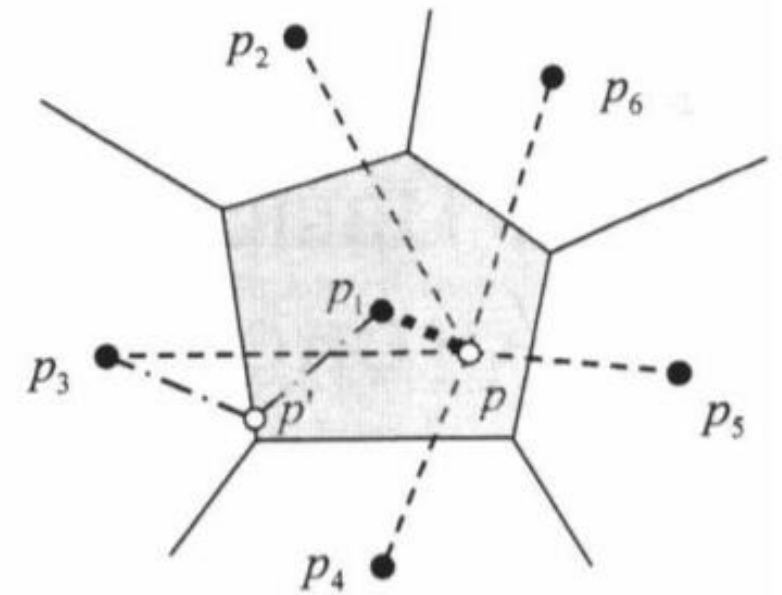
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Voronoi Diagrams

There are n – dimensional Voronoi diagrams for any positive integer n [BAE2012].

A Voronoi diagram of a set of **sites** or **generators** (points) is a collection of regions that divide the plane into sections. Each region corresponds to one of the generators or sites, and any point within that region is closer to that site than any other site.



[BAE2012]

Voronoi Diagrams



Voronoi diagram basic definition has a broad variety of uses both within and outside of mathematics. Voronoi diagrams used as a form of problem solving or as a model for existing instances.

They are useful in ***computational geometry***, particularly for representation or quantization problems.

Beneficial in plant competition (ecology and forestry), animal territories (zoology), neolithic clans and tribes (anthropology and archaeology), and urban settlement patterns (ecology and forestry) (geography).

Voronoi Diagrams

Formal Definition of the Voronoi 2D Diagram.

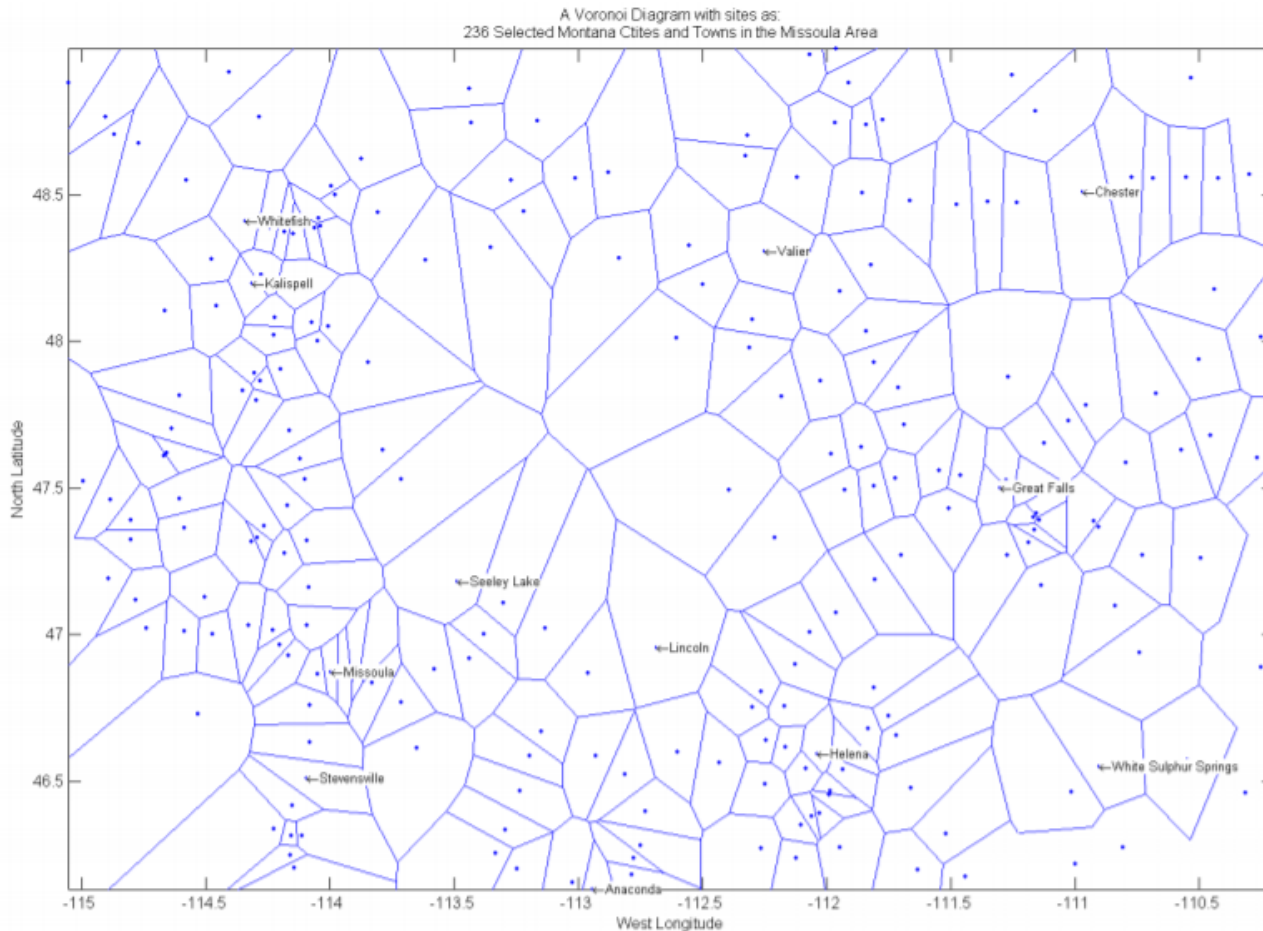
Indicate the location of a point p_i as (x_{i1}, x_{i2}) , and the corresponding vector will be \vec{x} . Let $P = \{p_1, p_2, \dots, p_n\} \in R^2$, where $2 \leq n < \infty$ and $p_i \neq p_j, i \neq j$ and $\forall i, j = 1, 2, \dots, n$ be the set of generator points, or generators. We call the region given by $V(p_i) = \{\vec{x} \mid \|\vec{x} - \vec{x}_i\| \leq \|\vec{x} - \vec{x}_j\| \forall j \ni i \neq j\}$

Voronoi Diagrams

Formal Definition of the Voronoi 2D Diagram.

The Voronoi region of p_i , where $\|\cdot\|$ is the usual Euclidean distance. $V_{(p_i)}$ may also be referred to as V_i . In a regular Voronoi, all of the Voronoi regions are connected and convex. We call the set given by $bV = \{V_{(p_1)}, V_{(p_2)}, \dots, V_{(p_n)}\}$ the Voronoi diagram of P .

Voronoi Diagrams



[MUM2004]

- Voronoi diagram of places with local geographical significance in the Missoula region.
- The actual diagram was created in MATLAB using the Voronoi package.
- A few major communities were categorized based on population size and disbursement range in order to obtain a simple geographic picture of how the data was distributed.

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Delaunay Triangulation

- If edges are introduced with the condition that no new edge intersects an existing edge, any set of points can be triangulated [LEE1980].
- The Delaunay triangulation.
- Has the property that the circumcircle of any triangle in the triangulation contains no point of V in its interior. (Let V be a set of $N \geq 3$ distinct points in the Euclidean plane.)

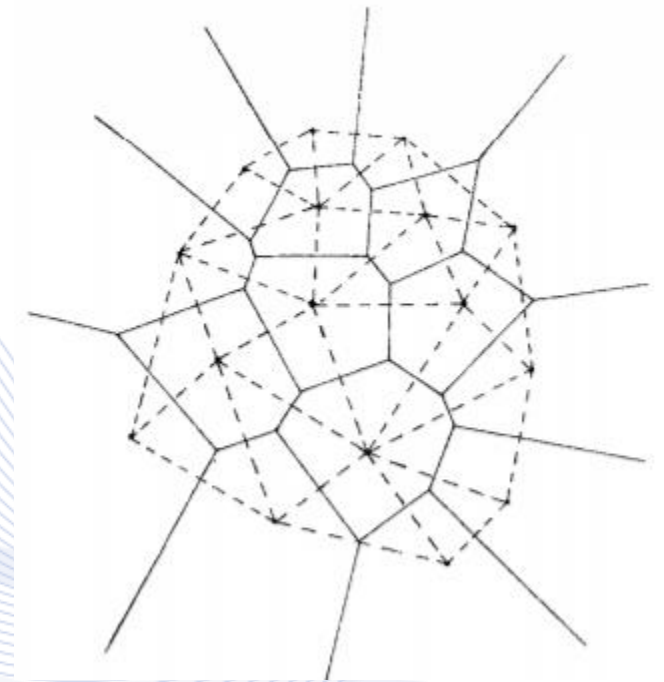
Delaunay Triangulation

Definition and properties of the Delaunay Triangulation.

Voronoi neighbors are Voronoi cells that share a similar edge. The region within the convex hull of the point set is tessellated by the sum of triangles created by connecting the growth centers of all Voronoi neighbors. The ***Delaunay Triangulation*** $DT(V)$ of V is the name given to this tessellation.

Delaunay Triangulation

- ***Delaunay Triangulation***; Voronoi diagram for a set of 16 points (solid lines) (dashed lines).
- Each Voronoi point is a Delaunay edge, and each Voronoi edge is a triangle. As a result of the fact that the number of Voronoi points and edges are both $O(N)$, the number of Delaunay triangles and edges are $O(N)$.



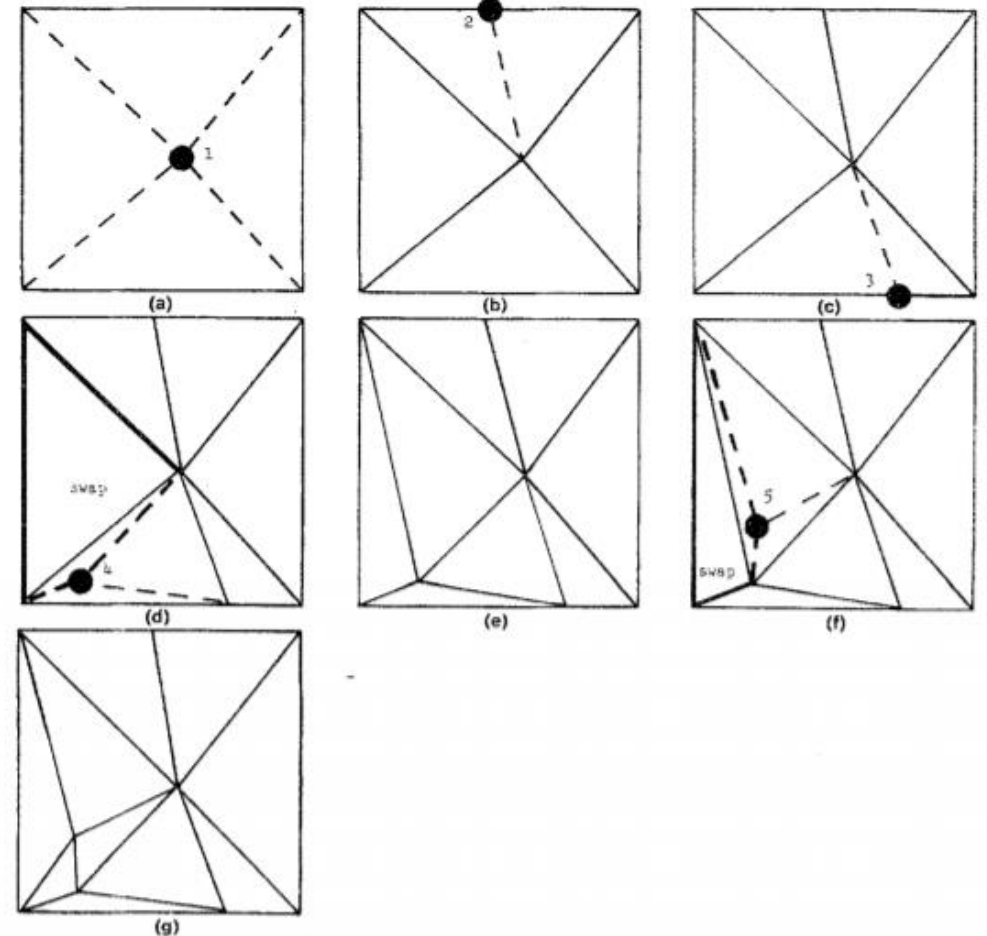
[LEE1980]

Delaunay Triangulation

Algorithm **TRIBUILD-ITERATION**

Step 5: This switching technique has the potential to spread further. Lawson has shown that this mechanism will always come to an end.

Step 6: STOP TRIBUILD if all points in V have been used; otherwise, go to phase 1.



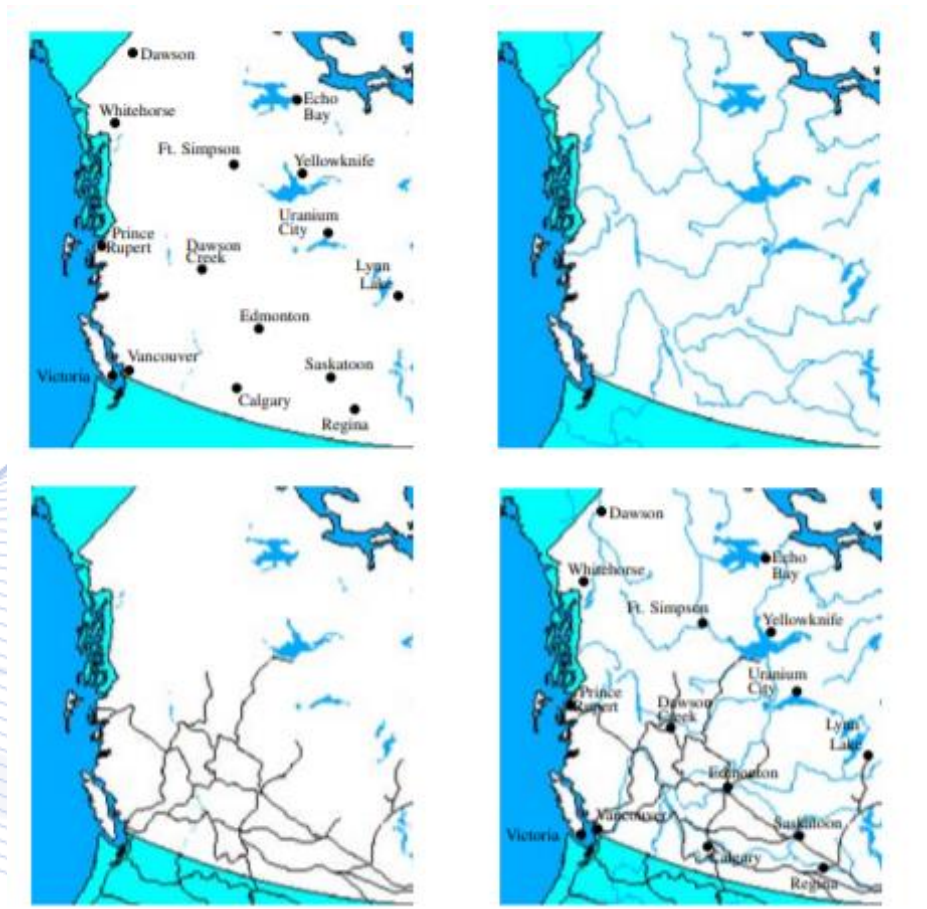
[LEE1980]

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Line Segment Intersection

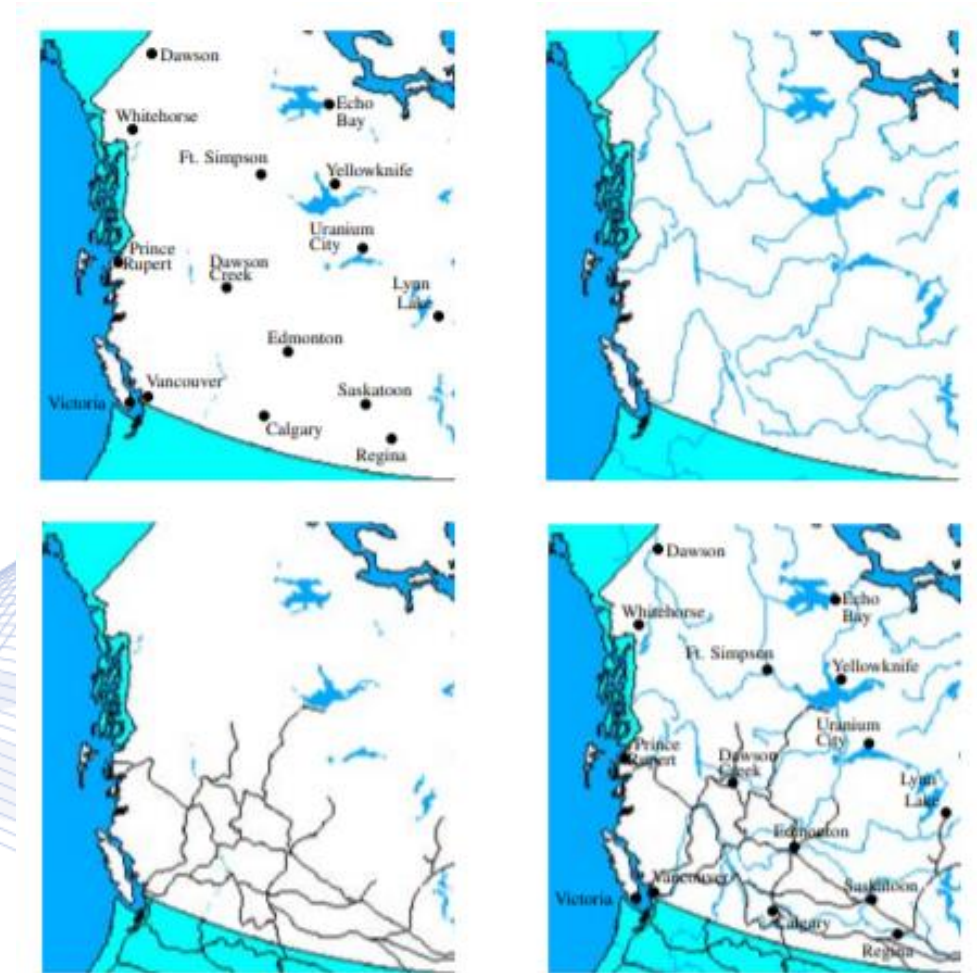
- Users of geographic information systems will see an overlay of several maps [KREV2000].
- Intersections in the overlay are points of particular interest when two or more thematic map layers are shown together.
- Western Canada's towns, rivers, railroads, and their intersections.



[KREV2000]

Line Segment Intersection

- If the intersections were clearly marked, this would be very helpful.
- The intersections of the two maps are points, and the two maps are essentially networks.
- The intersection of complete regions can pique one's interest.



[KREV2000]

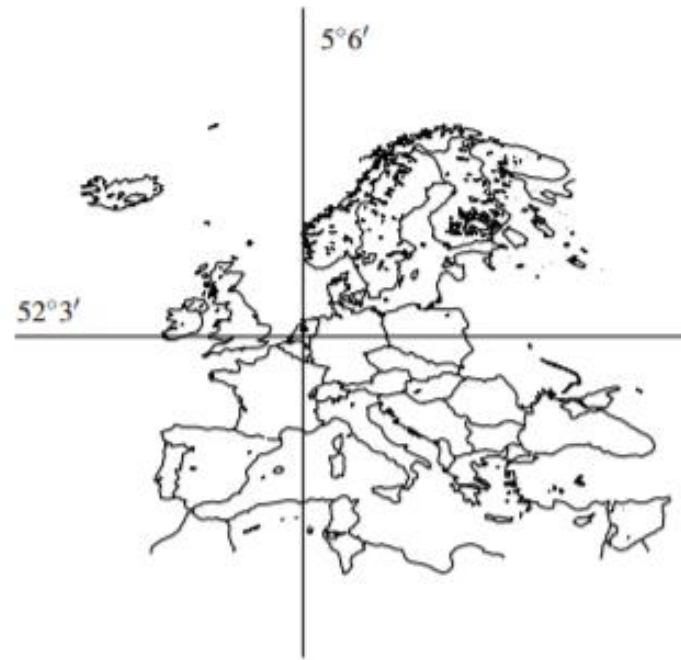
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Point Location

Find the area of the map that contains q given a map and a query point q defined by coordinates. A map is, after all, just a planar subdivision of the plane into regions [KREV2000].



[KREV2000]

Point Location

In a variety of situations, questions about point position emerge. Assume you're cruising on a sea that has sand banks and unpredictable winds in different areas. You must know the latest at your current location in order to navigate safely.

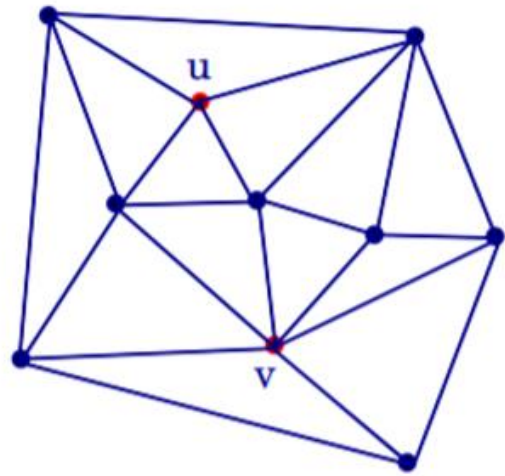
Once you've decided your position's coordinates, you'll need to find the point on the chart that shows the tides, or find the sea area.

This move could be taken a step further by automating it: save the map electronically and let the machine locate the points for you.

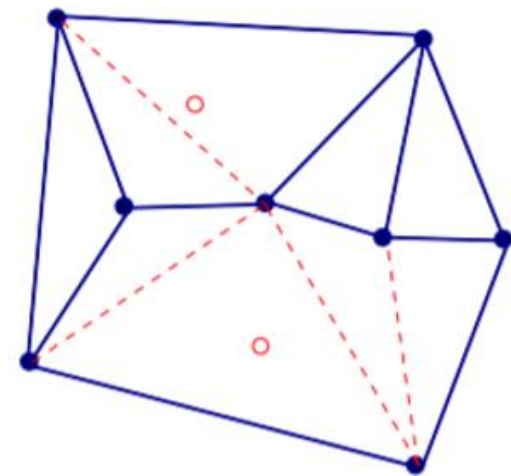
Point Location

Kirkpatrick's Algorithm

- Vertex deletion and re-triangulation



[USC]



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Application Domains

Computer Graphics

- Create images of designed scenes for viewing on a computer screen, printer, or other output unit [KREV2000].
- Geometric artifacts make up scenes, and geometric algorithms play an important role in computer graphics.

Application Domains

Computer Graphics

- The intersection of such primitives, deciding the primitive pointed to with the cursor, or determining the subset of primitives that fall within a given area are all common questions in 2D graphics.
- It's critical to detect collisions between objects while working with moving objects or in virtual reality applications.

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Application Domains



Robotics

- The field of robotics studies the design and use of robots [KREV2000].
- Geometric problems exist in many places because robots are geometric structures that function in a three-dimensional vacuum.
- The motion planning problem.
- In the construction of robotics and the work cells in which they reside, geometric issues arise.
- Geometric problems, with a kinematic part on occasion.

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Application Domains

Geographic Information Systems

- A geographic information system, or GIS, stores geographical details such as country shapes, mountain heights, river courses, vegetation types at various locations, population density, and rainfall [KREV2000].
- Cities, bridges, highways, power poles, and gas pipes are examples of human-made infrastructure.

Application Domains

Geographic Information Systems

- A GIS may be used to retrieve data from certain regions and to learn about the relationships between various forms of data.
- One of the most critical operations in a GIS is the combination of various types of data.

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Application Domains

CAD/CAM

- Computer aided design (CAD) concerns itself with the design of products with a computer. The products can vary from printed circuit boards, machine parts, or furniture, to complete buildings [KREV2000].
- CAD software helps with object intersections and unions, as well as decomposing structures and object borders into simplified shapes and visualizing the finished goods.

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Conclusion



Useful geometric tools

- OpenSCAD - The Programmers Solid 3D CAD Modeler,
<http://openscad.org/>
- J.R. Shewchuk - Adaptive Precision Floating-Point Arithmetic and Fast Robust Predicates, Effective implementation of Orientation and InCircle predicates,
<http://www.cs.cmu.edu/~quake/robust.html>

Conclusion

Useful geometric tools

- OpenMESH - A generic and efficient polygon mesh data structure, <https://www.graphics.rwth-aachen.de/software/openmesh/>
- VCG Library - The Visualization and Computer Graphics Library, <http://vcg.isti.cnr.it/vcglib/>

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Q & A

Thank you very much for your attention!

**More material in
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