Huffman Encoding Tutorial Exercise 2

Exercise

Consider a source with alphabet A, consisting of the symbols {a, b, c, d} with probabilities {0.5000, 0.2143, 0.1703, 0.1154} respectively. Suppose, also, that the conditional distribution of a sample $\mathcal{F}_n$ given its previous sample $\mathcal{F}_{n-1}$ is described by the following matrix:

$$Q = \begin{bmatrix} 0.6250 & 0.3750 & 0.3750 & 0.3750 \\ 0.1875 & 0.3125 & 0.1875 & 0.1875 \\ 0.1250 & 0.1875 & 0.3125 & 0.1250 \\ 0.0625 & 0.1250 & 0.1250 & 0.3125 \end{bmatrix},$$

where element i, j specifies the conditional probability $q(i|j)$, which is the probability that sample $\mathcal{F}_n$ is the i^{th} symbol, given that $\mathcal{F}_{n-1}$ is the j^{th} symbol. The joint probability density function of every two samples is given by:

$$p(f_{n-1}, f_n) = p(f_{n-1})q(f_n|f_{n-1}).$$

Apply Huffman encoding (using vector length = 2) in order to produce the codebook and estimate the average and minimum bit rate per vector sample for this source.