3D Image Registration summary

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Version 1.5.2

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3D Image Registration

- **3D solid motion models**
- Image registration
- 3D Point cloud registration

2D motion models

- 2D solid motion model:
- Motion from 2D point $[x, y]^T$ to point $[x', y']^T$:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \end{bmatrix}.$$

  - From the 6 relevant parameters, only 3 are independent:
  - 1 rotation parameter (rotation angle) and the 2 translation vector components).

2D motion models

Geometric image transforms

- ***2D Image translation***:

$$b[i][j] = a[i+k][j+1].$$

- ***2D Image rotation***: If the image point $a(x, y)$ is rotated by θ degrees, its new coordinates (x', y') are given by:

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}.$$

2D motion models

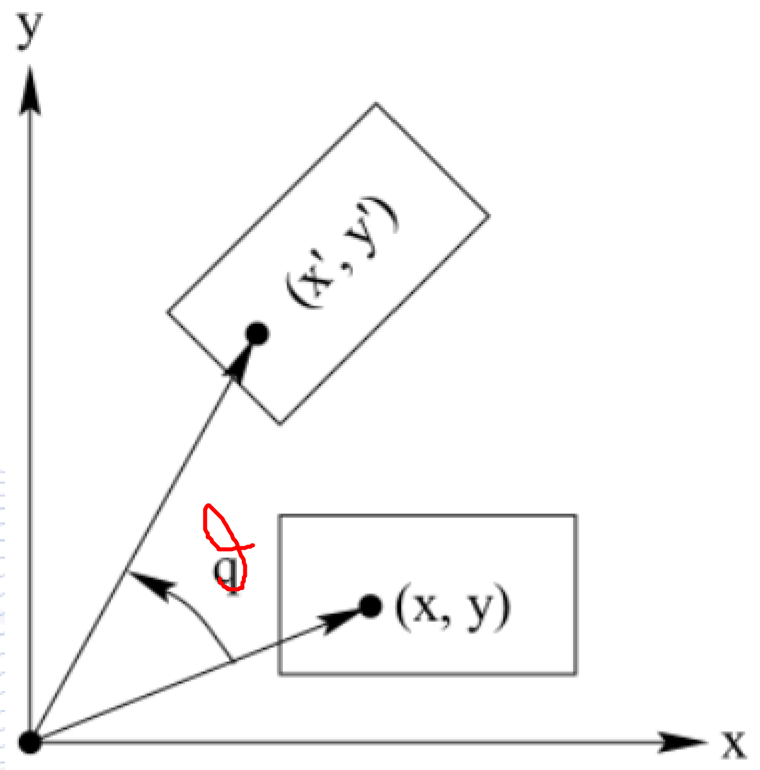

Image rotation.

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3D motion models

- 3D solid object motion can be described by the affine transformation:

$$\mathbf{X}' = \mathbf{R}\mathbf{X} + \mathbf{T},$$

where $\mathbf{T}$ is a 3×1 translation vector:

$$\mathbf{T} = \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix}$$

and $\mathbf{R}$ is a 3×3 rotation matrix (having various forms).

3D motion models

- In Cartesian coordinates, $\mathbf{R}$ can be described:
  - either by the Euler rotation angles about the three coordinate axes X, Y, Z.
  - or by a rotation axis and a rotation angle about this axis.
- The matrices describing the clockwise rotation around each axis in the three-dimensional space, are given by:

$$\mathbf{R} = \mathbf{R}_Z \mathbf{R}_Y \mathbf{R}_X.$$

- Their order ***does matter***.
- $\mathbf{R}$ is orthonormal, satisfying $\mathbf{R}^T = \mathbf{R}^{-1}$ and $det(\mathbf{R}) = \pm 1$.

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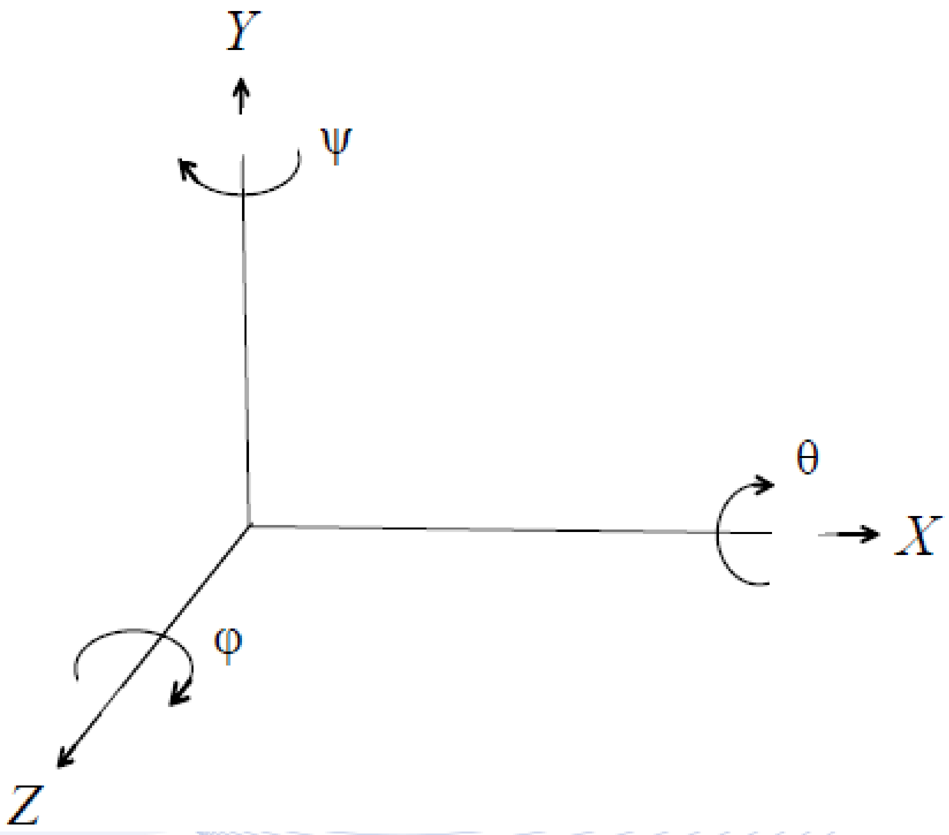
3D motion models



Euler rotation angles.

$$\mathbf{R}_X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix},$$

$$\mathbf{R}_Y = \begin{bmatrix} \cos\psi & 0 & \sin\psi \\ 0 & 1 & 0 \\ -\sin\psi & 0 & \cos\psi \end{bmatrix},$$

$$\mathbf{R}_Z = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

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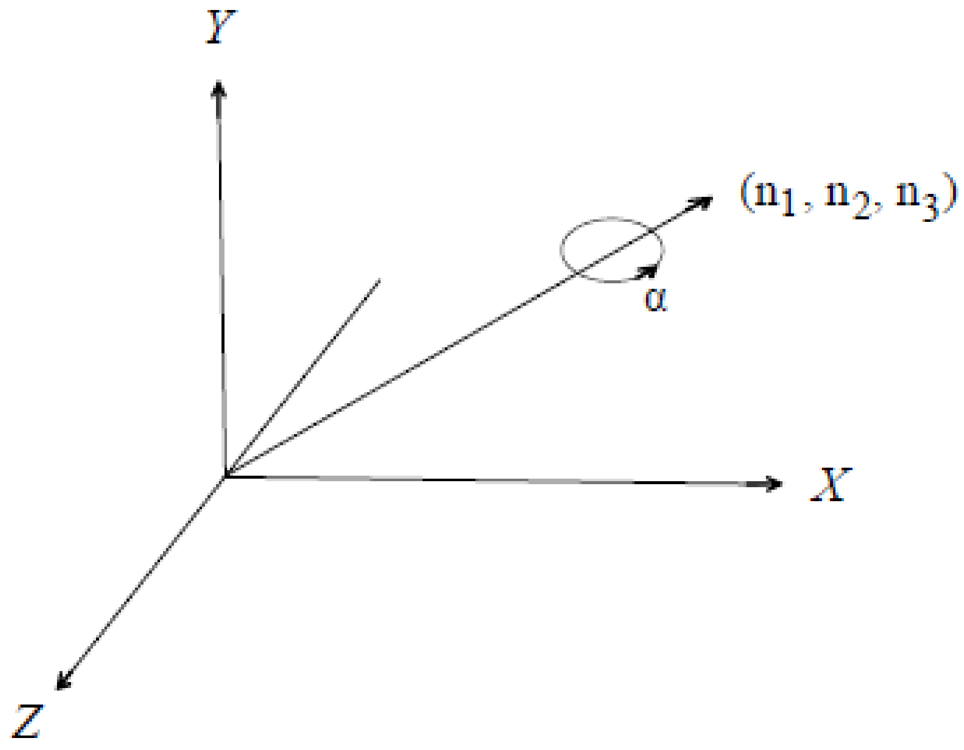
3D motion models



Object rotation about a rotation axis.

3D motion models

- 3D rotation can also be represented by ***quaternions*** that are extensions of complex numbers:

$$\mathbf{q} = q_0 + q_1\mathbf{i} + q_2\mathbf{j} + q_3\mathbf{k}$$

q_0, q_1, q_2, q_3 are real numbers and:

$$\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = \mathbf{ijk} = -1$$

- Unit quaternion $\mathbf{q}_R = [q_0\ q_1\ q_2\ q_3]^T$. It satisfies:

$$q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1.$$

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3D motion models

- 3D solid motion model:

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix}.$$

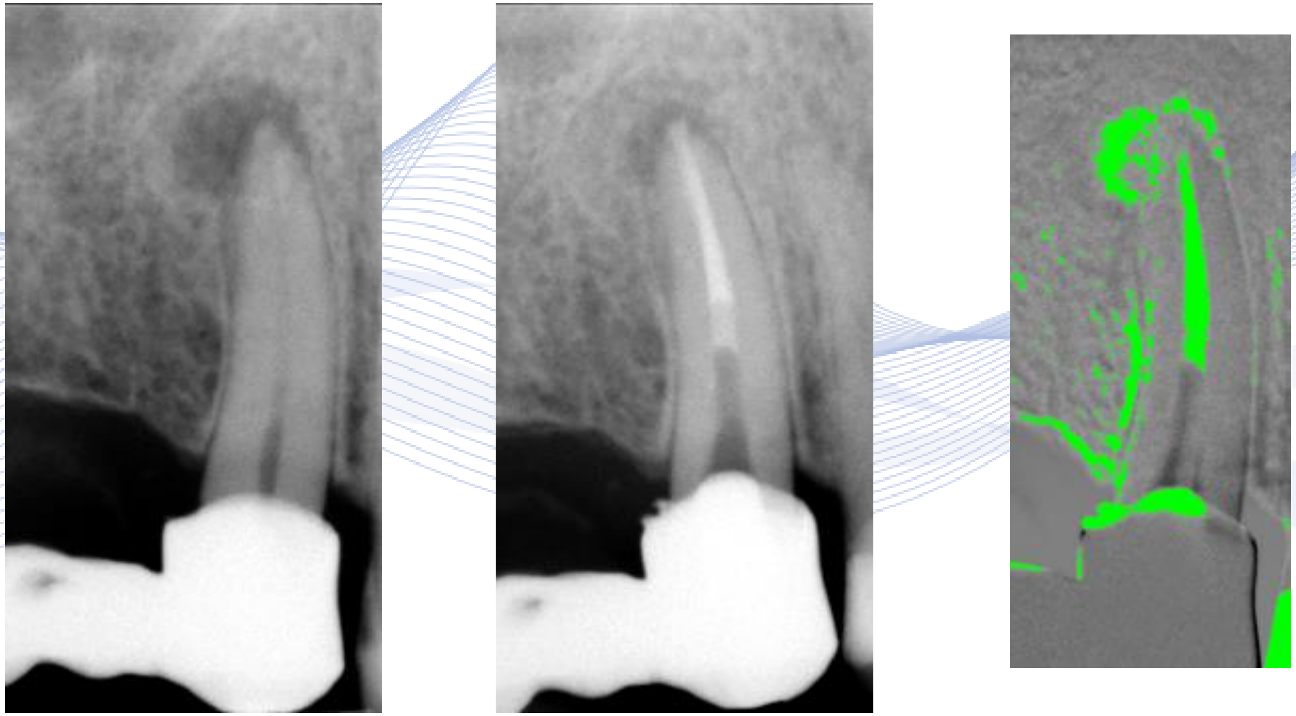
- From the 12 relevant parameters, only 6 are independent (3 rotation parameters and the 3 translation vector components).

Image registration

- 2D/3D solid motion models
- **Image registration**
  - **2D image registration**
  - 3D image registration
- Point cloud registration
  - 2D point cloud registration
  - 3D point cloud registration

2D Image registration

- 2D affine mapping transformation: it describes 2D rotation, translation and scaling.
- It can be used for 2D image registration and subtraction.



Subtractive radiography.

2D Image registration

- 2D affine transformation for image registration.
- Overlapping image regions are registered and mosaicking.

2D Image registration

- 2D image registration and mosaicking.

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Image registration

- 3D solid motion models
- Image registration
  - 2D image registration
  - 3D image registration
- **Point cloud registration**
  - 2D point cloud registration
  - **3D point cloud registration**

3D point cloud registration

- Registration of two 3D datasets is a common problem in 3D image analysis.
- Methods are distinguished by the type of geometric transforms they handle and whether the correspondence between matching points is known or not.
- The most used method is ***Iterative Closest Point*** (***ICP***).
  - It handles rigid transformations consisting of rotations and translations without a-priori knowledge of the correspondence between points.
  - The algorithm handles various types of data, including point sets, line segment sets, triangle sets and implicit/parametric surfaces.

3D point cloud registration

a) Model 3D tooth; b) Slice of model 3D tooth; c) Slice of data volume (3D tooth model rotated and translated along x-axis and translated along y-axis); d) Overlaid model (red channel) and registered (green channel) 3D tooth.

3D point cloud registration

The simplest case of the ICP algorithm is registration of two sets of geometrical data represented as point sets.

- Let N_p, N_x be the number of points in sets $\mathcal{P}, \mathcal{X}$ respectively:

$$\mathcal{P} = \{\mathbf{p}_i\}, \qquad i = 1, \dots, N_p,$$
$$\mathcal{X} = \{\mathbf{x}_i\}, \qquad i = 1, \dots, N_x,$$

- $\mathbf{p}_i, \mathbf{x}_i$ the 3D vectors (voxel coordinates):

$$\mathbf{x}_i = [X_i \; Y_i \; Z_i]^T.$$

ICP algorithm

ICP algorithm is an iterative algorithm that evaluates the correspondence between two 3D point sets.

- It estimates the optimum registration vector (the set of translation and rotation parameters that lead to the optimum registration)
- It applies the derived transformation to one of the sets.

The process is repeated until a certain dissimilarity measure (e.g., the mean square error) becomes smaller than a certain value.

SVD 3D Point Cloud Registration

- SVD-based methods demonstrate highest accuracy and stability when compared to quaternion-based methods [LIN2017].
- They are still inferior to ICP [BEL2015].

SVD 3D Point Cloud Registration

- First, it is assumed that the centroid of $\mathcal{P}$ is translated to the centroid of $\mathcal{X}$. Thus, restating the problem without translation, the points of the two sets can be rewritten as:

$$\mathcal{P}_c = \{\mathbf{p}_i - \boldsymbol{\mu}_p, i = 1, \dots, N_p\},$$
$$\mathcal{X}_c = \{\mathbf{x}_i - \boldsymbol{\mu}_x, i = 1, \dots, N_x\},$$

$$\boldsymbol{\mu}_p = \frac{1}{N_p}\sum_{i=1}^{N_p} \mathbf{p}_i, \qquad \boldsymbol{\mu}_x = \frac{1}{N_x}\sum_{i=1}^{N_x} \mathbf{x}_i.$$

- Using the points of the two sets $\mathcal{P}_c$, $\mathcal{X}_c$ the matrices $\mathbf{P}_c \in \mathcal{R}^{N_p \times 3}$, $\mathbf{X}_c \in \mathcal{R}^{N_x \times 3}$ are created. From now on we assume $N_p = N_x = N$.

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SVD 3D Point Cloud Registration

- Optimal rotation $\mathbf{R}$ implies the minimization of the transformation error:

$$\mathbf{R} = \text{argmin}_{\mathbf{R}} \|\mathbf{X}_c - \mathbf{P}_c\mathbf{R}\|_F^2.$$

- This is an ***orthogonal Procrustes problem.***
  - Matrix approximation problem.
- ***Frobenius norm*** $\|\cdot\|_F$ is defined as:

$$\|A\|_F = \sqrt{tr(A^T A)} = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^2}.$$

Q & A

Thank you very much for your attention!

More material in
http://icarus.csd.auth.gr/cvml-web-lecture-series/

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