3D Surface and Volume Rendering summary

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3D Surface and Volume Rendering

- **3D Geometric Modeling**
- Curves and Surfaces
- Surface Simplification Methods
- Illumination and Shading
- Hidden Surface Removal
- Surface Normal Estimation
- Volume Rendering
- 3D Surface vs Volume Rendering

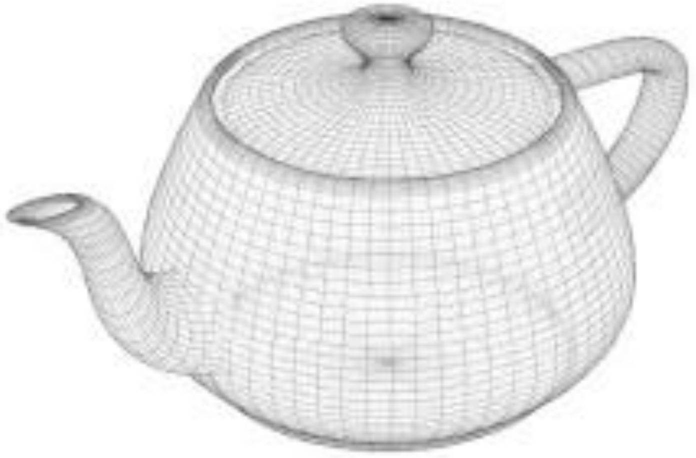
3D Geometric Modeling

- Geometric Modeling is called the mathematical representation of a physical object using software principles.

- Curves and surfaces are used in order to control the shape of the object and the topology.
- Can be divided into three main categories:
  - Wireframe Modeling.
  - Solid Modeling.
  - Surface Modeling.

Wireframe Modeling

- Wireframe modeling or also called polygonal mesh, consists of vertices and polygons, where vertices are connected to edges and polygons that are generated by them.
- Each vertex is defined by x, y, z coordinates.
- Edges are defined by a set of vertices.
- Faces are constructed by three or more edges.



Source: [CHR2017]

Triangle Reconstruction

- In Triangle-based surface reconstruction, the shape of an object can be represented as a set of triangles identifying in that way the surface of it.

- Two contours are represented as a sequence of contour points $P_0, \dots, P_{m-1}$ and $Q_0, \dots, Q_{n-1}$ respectively.

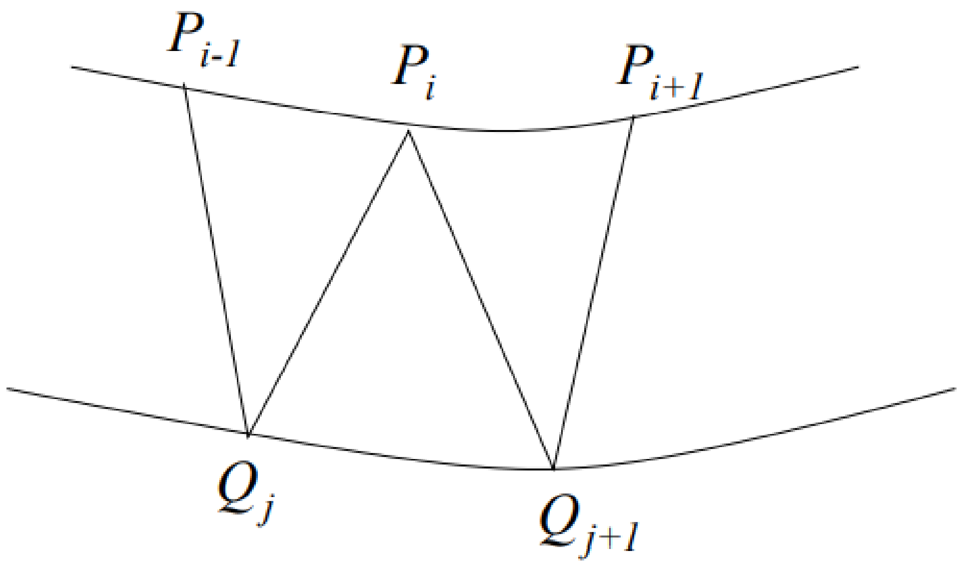
- Each triangle is bounded by a contour segment in the one contour and two linear segments, each connecting an end of the contour segment to a single point on the other.

Triangle Reconstruction

- Triangle denoted as:

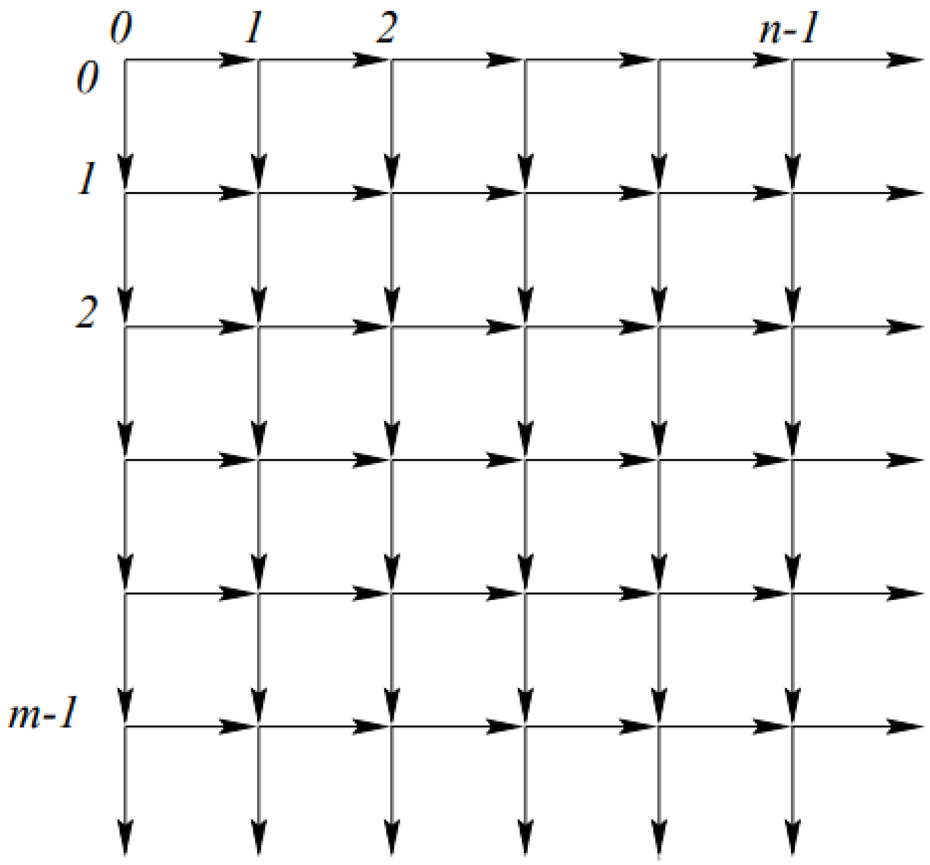
$$\{P_{i-1}, Q_j, P_i\},$$

where the triangles left linear segment (span) is $Q_j P_{i-1}$ the right span is $P_i\ Q_j$ and the contour segment is $P_{i-1} P_i$



Triangle Reconstruction

- Graph is constructed.
- The vertices of the graph represent the spans that connect the $P_0, \ldots, P_{m-1}$ and $Q_0, \ldots, Q_{n-1}$.
- The arcs correspond to the set of all possible triangles.



Solid Modeling

- 2D wireframe representations of an object can be used in order to create solid models, such models ensuring the geometrically correctness of the given surface.

- Essentially, solid modeling is the visualization of the geometry of a physical object.

- Boundaries of an object can be defined using points, curves and surfaces.

Solid Modeling

- ***Boundary Representation (BRep)*** is defined as a set of non-overlapping faces and the concatenation of them represent the object's boundaries.

- ***Constructive Solid Geometry (CSG):*** Is a representation of parameterized solids which may be simple shapes (e.g. cylinder, cones, boxes) or more complex shapes which are grouped in a hierarchical order.

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Curves and Surfaces

- Spline surfaces are piecewise polynomial parametric surface representation which can provide simplicity on their construction, ease and accuracy in evaluation.

- Basic concept in those representations are the control points, which are mostly lying in a regular grid. Knots is the control points located in the curve.

- When curve passes through all those control points, it is known as interpolating curve.

B-spline Surfaces

- B-spline curves can generate the B-spline surfaces, they denoted as a tensor product surface of B-spline curves in the form:

$$S(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} N_{i,p}(u) N_{j,q}(v) P_{i,j} \ ,$$

where $P_{i,j}$ is the control points in a grid with $m+1$ rows and $n+1$ columns. And knot vectors $U = \{u_0, u_1, \ldots, u_h\}$ and $\mathrm{V} = \{v_0, v_1, \ldots, v_h\}$. And p, q are the degrees of the u, v directions.

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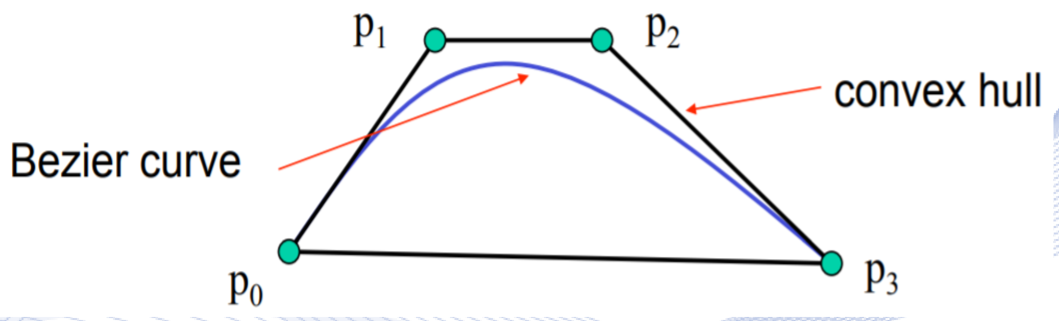
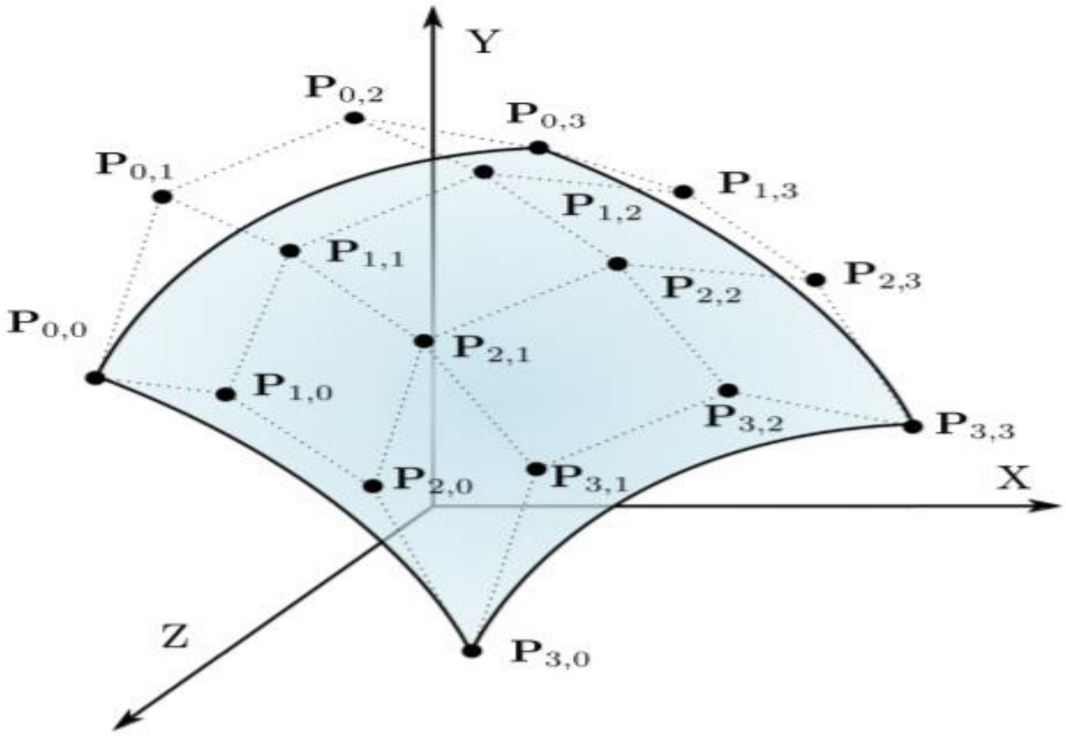
Bezier Surfaces

- Bernstein polynomials are used to assure us that all Bezier curves will be inside the convex hall, defined by the control points.



Bezier Surfaces

- Bezier Surface $S(u,v)$ is C^{p-k} continuous in the u direction at a u knot of multiplicity k and same for v direction.



Source: [PAL2018]

NURBS Surfaces

- Nonuniform Rational B-Spline Surfaces (NURBS) are constructed by adding a weight in the control points of the B-spline surface and then by normalizing it:

$$S(u,v)=\frac{\sum_{i=0}^{n}\sum_{j=0}^{m}N_{i,p}(u)\,N_{i,q}(v)w_{i,j}P_{i,j}}{\sum_{i=0}^{n}\sum_{j=0}^{m}N_{i,p}(u)\,N_{i,q}(v)w_{i,j}}.$$

- Perspective division is required.

  NURBS works well for perspective projection.

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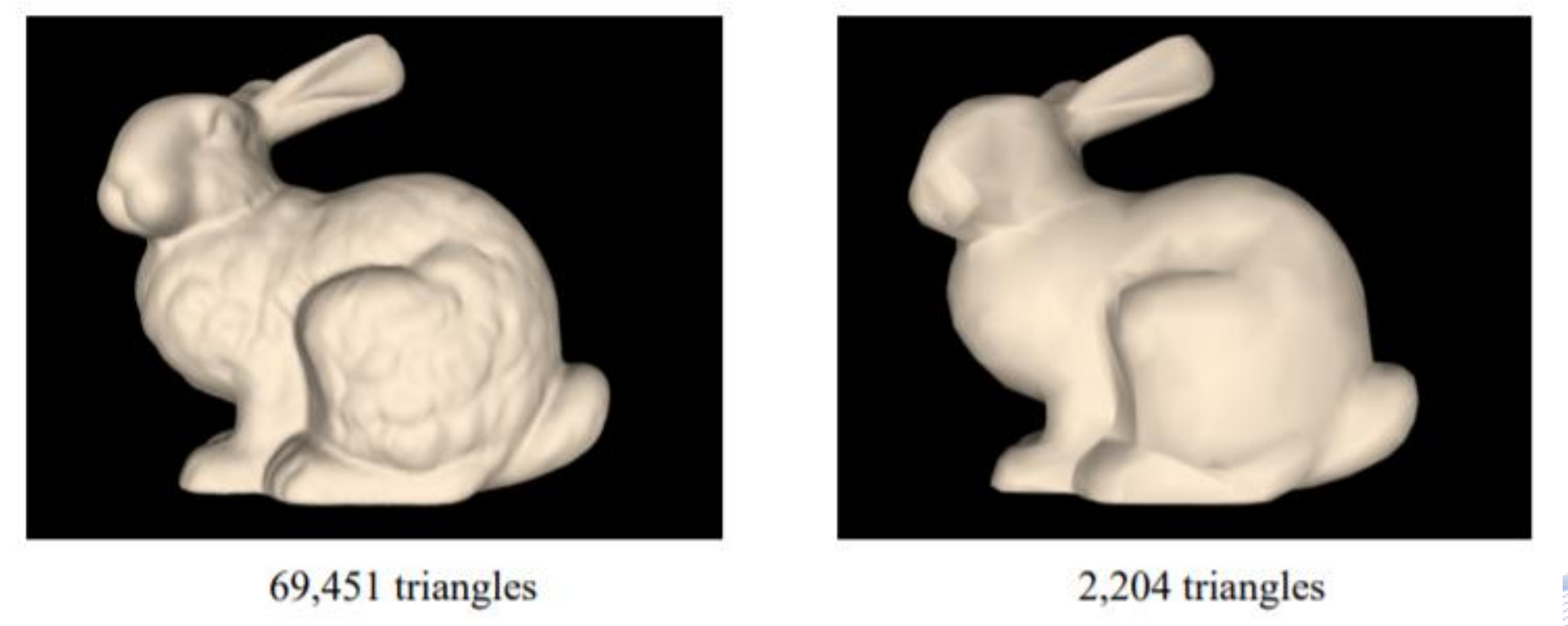
Surface Simplification



High and Low resolution representation.

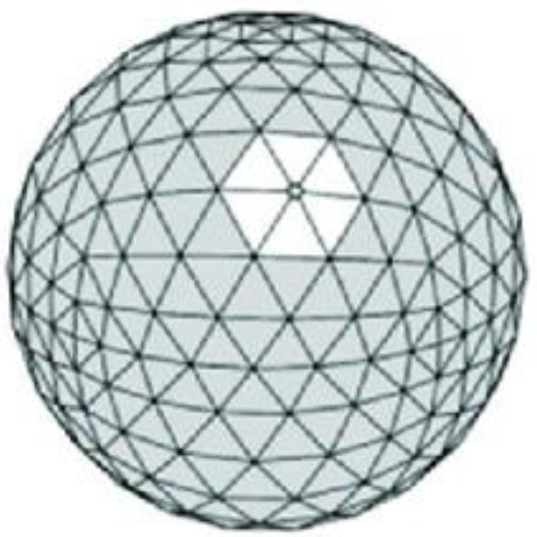
Source: [COH2000]

Manifold Mesh

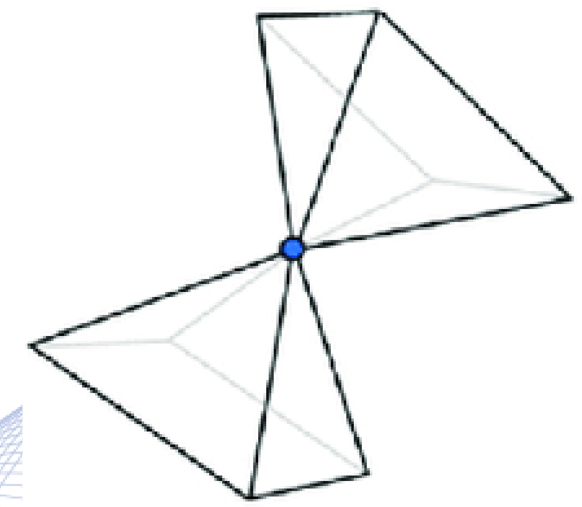
- Manifold triangle mesh has two topology properties:
  - Each vertex is adjacent to a set of triangles which form a single, complete cycle around the vertex.
  - Each edge is adjacent to exactly two triangles.

- Borders in manifold mesh:
  - An edge might be adjacent to one or two triangles.
  - A vertex might be surrounded by a single incomplete cycle.

Non-Manifold Mesh



Manifold Mesh.



Non Manifold Mesh.

Source: Wikipedia.org

Level of Detail

- Level of Detail describes complexity of a 3D model visualization by measuring the number of polygons in the object's surface. LoD is decreasing if the object is far of the virtual camera.

- LoD models can be divided in two categories:
  - Static LoD: Representation of an object as a set of independently meshes where everyone has a different number of triangles.

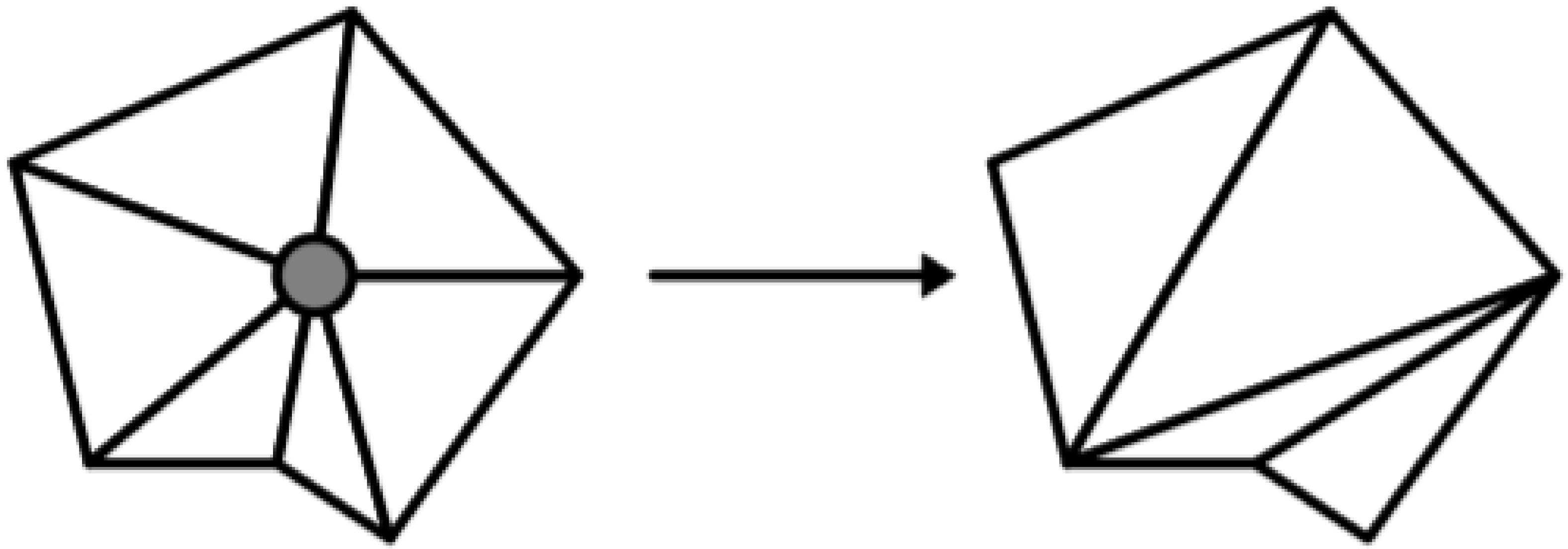
  - Dynamic LoD: Provide representations that are more carefully tuned to the viewing parameters of each particular rendered frame.

Simplification Operations

- Simplification of a 3D object can be achieved by performing local operations which are reducing the polygonal model complexity in a smaller amount.

- Simplification algorithms usually apply those operations in the input surface until the desired output complexity of the surface achieved. Such operations are:
  - Vertex Removal.
  - Vertex Clustering.
  - Edge Collapse.

Vertex Removal

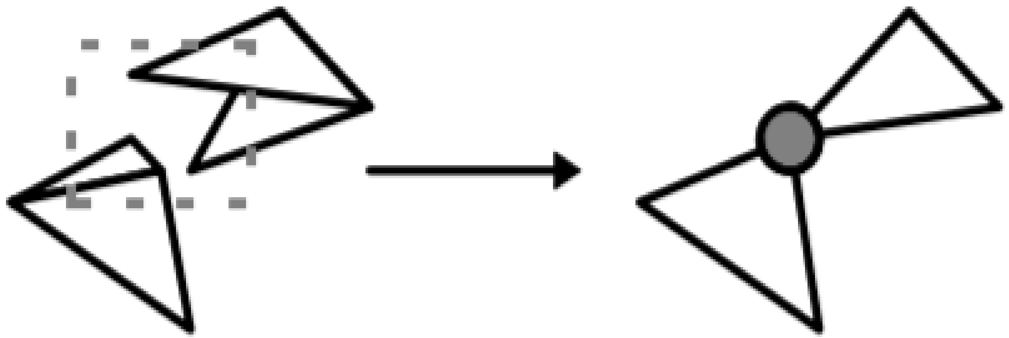


Vertex Remove operation.

Source: [COH2000]

Vertex Clustering

- Usual approach in this operation is to divide the object's bounding box into a small cells (cubes) and merge all vertices within each cube into one vertex.

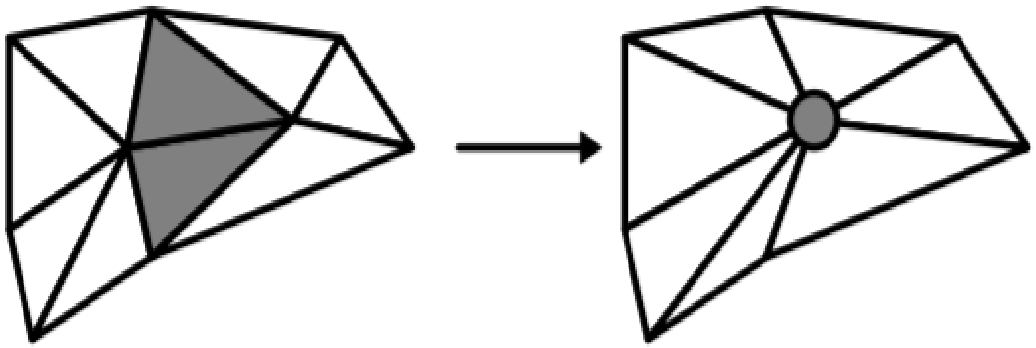


Source: [COH2000]

Edge Collapse

- The removal can be animated for a smooth transition.

- We move vertices toward the new location, then delete them.



Source: [COH2000]

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Illumination and Shading

- Illumination and Shading methods are two-step operations:
  - ***Illumination:*** As input we have parameters (e.g. light source position or intensity, surface, view point) which we are feeding them in the Illumination model and we produce as output the light intensity on a certain point.

  - ***Shading:*** Decision about which exactly points of the 3D scene will be used in order to calculate the Illumination model. All surface points must have assigned a given colour or gray level value in order to achieve illumination in scene.

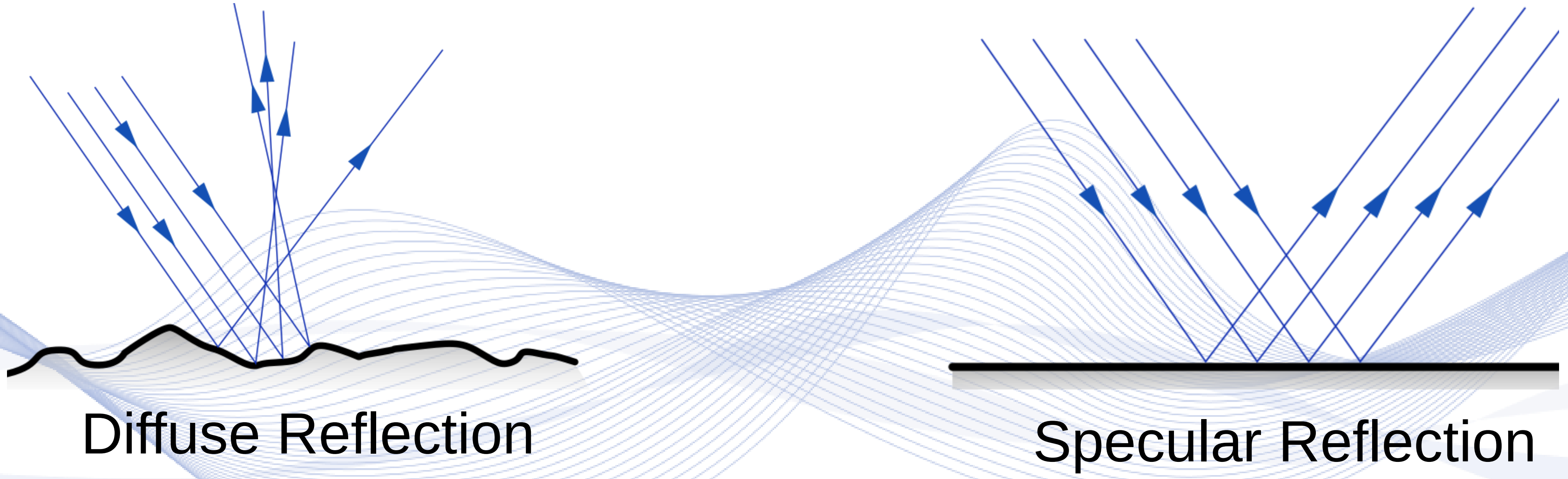
Lighting

- Surface intensity is determined by considering that an object has a instruct intensity associated with it. The illumination equation is defined as:

$$I = k_i,$$

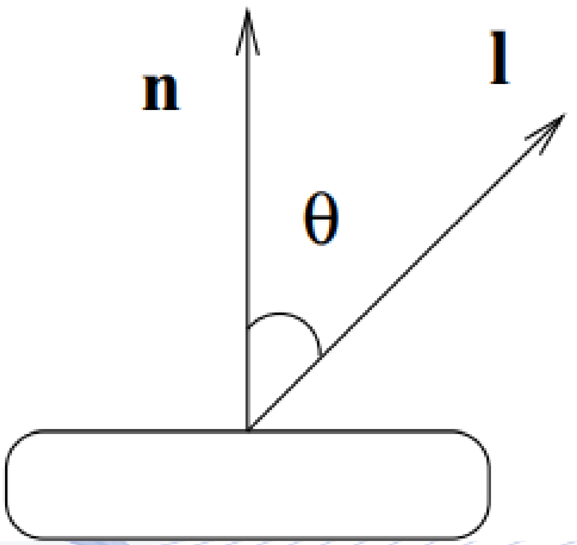
where k_i is the intrinsic intensity of an object.

Light Reflection

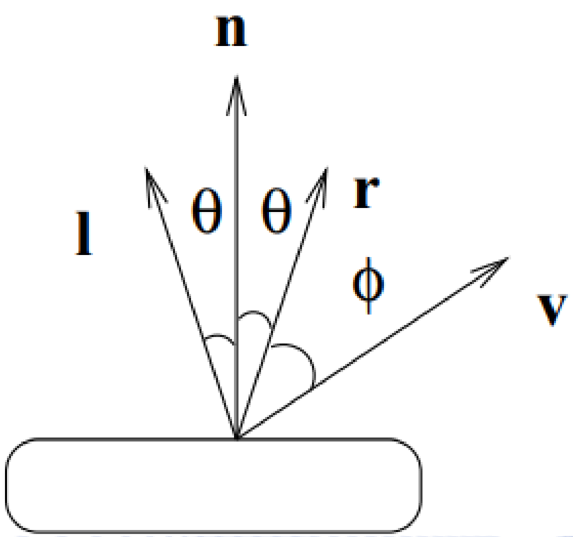


Source: Wikipedia.org

Diffuse vs Specular



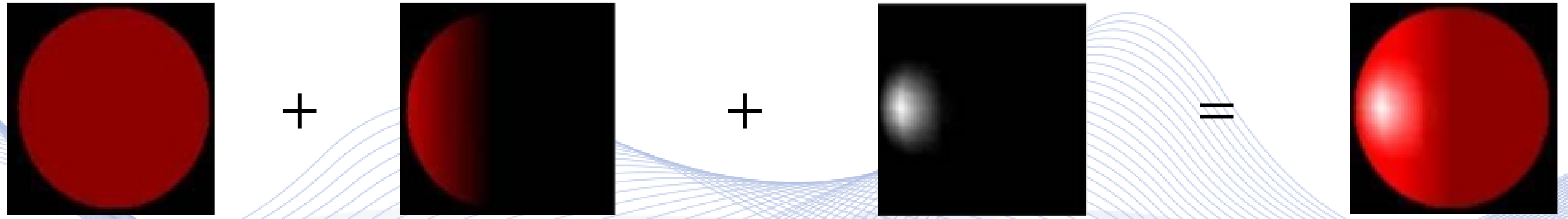
Diffuse Reflection



Specular Reflection

Phong Illumination

- By combining the above models (ambient, diffuse, specular) we can achieve the Phong illumination model.



Source: Wikipedia.org

Phong Illumination

- The phong illumination's model equation can be defined as:

$$I = I_a k_a + f_{att} I_p [k_d \cos\theta + k_s cos^n(\varphi)],$$

where the f_{att} represents the light source attenuation factor.

- Attenuation factor refers to the fact that light attenuates as it travels from its source to the object.

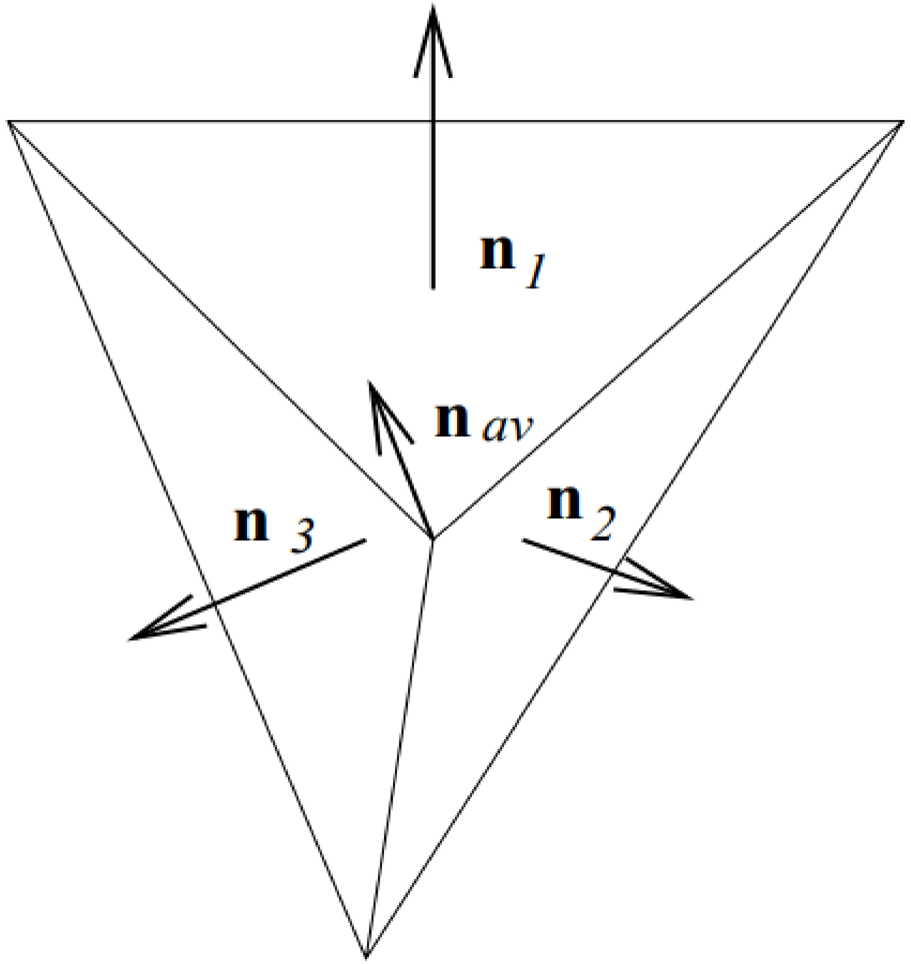
- f_{att} is inversely proportional to a function of the source-object distance.

Flat Shading

- Flat Shading is the simplest method where each polygon is assigned to a single intensity value and then use that value to shade the entire polygon.

- This method assumes that:
  - The light source is at infinity.
  - The viewer is at infinity.
  - The polygon is not a approximation of a curved surface.

Gouraud Shading



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Phong Shading

- Phong shading is designed in order to handle better the specular lighting than the Gouraud shading.
- Operates by deferring the illumination calculation until the fragment shading step.
- Thus, illumination values are calculated per pixel rather than per vertex.

Shading Models

Source: Wikipedia.org

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Hidden Surface Removal

- Once an object surface determined and illuminated then the visual parts of the surface must be found and projected on the view plane.

- Given n polyhedral faces in a 3D scene, we need a way to determine which parts are visible when viewed in a perpendicular direction to the projected plane.

Z-buffer algorithm

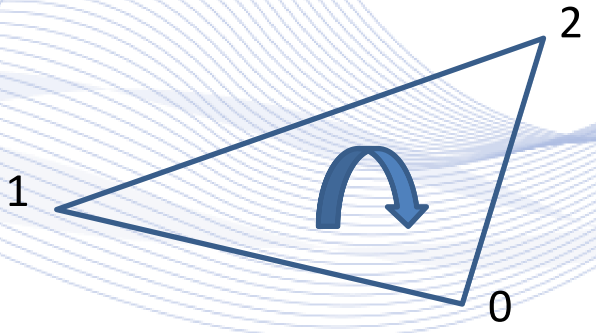
- Z-buffer also called Depth buffer algorithm is the simplest method of management the image depth coordinates.
- An image-space approach where Z-buffer algorithm is used to detect the visible surface by using z-depth operation in each surface.
- Each surface is processed pixel-by-pixel at time where the depth value of each one is compared and the smallest z surface determine the colour to be displayed in the frame buffer.

Backface Removal

- Backface removal or backface culling is working on solid objects where the view point is from the outside. Simplest method to discard polygons that face away from viewer.

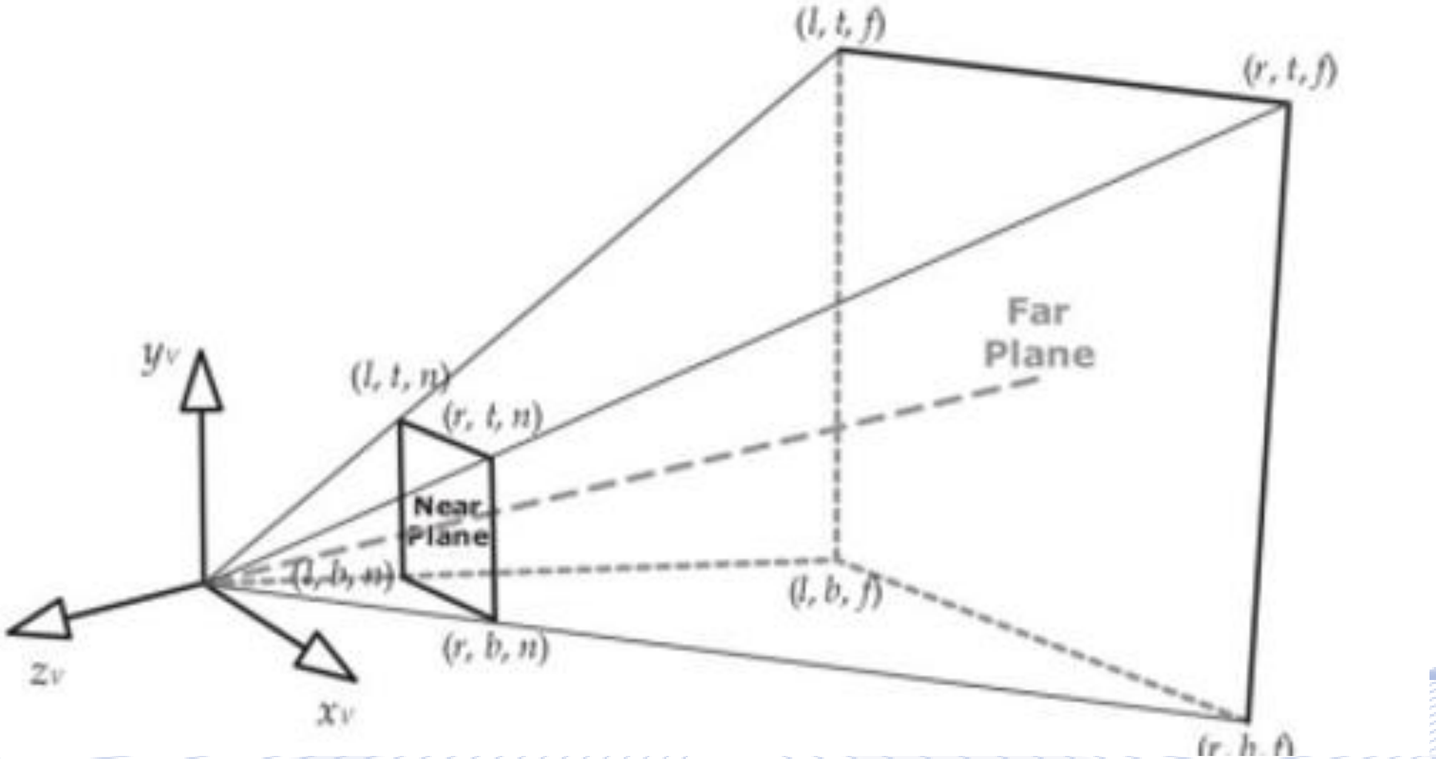
- Consistently oriented polygons are needed, where when we see the outer side the vertices, they appear counter clockwise.

Front facing



Back facing

Backface Removal



Space screen projection.

Source: [MOL2002]

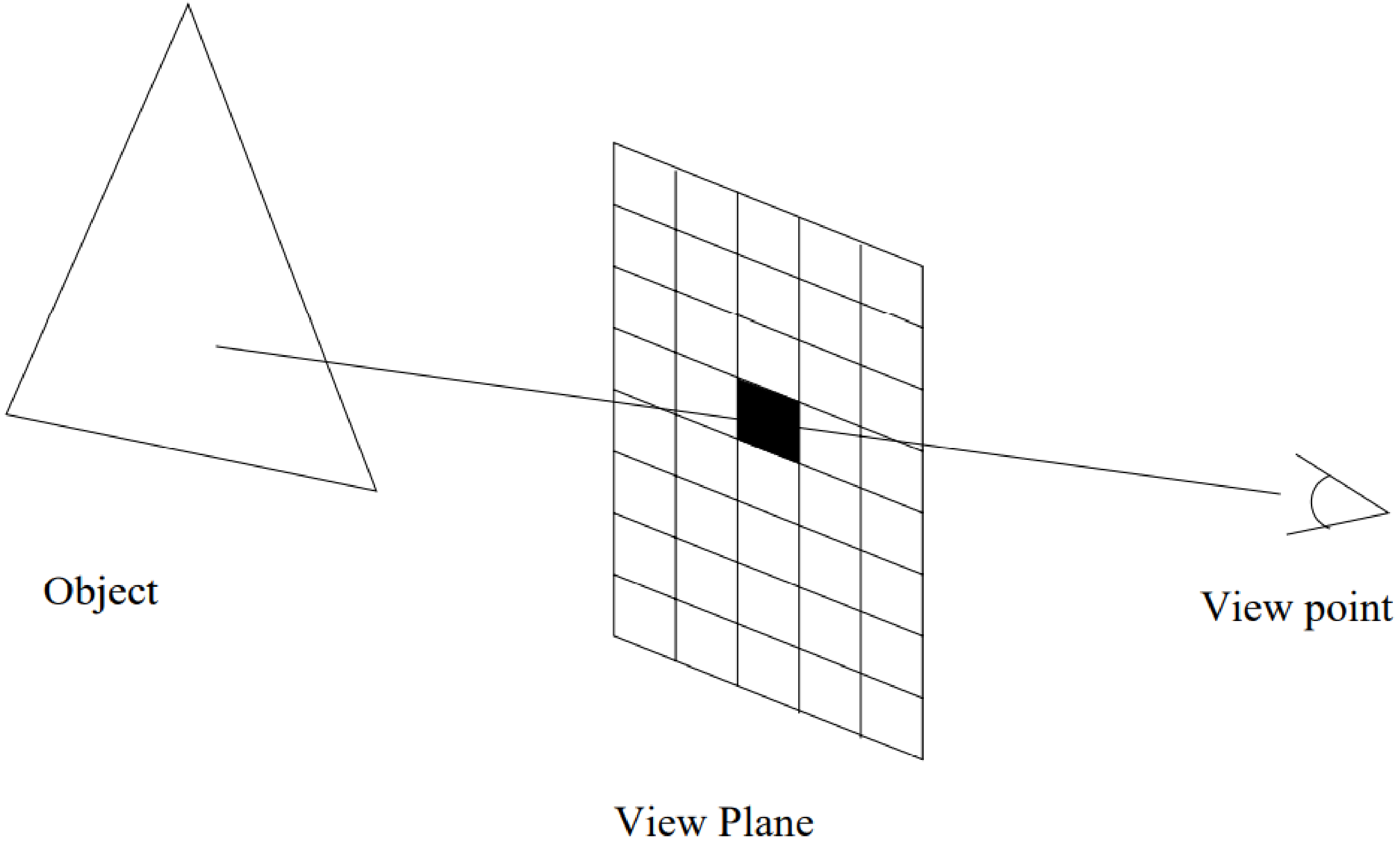
Ray tracing

- Ray tracing is an image space method, where a view ray is casted from each pixel.

- Those rays might be parallel to each other or they might converge at the same point known as view point (or center of projection).

- The intersection of the ray with the object surface is calculated and the view plane pixel is assigned an appropriate colour or gray value.

Ray tracing



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Z-buffer Gradient Algorithm

- Using the Z-buffer $Z(x, y)$ of a 3D volume we can determine the surface normal by finding the depth variations in the neighbour pixels.

- Neighbouring voxels on the object surface are projected on neighbouring pixels on the Z-buffer. Normal vector at a certain surface voxel (z, y, x) can be calculated with the gradient vector:

$$\nabla z = \left(\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, 1\right).$$

Gray Level Gradient Algorithm

- This method displays more accurate estimations for the gradient vector.

- Surface orientation is estimated by the gray values of the voxels in this neighbourhood. Given a voxel (z, y, x) on the object surface, the intensity gradient is calculated as follows:

$$
\begin{aligned}
G_x &= g(z, y, x+1) - g(z, y, x-1), \\
G_y &= g(z, y+1, x) - g(z, y-1, x), \\
G_z &= g(z+1, y, x) - g(z-1, y, x).
\end{aligned}
$$

Gray Level Gradient Algorithm

- The normalization of the gradient then evaluated as:

$$N_k = \frac{G_k}{\sqrt{G_x^2 + G_y^2 + G_z^2}} \qquad k = z, y, x.$$

- The evaluation of intensity gradient is similar to that of edge detection, with the difference that in edge detection we are only interested in gradient magnitude.

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Volume Rendering

- Volumetric Data is described as a set of 2D image slices stacked together in order to form a volume. Usually those slices are acquired by CT, MRI scanners.

- Volume Rendering techniques can be used in order to visualize either surfaces with shading or part of the volume with grey level values or a combination of them.

- Main idea is to display the data directly from the grey scale volume.

Ray Casting

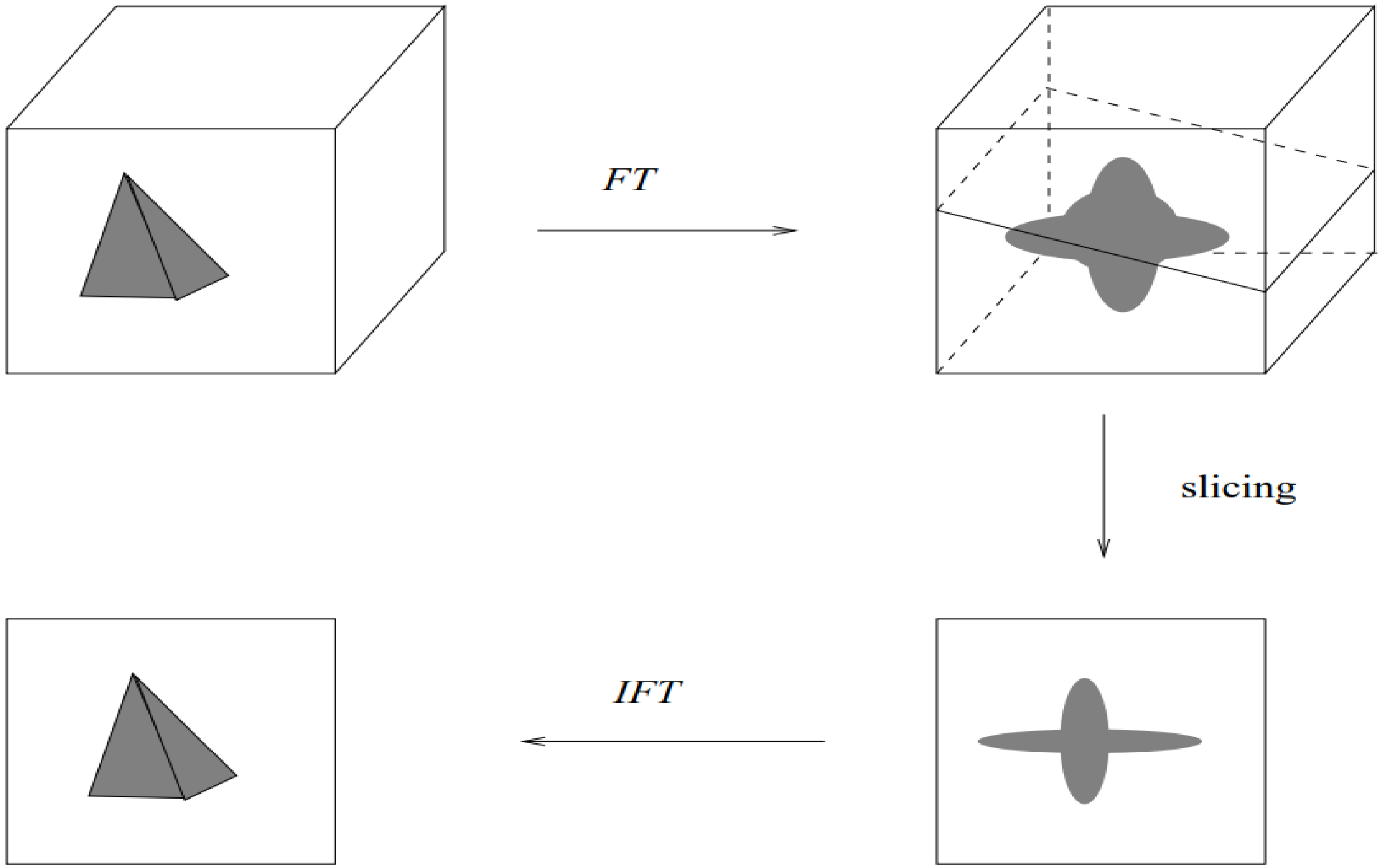
- Ray casting rendering method uses the weighted sum of the voxel colours along the ray, weights are considered the voxel opacities:

$$I(x,y) = \sum_{z=z_{min}}^{z=z_{max}} g(z,y,x)a(z,y,x)\,.$$

- A light source is placed at the view point, the transaction of the light is calculated for each voxel in the view ray.

- Voxel's examination is achieved in a front to back model.

Fourier Volume Rendering



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Surface vs Volume Rendering

- Surface rendering usually needs the definition of a threshold, specification of the intensity of the surface and then renders the smooth surface by passing all the pixels with the defined intensity.

- The above structure procceds by the simplification operation and if it fails, errors are appearing in the rendering.

- High noise if the threshold is low, non-physical holes if the threshold is high.

Bibliography

[PIT2021] I. Pitas, “Computer vision”, Createspace/Amazon, in press.
[PIT2017] I. Pitas, “Digital video processing and analysis” , China Machine Press, 2017 (in Chinese).
[PIT2013] I. Pitas, “Digital Video and Television” , Createspace/Amazon, 2013.
[NIK2000] N. Nikolaidis and I. Pitas, “3D Image Processing Algorithms”, J. Wiley, 2000.
[PIT2000] I. Pitas, “Digital Image Processing Algorithms and Applications”, J. Wiley, 2000.

Bibliography

[CHR2017] Christensson, Per. "Wireframe Definition." TechTerms. (March 22, 2017).

[PAL2018] Palomar, R., Gómez-Luna, J., Cheikh, F.A. et al. High-Performance Computation of Bézier Surfaces on Parallel and Heterogeneous Platforms. Int J Parallel Prog 46, 1035–1062 (2018).

[ROS1999] J. Rossignac and A. Requicha, Solid Modeling, Chapter in the Encyclopedia of Electrical and Electronics Engineering, Ed. J. Webster, John Wiley & Sons. 1999.

[LLO2013] Lloyd, Christopher. (2013). Surface modeling: high accuracy and high speed methods, by Tian-Xiang Yue. International Journal of Geographical Information Science.

[POU2020] O. Poursaeed, M. Fisher, N. Aigerman, Vladimir G. Kim, Coupling Explicit and Implicit Surface Representations for Generative 3D Modeling, arXiv, 2020.

[FAb2010] Fabry, Thomas & Smeets, Dirk & Vandermeulen, Dirk. (2010). Surface representations for 3D face recognition.

[MOL2002] Tomas Akenine-Möller, Spatial Data Structures and Speed-Up Techniques, Department of Computer Engineering Chalmers University of Technology, 2002.

Bibliography

[HEC1997] Heckbert, Paul & Garl, Michael. (1997). Survey of Polygonal Surface Simplification Algorithms.

[THE2001] Theoharis, Papaioannou, Karabassi A.. (2001). The Magic of the Z-Buffer: A Survey.

[KUM1996] Kumar, Subodh & Manocha, Dinesh & Garrett, Bill & Lin, Ming. (1996). Hierarchical Back-Face Culling.

[ICH2013] Ichikawa, Tsubasa & Yamaguchi, Kazuhiro & Sakamoto, Yuji. (2013). Realistic expression for full-parallax computer-generated holograms with the ray-tracing method. Applied optics.

[SCH2012] Schunke, Anja & Bromiley, Paul & Tautz, Diethard & Thacker, Neil. (2012). TINA manual landmarking tool: Software for the precise digitization of 3D landmarks. Frontiers in zoology. 9. 6. 10.1186/1742-9994-9-6.

[PIT2000] I.Pitas, Digital Image Processing Algorithms and Applications, Wiley, 2000.

[COH2000] Cohen, J.. "Concepts and Algorithms for Polygonal Simplification." (2000).

Q & A

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