Motion and Fourier Transform Tutorial Exercise

Exercise
The impulse response of a camera can be modelled by:

$$h(x,y,t) = \begin{cases} \frac{1}{T_x T_y T_e} & |x| < \frac{T_x}{2}, \quad |y| < \frac{T_y}{2}, \quad t \in (0, T_e), \\ 0 & , \quad \text{otherwise.} \end{cases}$$

where T_x, T_y are the horizontal and vertical size of the camera aperture and T_e is the exposure time. The camera is looking at a scene consisting of a cube, with edge length equal to B, moving in parallel with the camera imaging plane. The image projected on the camera plane can be described by:

$$\psi(x,y,t) = \begin{cases} 1 & \text{if } -B/2 + v_x t < x < B/2 + v_x t, -B/2 < y < B/2, \\ 0 & \text{otherwise .} \end{cases}$$

where v_x is the horizontal movement speed. The Fourier Transform of $h(x,y,t)$ is:

$$H(f_x, f_y, f_t) = \frac{\sin(\pi f_x T_x)}{\pi f_x T_x} \frac{\sin(\pi f_y T_y)}{\pi f_y T_y} \frac{1}{T_e} \frac{e^{-i2\pi f t \frac{T_e}{2}} - e^{i2\pi f t \frac{T_e}{2}}}{2i\pi f_t} e^{i\pi f_t T_e} =$$
$$= \frac{\sin(\pi f_x T_x)}{\pi f_x T_x} \frac{\sin(\pi f_y T_y)}{\pi f_y T_y} \frac{\sin(\pi f_t T_e)}{\pi f_t T_e} e^{i\pi f_t T_e}.$$

Assuming that $B >> T_x$ and $B >> T_y$, derive camera signal and its spectrum.