

# Conditional Huffman Encoding Tutorial Exercise

## Exercise

Consider a source with alphabet A, consisting of the symbols {a, b, c, d} with probabilities {0.5000, 0.2143, 0.1703, 0.1154} respectively. Suppose, also, that the conditional distribution of a sample  $\mathcal{F}_n$  given its previous sample  $\mathcal{F}_{n-1}$  is described by the following matrix:

$$Q = \begin{bmatrix} 0.6250 & 0.3750 & 0.3750 & 0.3750 \\ 0.1875 & 0.3125 & 0.1875 & 0.1875 \\ 0.1250 & 0.1875 & 0.3125 & 0.1250 \\ 0.0625 & 0.1250 & 0.1250 & 0.3125 \end{bmatrix}.$$

where element  $i, j$  specifies the conditional probability  $q(i|j)$ , which is the probability that sample  $\mathcal{F}_n$  is the  $i^{\text{th}}$  symbol, given that  $\mathcal{F}_{n-1}$  is the  $j^{\text{th}}$  symbol. The joint probability density function of every two samples is given by:

$$p(f_{n-1}, f_n) = p(f_{n-1})q(f_n|f_{n-1}).$$

Design a codebook using conditional Huffman encoding and calculate the average and minimum bit rate per vector sample for this source.