Fast Convolution Algorithms for deep learning and computer vision

Sample slides only

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Artificial Intelligence & Information Analysis Lab

Outline

- 1D convolutions

  Linear & Cyclic 1D convolutions

  Discrete Fourier Transform, Fast Fourier Transform

  Winograd algorithm

- Linear & Cyclic 2D convolutions

- Applications in deep learning

  Convolutional neural networks

Motivation

- Fast implementation of 1D and 2D digital filters

  Image filtering

  Image feature calculation

  - Gabor filters

- Fast implementation of 1D and 2D correlation

  Template matching

  Correlation tracking

- Machine learning

  Convolutional Neural Networks

Linear 1D convolution

- The one-dimensional (linear) convolution of:
  - an input signal x and
  - a convolution kernel h (filter finite impulse response) of length N:

$$y(k) = h(k) * x(k) = \sum_{i=0}^{N-1} h(i)x(k-i)$$

- For a convolution kernel centered around 0 and $N = 2v + 1$, it takes the form:

$$y(k) = h(k) * x(k) = \sum_{i=-v}^{v} h(i)x(k-i)$$

Linear 1D convolution - Example

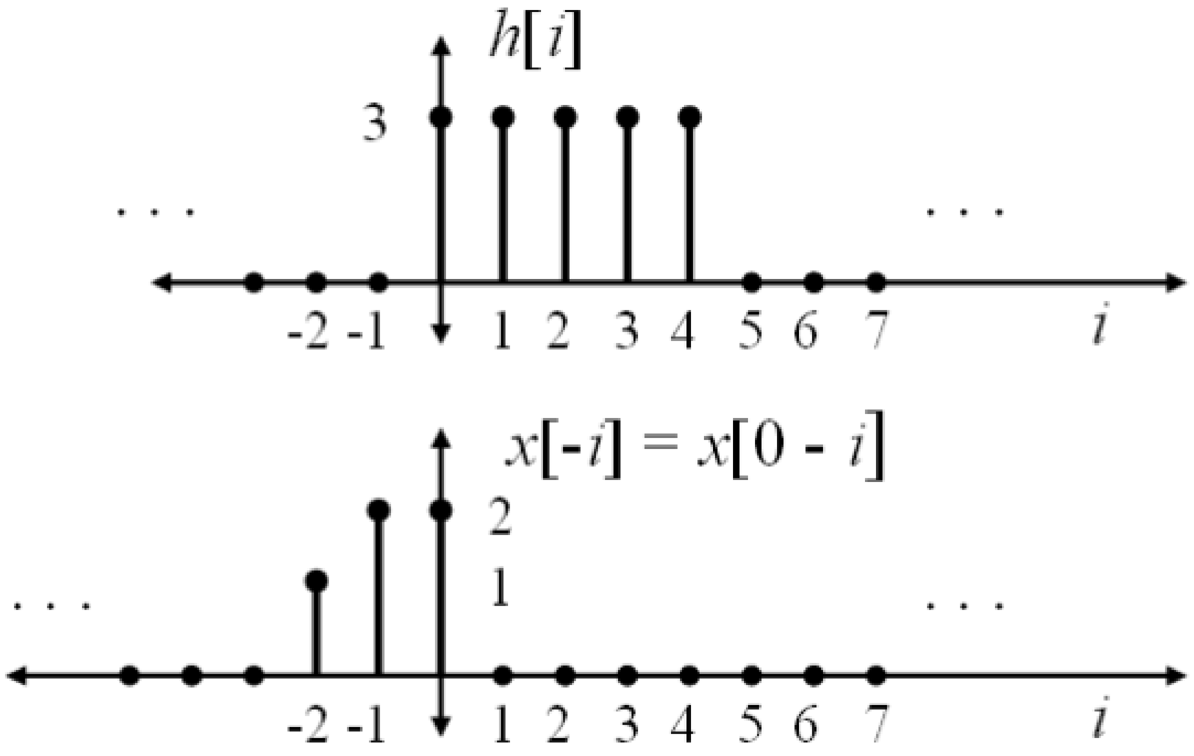


Image source: http://electricalacademia.com/signals-and-systems/example-of-discrete-time-graphical-convolution/

Linear 1D convolution - Example

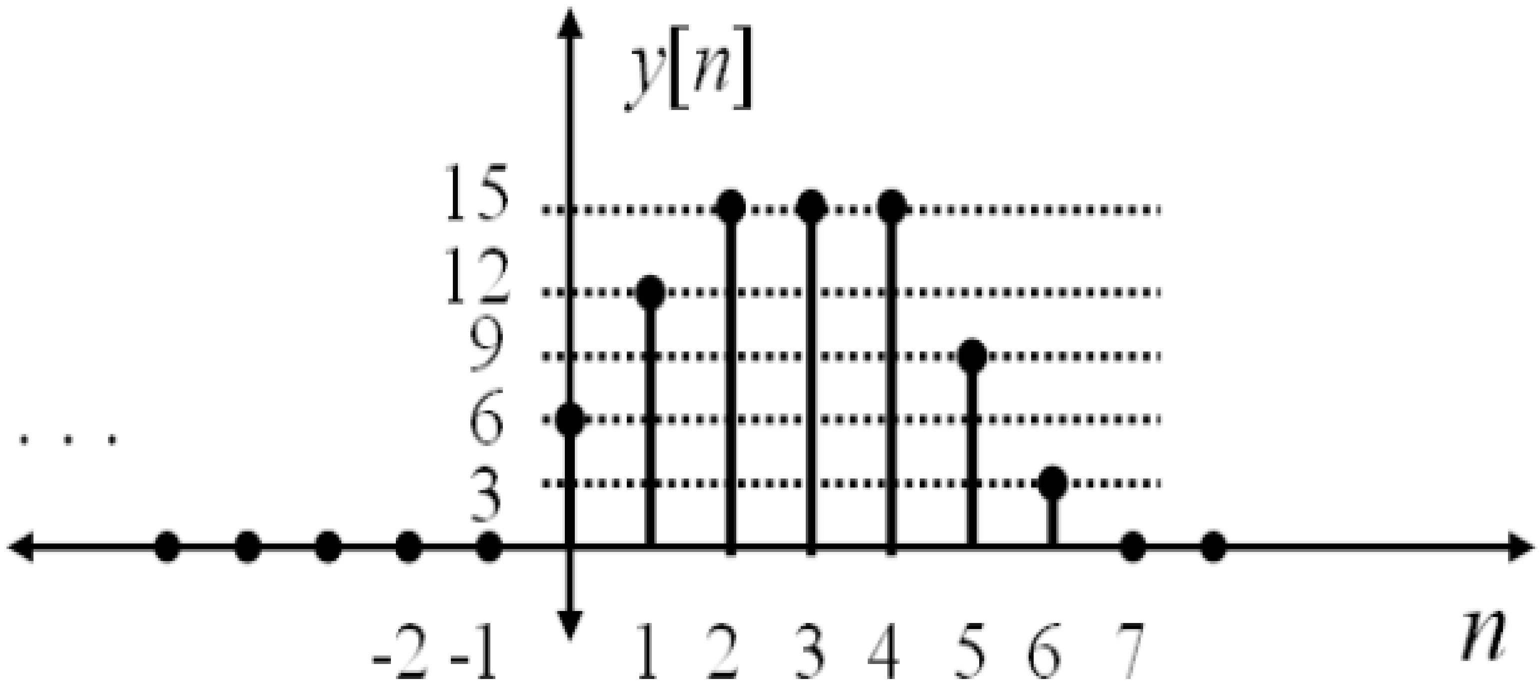


Image source: http://electricalacademia.com/signals-and-systems/example-of-discrete-time-graphical-convolution/

Linear 1D correlation

- Correlation of template h and input signal $x(k)$:

$$r(k) = \sum_{i=0}^{N-1} h(i)x(k+i)$$

- Input signal is not flipped.
- It is used for template matching and for object tracking in video.
- It is often confused with convolution: they are identical only if h is centered at and is symmetric about $i=0$.

Cyclic 1D convolution

- One-dimensional cyclic convolution of length N , $(k)_N = k\ mod\ N$:

$$y(k) = x(k) \circledast h(k) = \sum_{i=0}^{N-1} h(i)x(((k-i)_N))$$

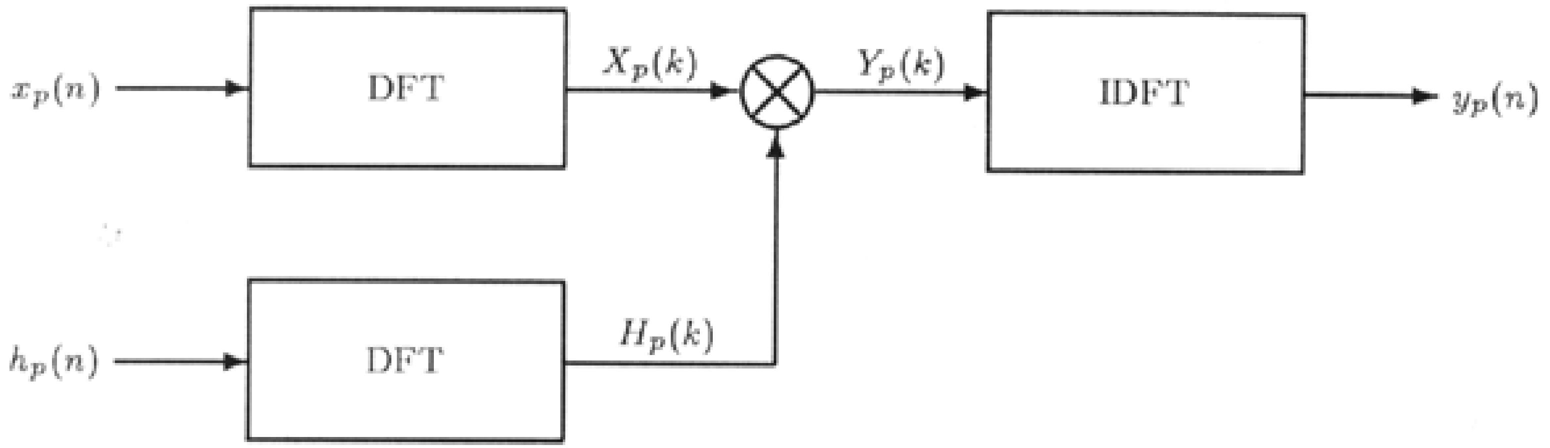
- Embedding linear convolution in a cyclic convolution $y(n) = x(x) \otimes h(n)$ of length $N \geq L + M - 1$ and then performing a cyclic convolution of length N:

$$y(k) = x(k) \circledast h(k) = \sum_{i=0}^{N-1} x_N(i)\, h_n(((k-i)_N))$$

Cyclic Convolution via DFT

Cyclic convolution can also be calculated using 1D DFT:

$$\boldsymbol{y} = IDFT(DFT(\boldsymbol{x})DFT(\boldsymbol{h}))$$



1D FFT

- There are a few algorithms to speed up the calculation of DFT.
- The most well known is the **radix-2** decimation-in-time (**DIT**) Fast Fourier Transform (**FFT**) (Cooley-Tuckey).

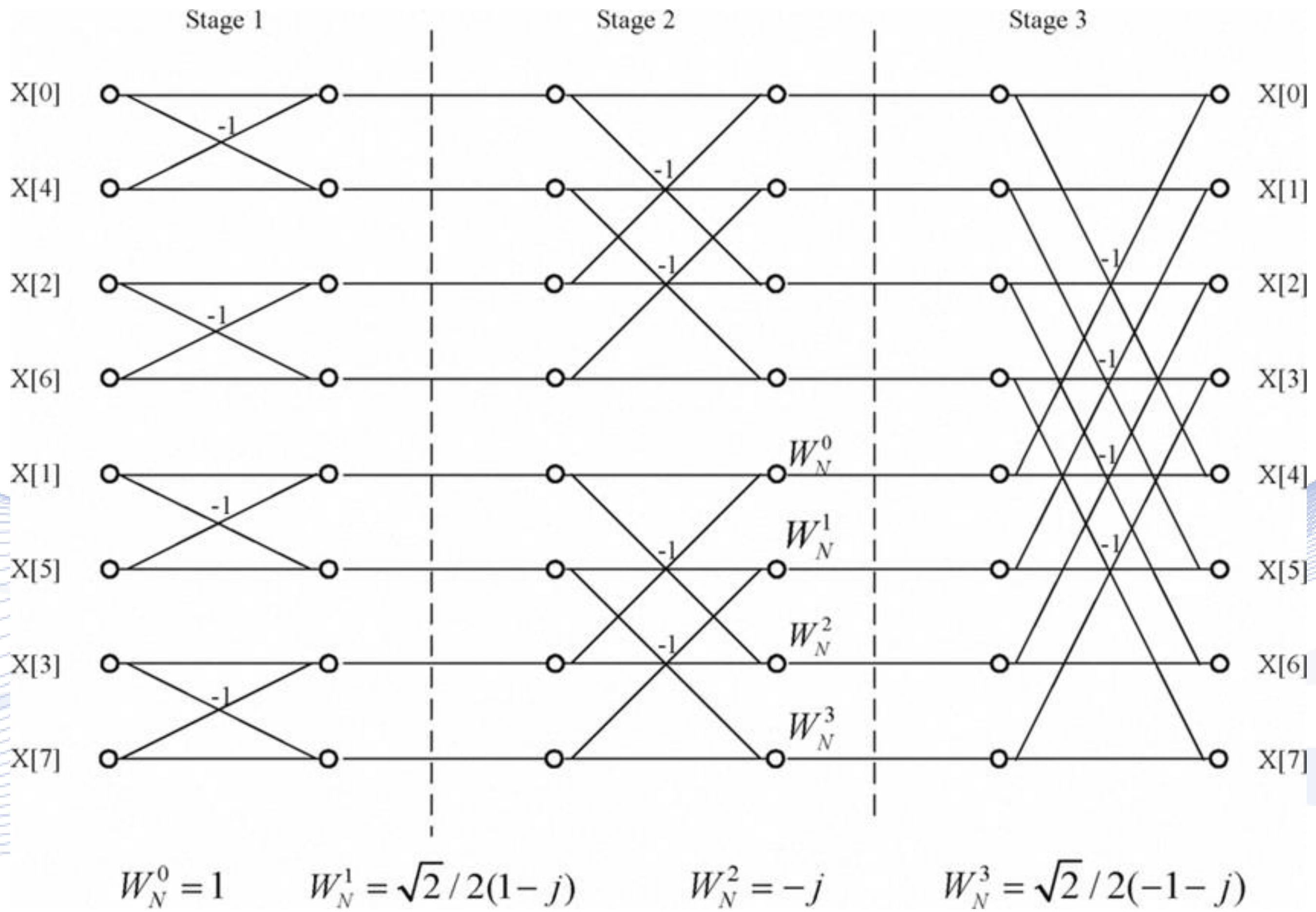
1. The DFT of a sequence $x(n)$ of length N is:

$$X(k) = \sum_{n=0}^{N-1} x(n) \; e^{-\frac{2\pi i}{N}nk}$$

where k is an integer ranging from 0 to $N - 1$.

1D FFT

- radix-2 FFT breaks a length-N DFT into many size-2 DFTs called "butterfly" operations.
- There are log_2N stages.



Z-transform

The Z-transform of a signal (function) $x(n)$ having domain $[0, \ldots, N]$ is given by:

$$X(z) = \sum_{n=0}^{N-1} x(n) z^{-n}$$

The domain of Z-transform is the complex plane, since z is a complex number.
The following relation holds for the Z-transform:

$$y(n) = x(n) * h(n) \Leftrightarrow Y(z) = X(z)H(z)$$

Cyclic convolution and Z-transform

$$y(k) = x(k) \circledast h(k) = \sum_{i=0}^{N-1} h(i)x((k-i)_N)$$

Where : $(k)_N = k \bmod N$

$$y(n) = x(n) \circledast h(n) <=> Y(z) = X(z)H(z) \bmod(z^N - 1)$$

Winograd algorithm
Fast 1D cyclic convolution with minimal complexity

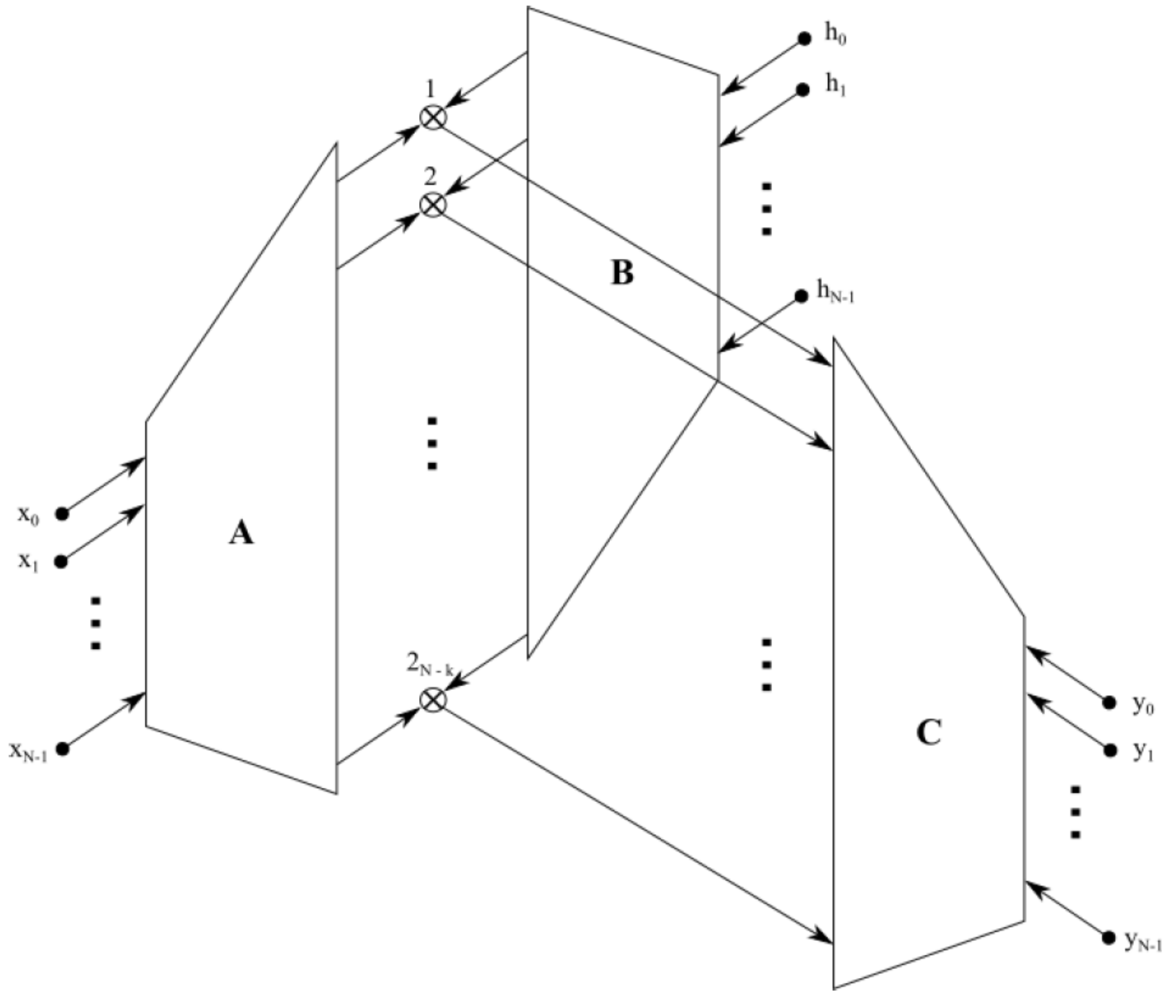
- The Winograd algorithm works on small tiles of the input image.
- The input tile and filter are transformed
- The outputs of the transform are multiplied together in an element-wise fashion
- The result is transformed back to obtain the outputs of the convolution.

Winograd algorithm
Fast 1D cyclic convolution with minimal complexity

- Winograd convolution algorithms or fast filtering algorithms:

$$Y = \mathbf{C}(\mathbf{Ax} \otimes \mathbf{Bh})$$

- They require only $2N - \nu$ multiplications in their middle vector product, thus having minimal complexity.
- ν: number of cyclotomic polynomial factors of polynomial $z^N - 1$ over the rational numbers Q.
- GEneral Matrix Multiplication (GEMM) BLAS or CUBLAS routines can be used.



Linear and cyclic 2D convolutions

- Two-dimensional linear convolution with convolutional kernel h of size $N_1 \times N_2$ is given by:

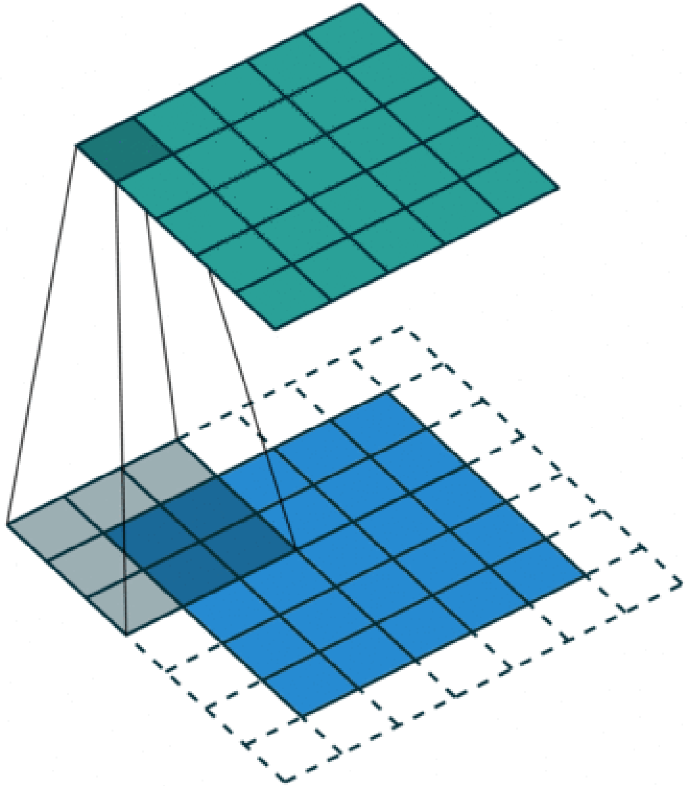
$$y(k_1, k_2) = h(k_1, k_2) ** x(k_1, k_2) = \sum_{i_1}^{N_1} \sum_{i_2}^{N_2} h(i_1, i_2) x(k_1 - i_1, k_2 - i_2)$$

- Its two-dimensional cyclic convolution counterpart of support $N_1 \times N_2$ is defined as:

$$y(k_1, k_2) = h(k_1, k_2) \circledast\circledast x(k_1, k_2) = \sum_{i_1}^{N_1} \sum_{i_2}^{N_2} h(i_1, i_2) x((k_1 - i_1)_{N_1}, (k_2 - i_2)_{N_2})$$

2D Convolution - Example

- With Padding



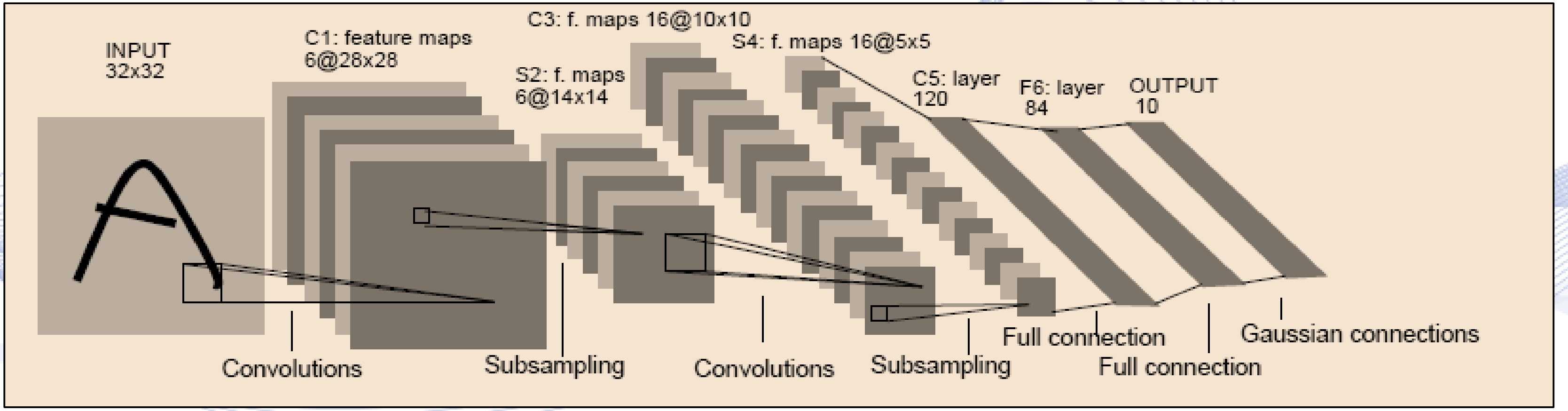
Applications

- Convolutional neural networks
- Signal processing
  - Signal filtering
  - Signal restoration
  - Signal deconvolution
- Signal analysis
  - Time delay estimation
  - Distance calculation (e.g., sonar)
  - 1D template matching

Convolutional Neural Networks

Convergence of machine learning and signal processing processing

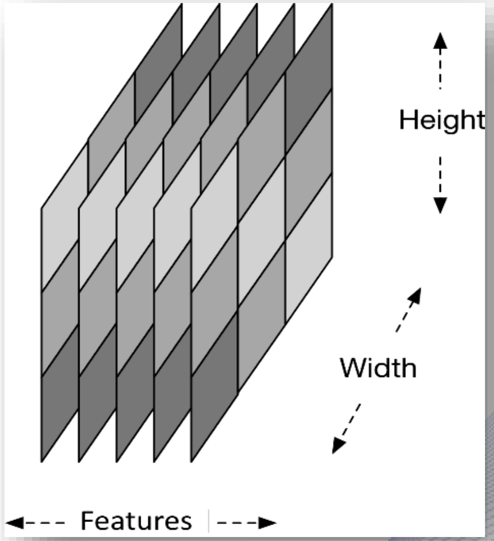
- Two step architecture:
  - First layers with sparse NN connections: convolutions.
  - Fully connected final layers.
- Need for fast convolution calculations.



Convolutional Layer

For RGB images

- For a convolutional layer l with an activation function $f_l(\cdot)$, multiple incoming features d_{in} and one single output feature o.



Multiple input features to single feature o transformation

$$y^{(l)}(i,j,o) = f_l\left(b^{(l)} + \sum_{r=1}^{d_{in}} \sum_{k_1=-q_1}^{q_1^{(l)}} \sum_{k_2=-q_2}^{q_2^{(l)}} w^{(l)}(k_1,k_2,r,o)\, x^{(l)}(i-k_1, j-k_2, r) \right)$$

Convolutional Layer Activation Volume (3D tensor)

$$a_{ij}^{(l)}(o) = f_l\left(b^{(l)}(o) + \sum_{r=1}^{d_{in}} \boldsymbol{W}^{(l)}(r,o) * \boldsymbol{X}_{ij}^{(l)}(r) \right) \qquad \boldsymbol{A}^{(l)} = [a_{ij}^{(l)}(o): i = 1,..,n^{(l)}, j = 1,..,m^{(l)}, o = 1, \dots, d_{out}]$$

where $\boldsymbol{A}^{(l)}$ is the activation volume for the convolutional layer l, $\boldsymbol{W}^{(l)}(r,o)$ is a 2D slice of the convolutional kernel $\boldsymbol{W}^{(l)} \in \mathbb{R}^{h_1 \times h_2 \times d_{in} \times d_{out}}$ for input feature r and output feature o , $b^{(l)}(o)$ a scalar bias and $\boldsymbol{X}_{ij}^{(l)}(r)$ a region of input feature r centered at $[i,j]^T$, e.g. $\boldsymbol{X}^{(1)}(1)$ the R channel of an image $d_{in} = C = 3$.

Deep Learning Frameworks

Framework	User Interface	Data Parallelism	Model Parallelism
Caffe	protobuf, C++, Python	Yes	Limited
CNTK	BrainScript, C++, C#	Yes	No
TensorFlow	Python, C++	Yes	Yes
Theano	Python	No	No
Torch	LuaJIT	Yes	Yes

Image Source: Heehoon Kim, Hyoungwook Nam, Wookeun Jung, and Jaejin Le - Performance Analysis of CNN Frameworks for GPUs

Deep Learning Frameworks

- All 5 frameworks work with cuDNN as backend.
- cuDNN unfortunately not open source
- cuDNN supports FFT and Winograd

Framework	User Selectable	Heuristic-based	Profile-based	Default
Caffe	No	Yes	No	Heuristic-based
CNTK	No	No	Yes	Profile-based
TensorFlow	No	No	No	Heuristic-based†
Theano	**Yes**	Yes	Yes	GEMM
Torch	**Yes**	Yes	Yes	GEMM

†TensorFlow uses its own heuristic algorithm

Image Source: Heehoon Kim, Hyoungwook Nam, Wookeun Jung, and Jaejin Le - Performance Analysis of CNN Frameworks for GPUs

The Neon story

- Developed by Nervana in 2015
- Written in Python and C
- Doesn’t support Windows
- Uses MKL for CPU (highly optimized by Intel)
- Supports CUDA for GPU
- Known mostly to be the first to implement Winograd faster than others.

Q & A

Thank you very much for your attention!

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