Adaptive Filters summary

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Adaptive Filters

- **Adaptive Filters**
- Minimum Mean Square Error (MMSE)
- Widrow LMS Algorithm
- Properties of the LMS Algorithm
- CLMS Algorithm
- Nonlinear Feedforward Complex Adaptive Filters
- Kernel Adaptive Filters
- KLMS Algorithm

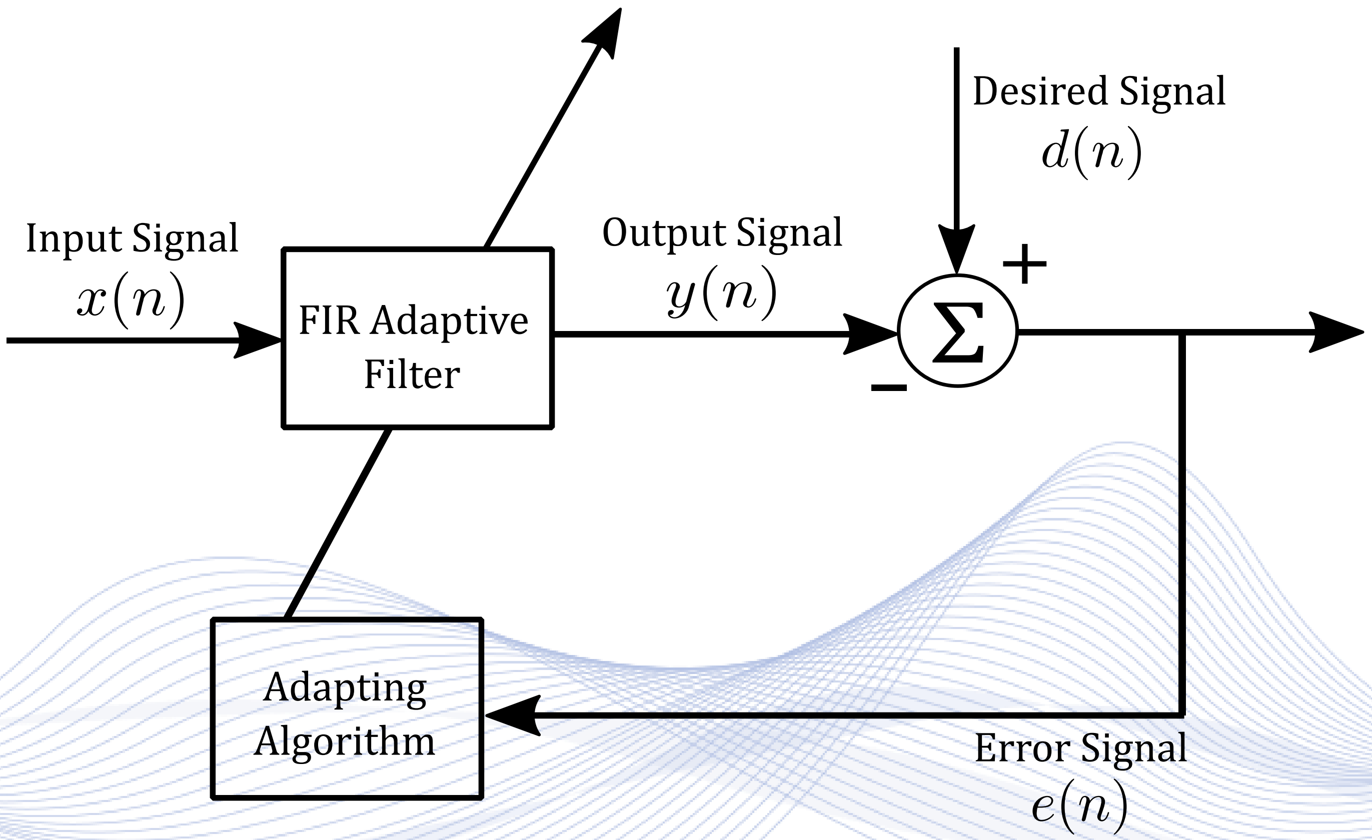
Adaptive Filters

An adaptive filter is a digital filter with variable parameters, that are adjusted according to an optimization algorithm.

For example, a high-speed modem designed to carry information in various channels, would use a channel equalizer to counterbalance distortion. The only way for the modem to transfer information efficiently is having adjustable coefficients that can minimize distortion in each of the different channels.

In order to achieve that, we use an adaptive filter.

Adaptive Filters



A detailed diagram of an adaptive filter.

Adaptive Filters

Adaptive filters can be classified according to the choices made in the following fields:

- Criteria of optimization
- Adjustment algorithm
- Structure of the programmable filter
- Type of processed signal (one or two dimensional)

Adaptive Filters

An example of an adaptive filter's use is the recognition of a system's identity. We have an unknown system called 'plant' that we want to identify. The system is represented by a FIR filter with variable parameters.

If $y(n)$ is the output of the plant and $\hat{y}(n)$ is the output of the model, then:

$$\hat{y}(n) = \sum_{k=0}^{M-1} h(k)x(n-k)$$

Adaptive Filters

The error sequence is:

$$e(n) = y(n) - \hat{y}(n)$$

We choose the coefficients $h(k)$ so we can minimize the equation:

$$E_M = \sum_{n=0}^{\infty} \left[y(n) - \sum_{k=0}^{M-1} h(k)x(n-k) \right]^2$$

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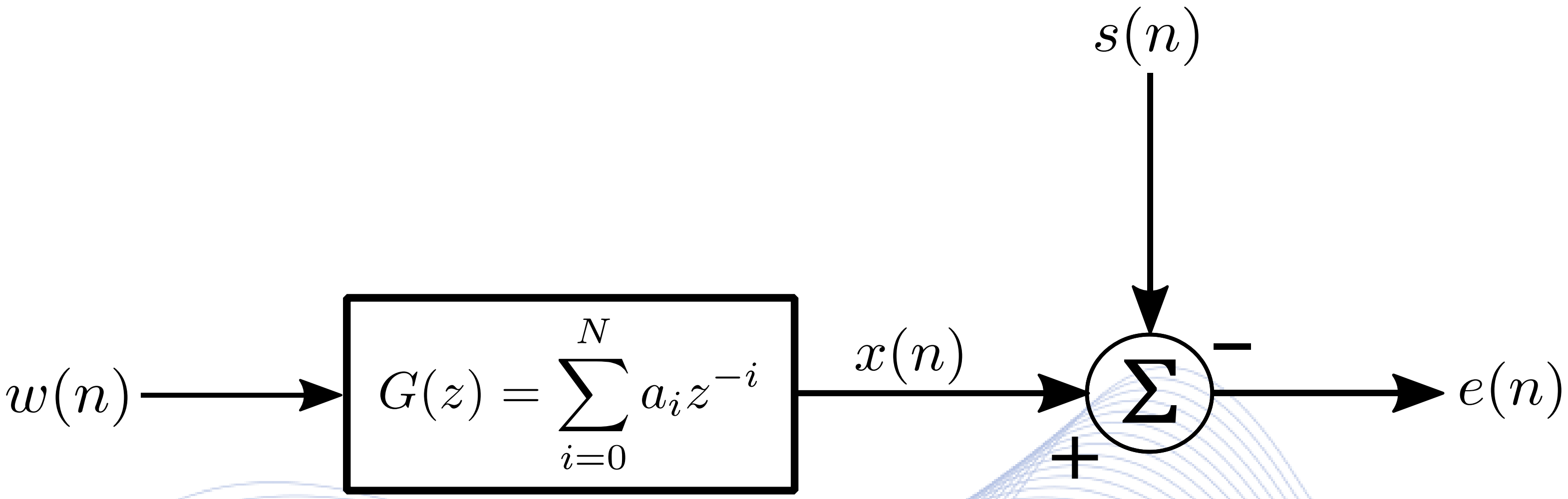
Minimum Mean Square Error



A block diagram of a Wiener Filter.

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Widrow LMS Algorithm

There are many algorithms that we can employ in order to find the best possible coefficients. One of those algorithms is the Widrow algorithm. Assuming that $J(\mathbf{h}_m)$ is a known function of the coefficients $h(n), 0 \leq n \leq M-1$.

The algorithms for the iterative computing of the filter's parameters are:

$$\mathbf{h}_M(n+1) = \mathbf{h}_M(n) + \frac{1}{2}\Delta(n)\mathbf{S}(n) \qquad n = 0,1,\ldots$$

Where $\Delta(n)$ is the step size of the nth iteration and $\mathbf{S}(n)$ is the direction vector of the nth iteration.

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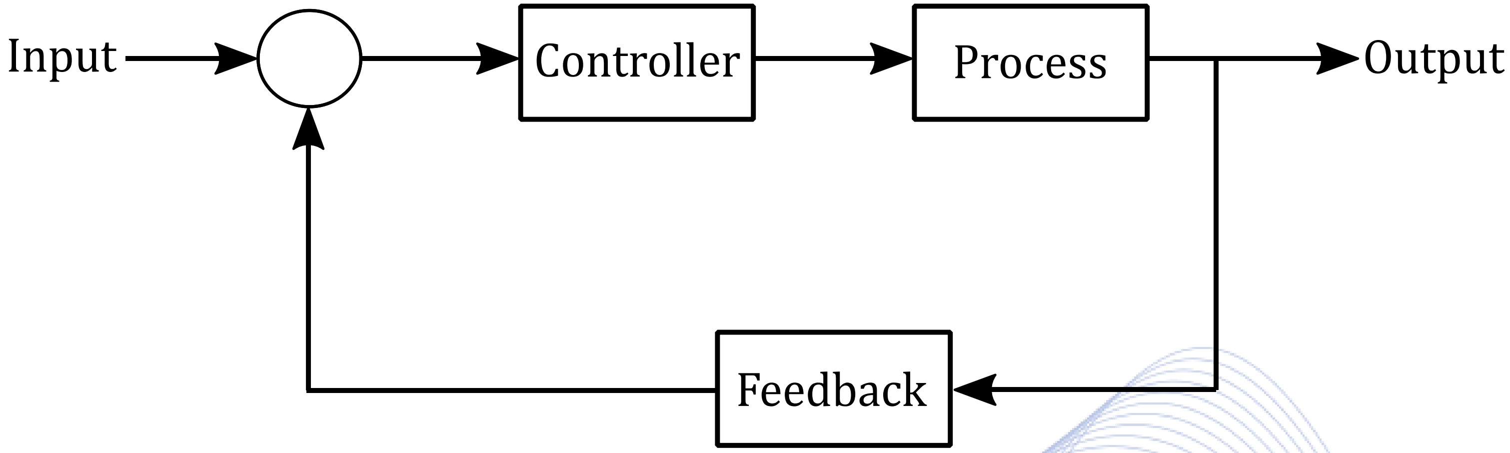
Properties of the LMS Algorithm



The previous equation can be represented as a closed loop system, like the above diagram.

The rate of convergence and the stability of this system is determined by the parameter Δ.

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CLMS Algorithm

Two formulations that are used often for the CLMS are:

$$y = \mathbf{w}^{\mathrm{H}}(n)x(n) \quad \rightarrow \quad \mathbf{w}(n+1) = \boldsymbol{w}(n) + \mu e^{*}(n)\mathbf{x}(n)$$

And

$$y = \mathbf{x}^{\mathrm{H}}(n)\mathbf{w}(n) \quad \rightarrow \quad \mathbf{w}(n+1) = \mathbf{w}(n) + \mu e(n)\mathbf{x}(n)$$

All of the above formulations are equivalent, but these two specific forms are mostly used for more convenient matrix manipulation.

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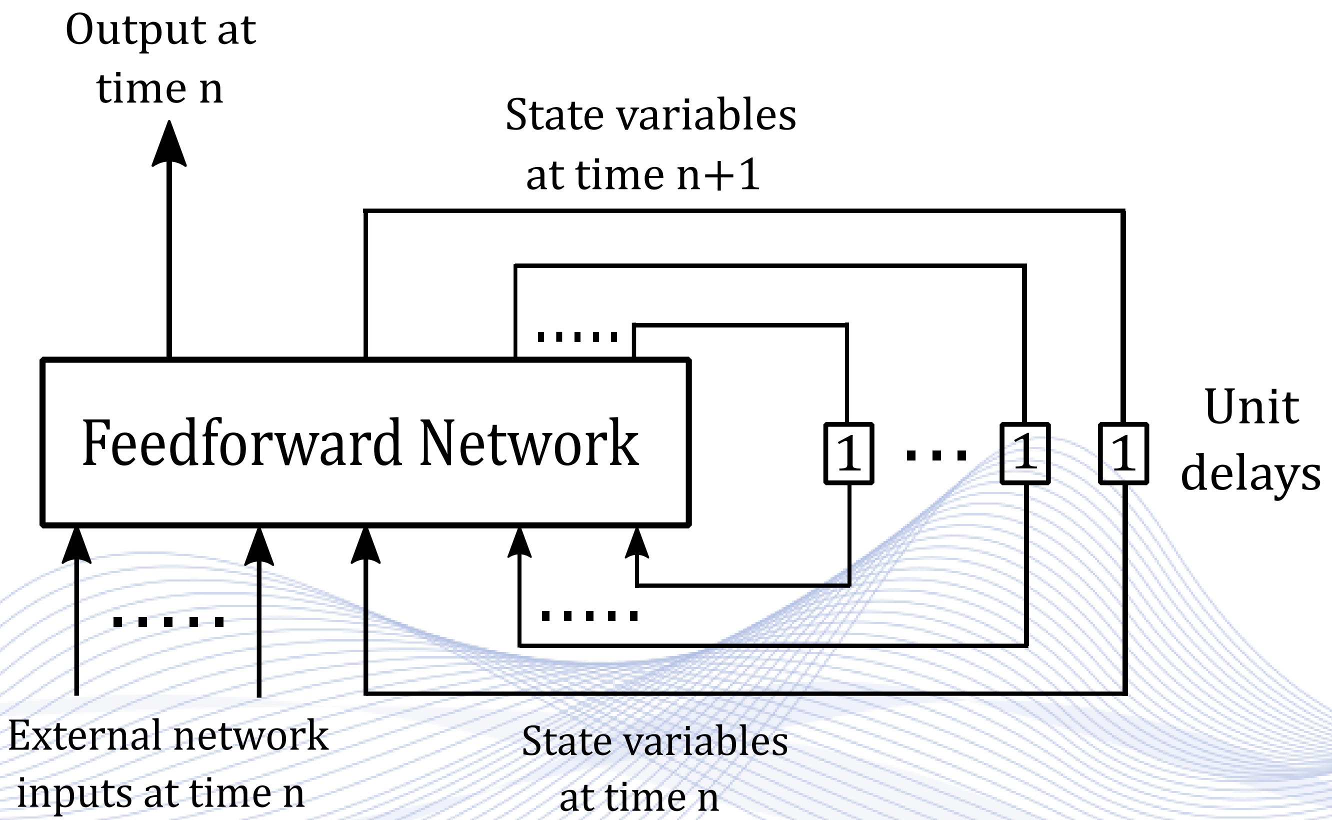
Nonlinear Feedforward Complex Adaptive Filters



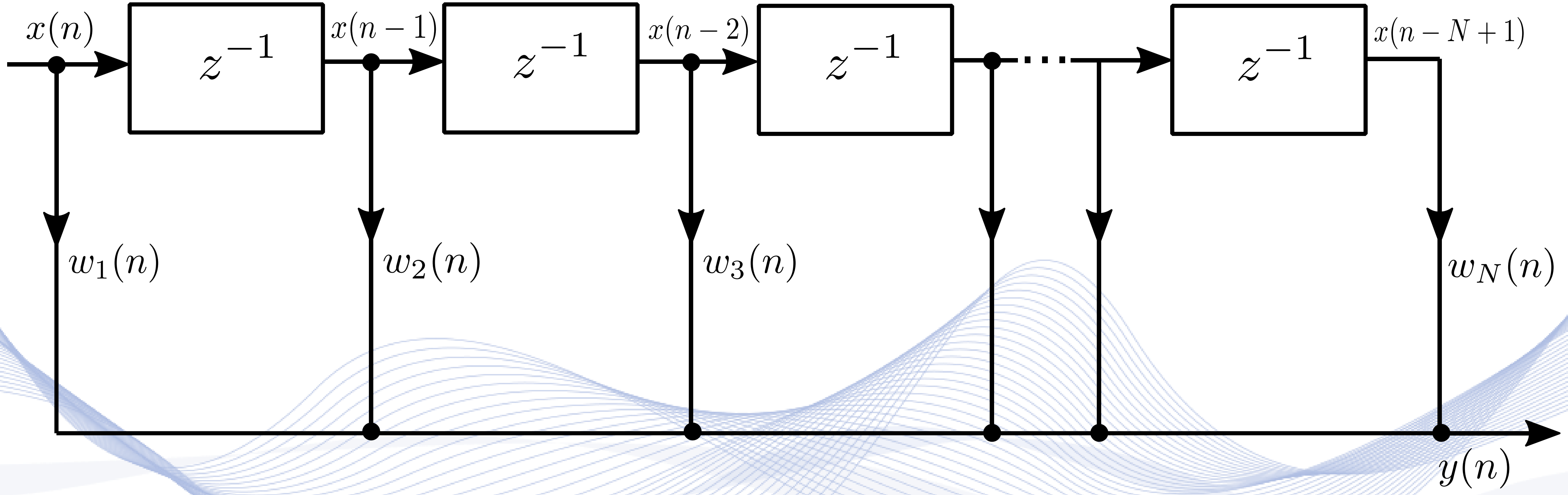
A diagram of a Feedforward network.

Nonlinear Feedforward Complex Adaptive Filters

A feedforward neural network is a network where the connections between each node do not form a circle. As such, information only moves towards one direction and it never returns backwards. The same principle is used in feedforward non-linear filters.

Following our previous derivation of the CLMS algorithm, we are going to find a stochastic learning algorithm that is able to train these fully complex filters, called Complex Nonlinear Gradient Descent (CNGD).

Nonlinear Feedforward Complex Adaptive Filters



A diagram of a Nonlinear adaptive filter.

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Kernel Adaptive Filters

In machine learning, kernel methods are popular modeling tools designed for pattern analysis. Kernel adaptive filters are a new subfield of adaptive filtering, that relies on online kernel learning. It includes:

- Kernel least mean square (KLMS) algorithm
- Kernel affine projection algorithms (KAPA)
- Kernel recursive least squares (KRLS) algorithm
- Extended kernel recursive least square (EX-KRLS) algorithm

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Q & A

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