Image acquisition, camera geometry

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Image acquisition

- A still image visualizes a still object or scene, using a still picture camera.
- A video sequence (moving image) is the visualization of an object or scene illuminated by a light source, using a video camera.
- The captured object, the light source and the video camera can all be either moving or still.
- Thus, moving images are the projection of moving 3D objects on the camera image plane, as a function of time.
- Digital video corresponds to their spatiotemporal sampling.

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Light reflection

- Objects reflect or emit light.
- Reflection can be decomposed in two components:
  - *Diffuse reflection* (distributes light energy equally along any spatial direction, allows perceiving object color).
  - *Specular reflection* (strongest along the direction of the incident light, incident light color is perceived).
- *Lambertian surfaces* perform only diffuse reflection, thus being dull and matte (e.g., cement surface).

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Light reflection

- *Ambient illumination* sources emit the same light energy in all directions (e.g., a cloudy sky).

- *Point illumination* sources emit light energy isotropically or anisotropically (e.g., ordinary light bulbs) along various directions.

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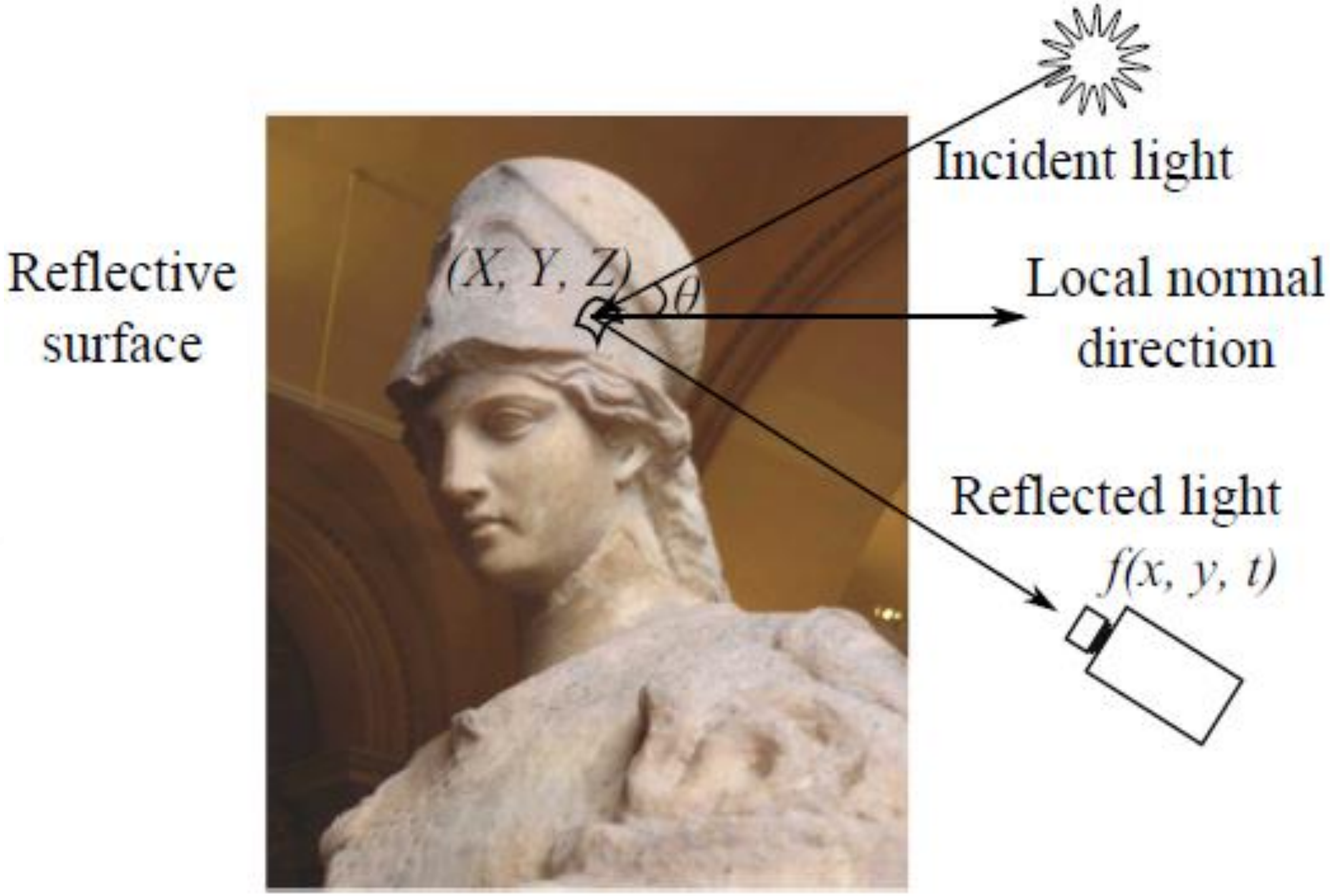
Light reflection



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Light reflection

- Reflected irradiance when object surface produces diffuse reflectance and incident light source comes from:
  - Ambient illumination:

$$f_r(X,Y,Z,t,\lambda) = r(X,Y,Z,t,\lambda) \cdot E_\alpha(t,\lambda)$$

  - Point light source:

$$f_r(X,Y,Z,t,\lambda) = r(X,Y,Z,t,\lambda) \cdot E_p(t,\lambda) \cdot \cos\theta$$

  - Distant point source and ambient illumination:

$$E(t,\lambda) = E_a(t,\lambda) + E_p(t,\lambda) \cdot \cos\theta$$

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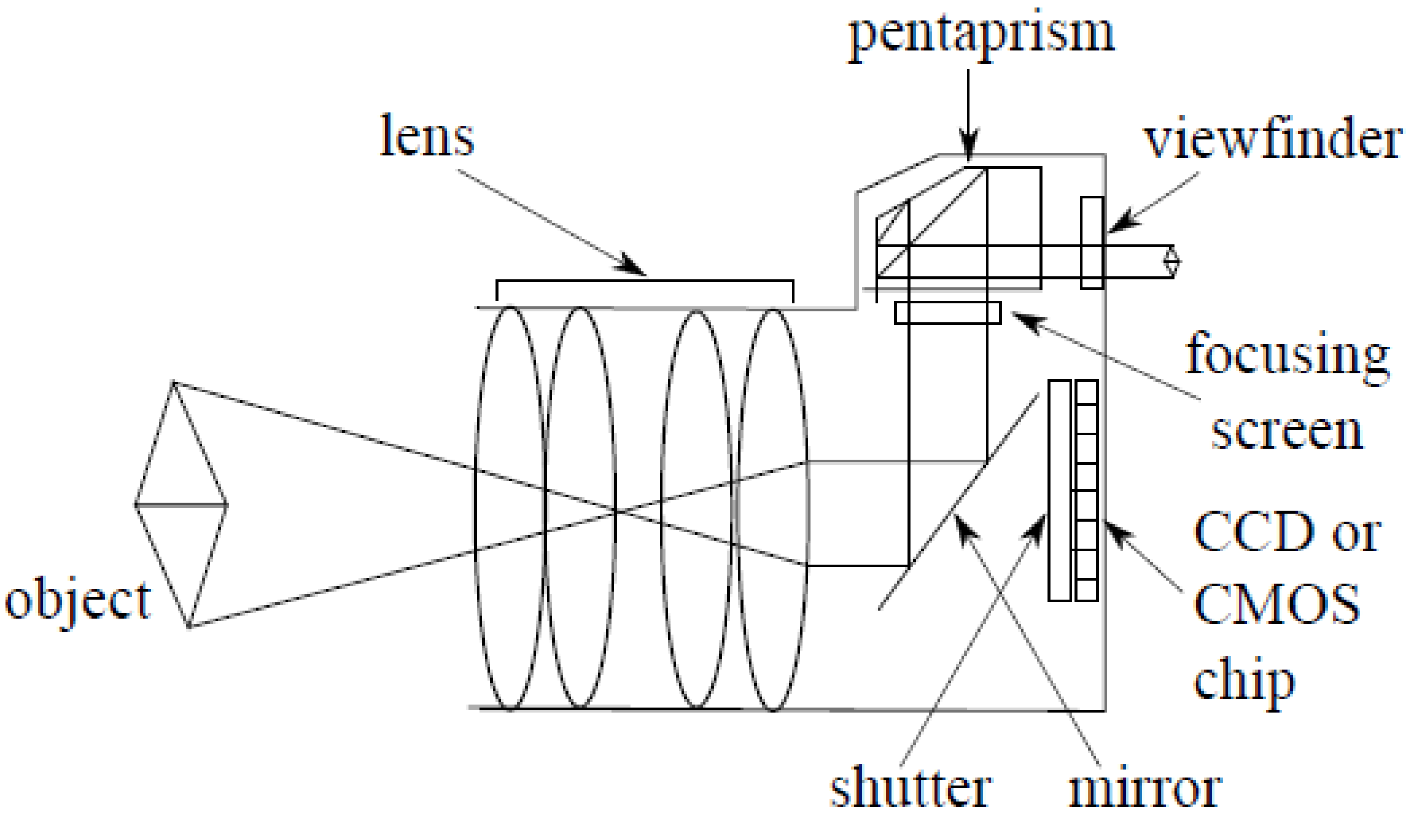
Camera structure



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Camera structure

- The *lens* is the most important part of the camera.
- Incident light rays pass through a lens (or a group of lenses) and get focused on the semiconductor chip.
- The distance between the lens center (*optical center*, $\mathbf{O}$) and the point of convergence of the light rays inside the camera (*focal point*, $\mathbf{F}$) is called *focal length.*
- Focal length characterizes the lens and determines the scene part to be captured as well as scene object sizes (*magnification*).

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Camera structure

- Two kinds of lenses:
  - Fixed (e.g., *prime*) and
  - Zoom (e.g., *telephoto*)

- Based on their focal length, lenses are categorized in wide-angle, normal and telephoto:
  - Wide-angle lenses have smaller focal length than normal, thus capturing wider parts of the scene and exaggerating differences in the relative distance and size between foreground and background objects.

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Camera structure

- The *shutter* opens and closes to control the time interval during which light rays can hit the CCD or CMOS chip.

- *Shutter speed* is the speed at which the shutter opens and closes and determines the amount of incoming light.
- Higher speed is required for capturing unblurred, fast moving scenes, while lower speed is used in night shooting, along with bigger *aperture* size.

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Camera structure

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- Aperture size is usually expressed in *f-numbers.* The bigger the f-number the smaller the aperture size.

- It controls the *depth of field* (DOF), the distance between the nearest and farthest focused objects in the image

- The smaller the aperture size is, the longer the depth of field, since less light rays are captured on the image for each visible 3D scene point.

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Pinhole Camera and Perspective Projection

- The rather naïve *pinhole camera system* can be used to accurately model the geometric and optical aspects of most modern cameras through the *pinhole perspective projection model* or *central perspective projection model.*
  - A very small aperture size is considered
  - Camera pinhole coincides with the *optical center*, or *center of projection* or *camera center.*

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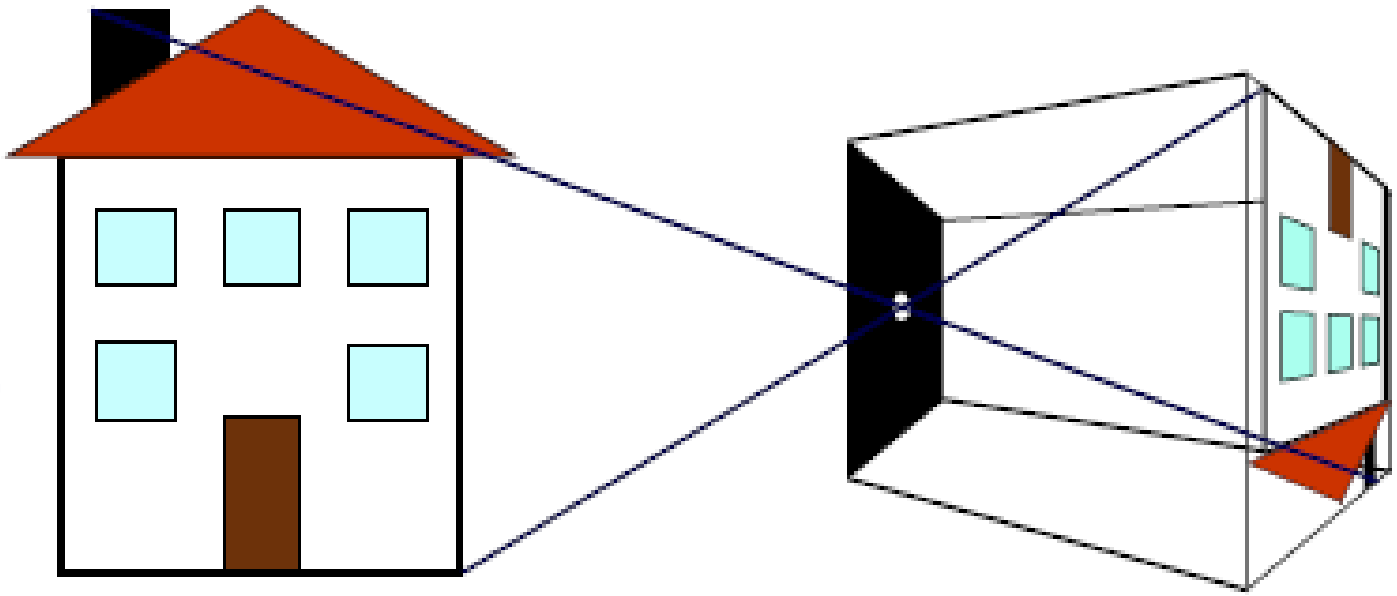
Pinhole Camera and Perspective Projection



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Pinhole Camera and Perspective Projection

- Let us consider 2 coordinate systems:
  - the *camera* (or *standard*) *coordinate system* $(\mathbf{O}_c, X_c, Y_c, Z_c)$ and
  - the image coordinate system $(\mathbf{o}, x, y)$.

- X_c, *and* Y_c define the plane $\mathcal{F}$ that is parallel to the camera image plane $\mathcal{I}$, lying at a focal length f behind the optical center $\mathbf{O}_c$ along the optical axis Z_c.

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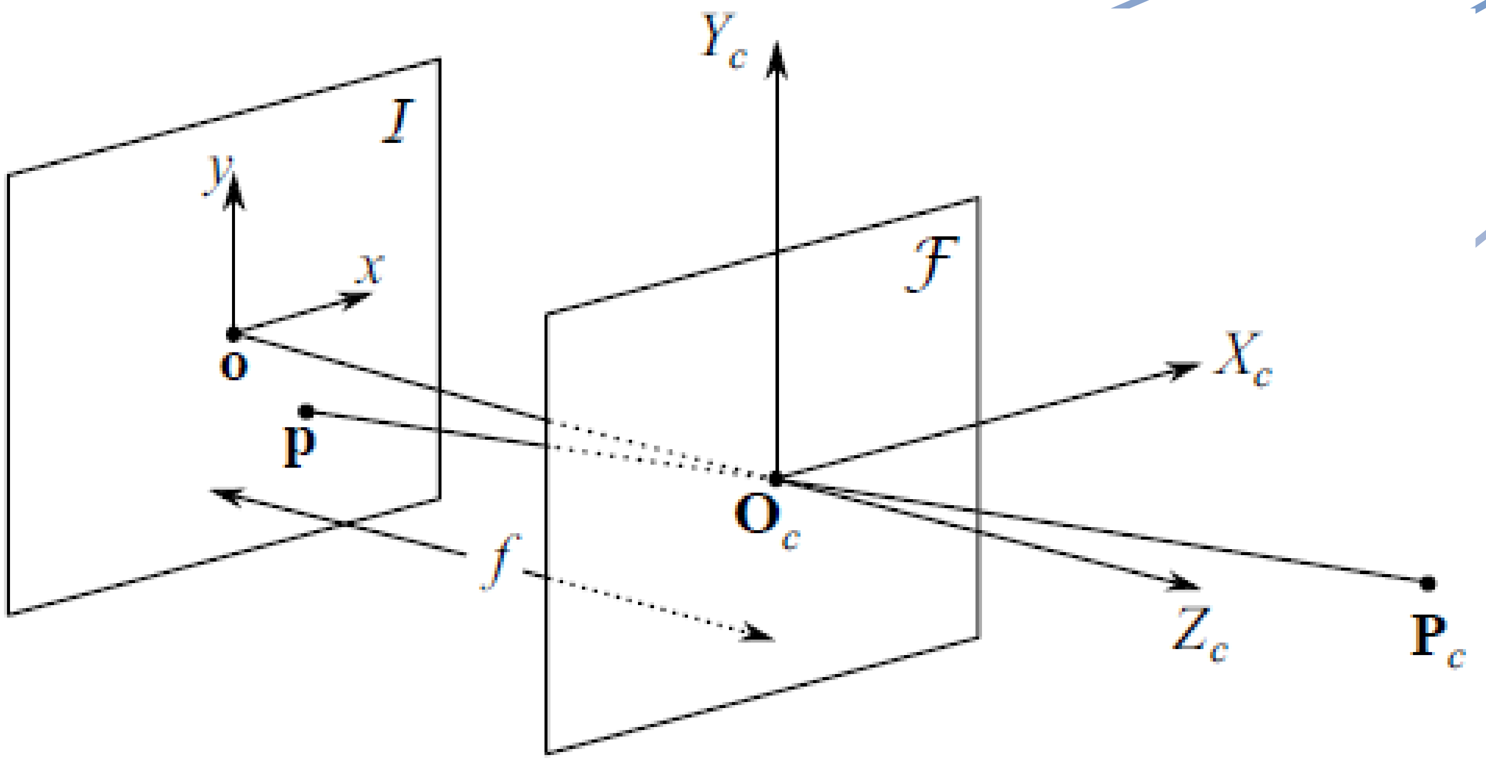
Pinhole Camera and Perspective Projection



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Pinhole Camera and Perspective Projection

- Points projected on the image plane are assigned camera coordinates of opposite sign:
  - images are inverted.

- In order to facilitate the mathematical treatment, we can define a virtual image plane $\mathcal{I}'$, in front of $\mathcal{F}$ at a positive distance f.

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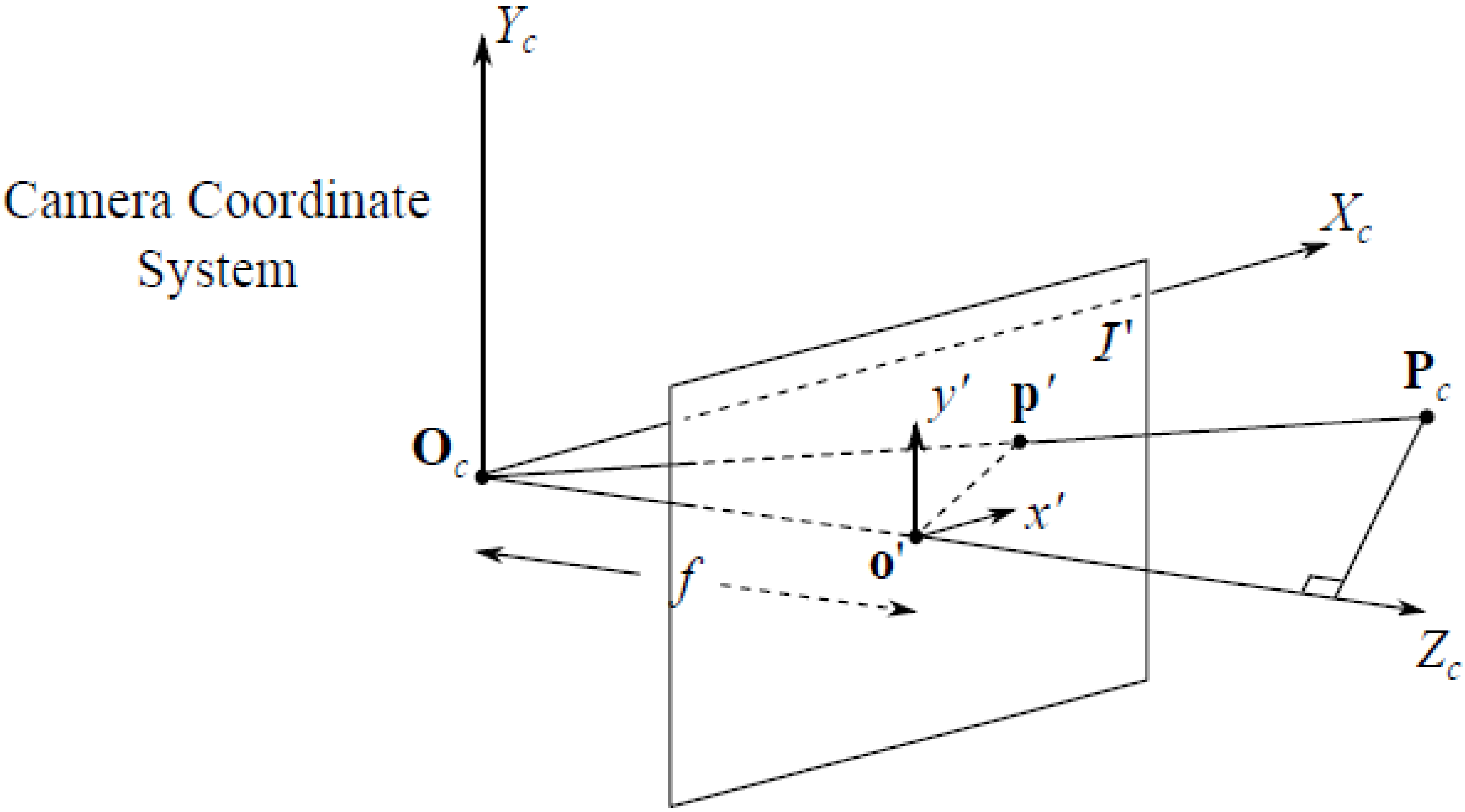
Pinhole Camera and Perspective Projection



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Pinhole Camera and Perspective Projection

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- We want to derive the equations that connect a 3D point (3D vector) $\mathbf{P}_c = [X_c, Y_c, Z_c]^T$ referenced in the camera coordinate system with its projection point (2D vector) $\mathbf{p}' = [x', y']^T$ on the virtual image plane.
- By employing the similarity of triangles $\mathbf{O}_c\mathbf{o}'\mathbf{p}'$ and $\mathbf{O}_c\mathbf{Z}_c\mathbf{P}_c$:

$$\frac{x'}{X_c} = \frac{y'}{Y_c} = \frac{f}{Z_c}, \qquad x' = f\frac{X_c}{Z_c}, \qquad y' = f\frac{Y_c}{Z_c}$$

- Coordinates on the real image plane are given by the same equations, differing only by a minus sign.

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The Weak-Perspective Camera Model

- Perspective projection equations are rational, rather than linear. Thus:
  - straight lines are mapped to straight lines but
  - distances between points and the angles between straight lines are not preserved after projection.

- We can linearise them applying two transformations:
  - *orthographic projection* $x' = X_c$ and $y' = Y_c$ and
  - *isotropic scaling* $f/\bar{Z}$

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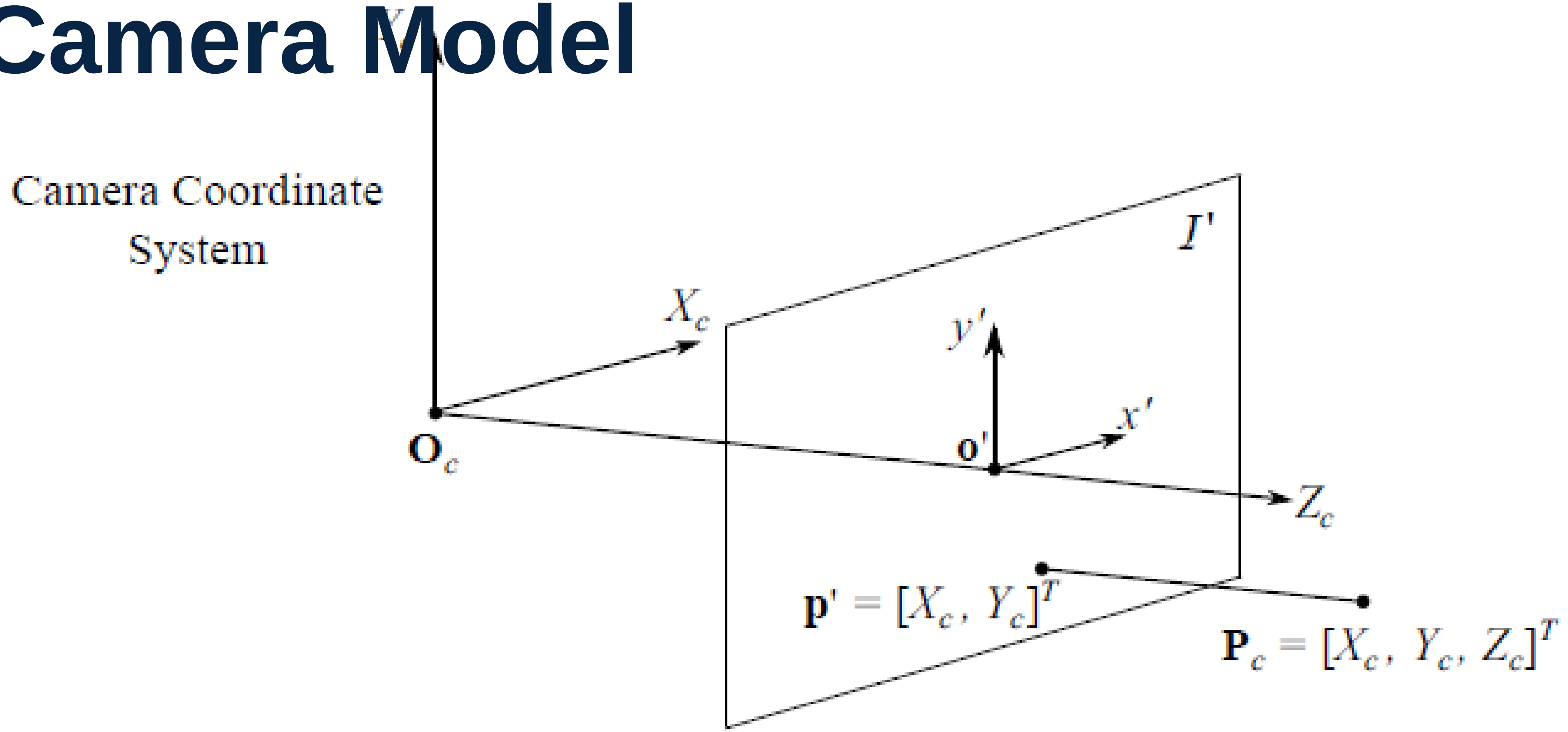
The Weak-Perspective Camera Model



Orthographic projection

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The Weak-Perspective Camera Model

- Isotropic scaling transformation leads to linearly approximate perspective projection equations, defining the so called *weak-perspective* camera model:

$$x' = f\frac{X_c}{Z_c} \approx \frac{f}{\bar{Z}}X_c, \qquad y' = f\frac{Y_c}{Z_c} \approx \frac{f}{\bar{Z}}Y_c$$

- The weak-perspective camera model is only an approximation of the pinhole camera imaging. It holds if the *relative distance* dZ_c for any pair of scene points along the optical axis is much smaller than $\bar{Z}$.

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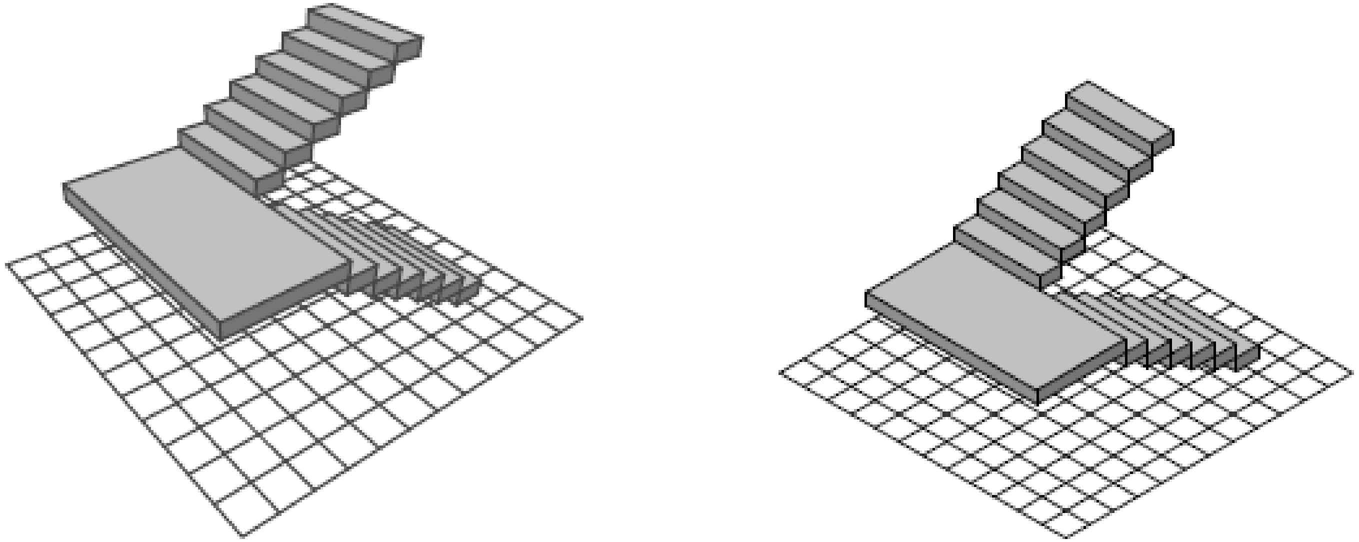
The Weak-Perspective Camera Model

- While a weak-perspective camera preserves parallelism in the projected lines, as orthographic projection does (b), perspective projection (a) does not.



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Central Projection and Homogeneous Coordinates

- A way to linearize the perspective projection equations is by using the so-called *homogeneous coordinates.*
  - A 2D image point is mapped to a 3D point:

    $\mathbf{p} = [x, y]^T \in \mathbb{R}^2 \longrightarrow \mathbf{p}_H = [x, y, 1]^T \in \mathbb{P}^2$.
  - A 3D scene point is mapped to a 4D point:

    $\mathbf{P} = [X, Y, Z]^T \in \mathbb{R}^3 \longrightarrow \mathbf{P}_H = [X, Y, Z, 1]^T \in \mathbb{P}^3$.

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Central Projection and Homogeneous Coordinates

- The linear relationship that connects $\mathbf{p}_H$ and $\mathbf{P}_H$ is:

$$Z\mathbf{p}_H = \begin{bmatrix} Zx \\ Zy \\ Z \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \mathcal{P}\mathbf{P}_H$$

where $\mathcal{P}$ is the so-called camera *perspective projection matrix,* a 3×4 full row rank homogeneous matrix with 11 *degrees of freedom.*

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Camera Parameters and Projection Matrix

- Object coordinates **P** in the camera coordinate system are, in most cases, unknown, whereas in the *world coordinate system* they may be known.
- The required transformation from the world to the camera coordinate system involves a translation followed by a rotation, based on the *extrinsic* camera parameters.
- Projection on the image plane requires the *intrinsic* camera parameters.

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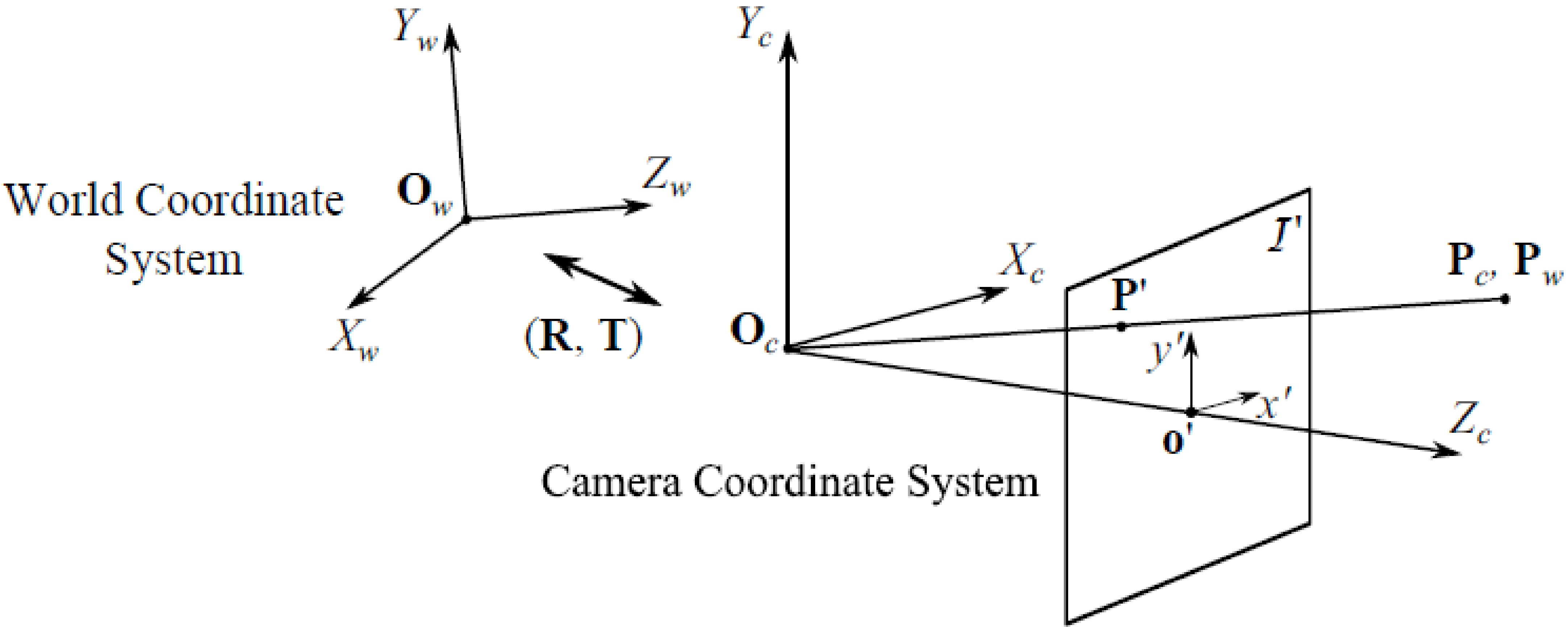
Camera Parameters and Projection Matrix



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Camera Parameters and Projection Matrix

- Extrinsic camera parameters:
  - *Translation vector* $\mathbf{T} \in \mathbb{R}^3$ (3 degrees of freedom)
  - *Orthonormal rotation matrix* $\mathbf{R} \in \mathbb{R}^{3\times3}$ (3 degrees of freedom: only 3 of the 9 rotation matrix entries are independent from each other).
- The relationship between a point $\mathbf{P}_w \in \mathbb{R}^3$ in world coordinates and its camera coordinate counterpart $\mathbf{P}_c \in \mathbb{R}^3$ is:

$$\mathbf{P}_c = \mathbf{R}(\mathbf{P}_w - \mathbf{T})$$

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Camera Parameters and Projection Matrix

- If the three rows of the rotation matrix and focal length *f* are known, the image coordinates *x'*, *y'* on the virtual image plane are given by:

$$x' = f\frac{\mathbf{R}_1^T(\mathbf{P}_w - \mathbf{T})}{\mathbf{R}_3^T(\mathbf{P}_w - \mathbf{T})}$$

$$y' = f\frac{\mathbf{R}_2^T(\mathbf{P}_w - \mathbf{T})}{\mathbf{R}_3^T(\mathbf{P}_w - \mathbf{T})}$$

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Camera Parameters and Projection Matrix

- In reality, the virtual image plane $\mathcal{I}'$ does not exist. The real 2D image plane (image sensor surface) is digitized.
- The transformation of an image point $\mathbf{p} = [x, y]^T$ on the image plane coordinates to the corresponding discrete point $\mathbf{p}_d = [x_d, y_d]^T$ in pixel coordinates, is given by:

$$x = -(x_d - o_x)s_x, \qquad y = -(y_d - o_y)s_y$$

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Camera Parameters and Projection Matrix

where:

- $[o_x, o_y]^{\mathrm{T}}$: the location of the principal camera point $\mathbf{o}$ in pixel coordinates.
- s_x, s_y: the effective pixel sizes in millimeters
- Coordinate system origin: at the top left corner of the image, not at the image center.

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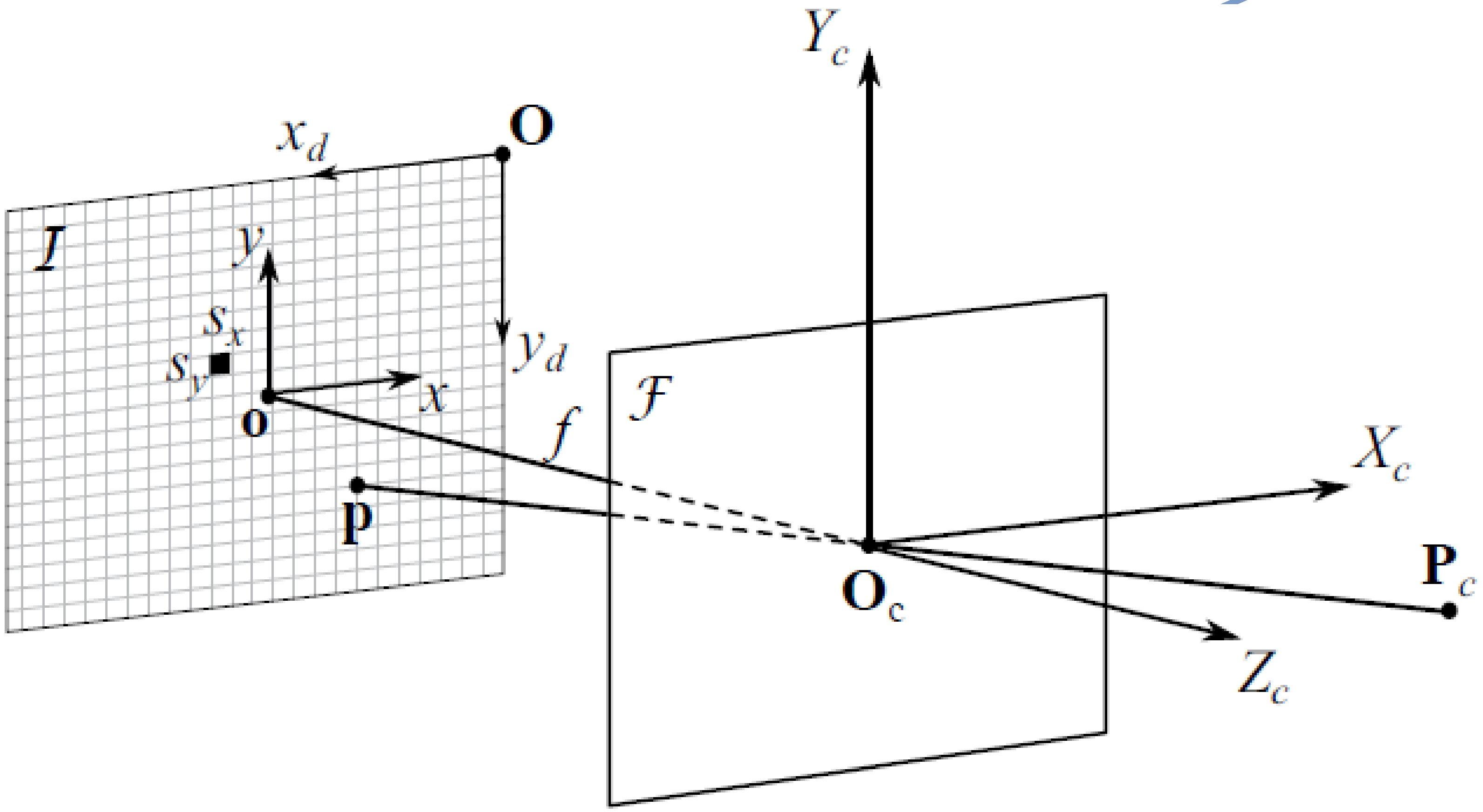
Camera Parameters and Projection Matrix



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Camera Parameters and Projection Matrix

- The transformation relating the image pixel coordinates with the world coordinates is:

$$x_d = o_x - \frac{f}{s_x} \frac{\mathrm{R}_1^T(\mathrm{P}_w - \mathrm{T})}{\mathrm{R}_3^T(\mathrm{P}_w - \mathrm{T})} \qquad y_d = o_y - \frac{f}{s_y} \frac{\mathrm{R}_2^T(\mathrm{P}_w - \mathrm{T})}{\mathrm{R}_3^T(\mathrm{P}_w - \mathrm{T})}$$

- It can be linearized in homogeneous coordinates, by decomposing the transformation into a sequence of two transformations:
  - Map a world coordinate point to camera coordinates.
  - Map the came coordinate point to homogeneous image pixel coordinates.

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Camera Parameters and Projection Matrix

- Definition of the 3×4 matrix of extrinsic parameters $\mathbf{P}_E$:

$$\mathrm{P}_E = \begin{bmatrix} r_{11} & r_{12} & r_{13} & -\mathrm{R}_1^T\mathrm{T} \\ r_{21} & r_{22} & r_{23} & -\mathrm{R}_2^T\mathrm{T} \\ r_{31} & r_{32} & r_{33} & -\mathrm{R}_3^T\mathrm{T} \end{bmatrix}$$

- Definition of the 3×3 matrix of intrinsic parameters $\mathbf{P}_I$:

$$\mathrm{P}_I = \begin{bmatrix} -\frac{f}{s_x} & 0 & o_x \\ 0 & -\frac{f}{s_y} & o_y \\ 0 & 0 & 1 \end{bmatrix}$$

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Camera Parameters and Projection Matrix

- The transformation of a point $\mathbf{P} \in \mathbb{P}^3$ to $\mathbf{p} \in \mathbb{P}^2$ is given by:

$$\begin{bmatrix} Zx_d \\ Zy_d \\ Z \end{bmatrix} = \mathrm{P}_I \mathrm{P}_E \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} \qquad \mathrm{p} = \mathrm{P}_I \mathrm{P}_E \mathrm{P} = \mathcal{P}\mathrm{P}$$

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Camera Parameters and Projection Matrix

- Where $\mathcal{P} = \mathbf{P}_I \; \mathbf{P}_E$ is the 3×4 *camera projection matrix*, also called *camera calibration matrix*

$$\mathcal{P} = \begin{bmatrix} -\frac{f}{s_x} & 0 & o_x \\ 0 & -\frac{f}{s_y} & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & -\mathbf{R}_1^T\mathbf{T} \\ r_{21} & r_{22} & r_{23} & -\mathbf{R}_2^T\mathbf{T} \\ r_{31} & r_{32} & r_{33} & -\mathbf{R}_3^T\mathbf{T} \end{bmatrix}$$

- $\mathbf{P}_E$ has the form $\mathbf{P}_E = [\mathbf{R}_1 | \mathbf{R}_2 | \mathbf{R}_3 | -\mathbf{RT}]$, since we assume that the camera coordinate system is first translated and then rotated. Otherwise it would be $\mathbf{P}_E = [\mathbf{R}_1 | \mathbf{R}_2 | \mathbf{R}_3 | \mathbf{T}]$.

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Camera Parameters and Projection Matrix

- For reasons of simplicity, it is common to assume that:
  - the origins of both the pixel coordinate system and the image plane coordinate system coincide with the principal point, $o_x = o_y = 0$ and
  - pixels are square having unit edge length $s_x = s_y = 1$.

- The projection matrix, can thus be rewritten as:

$$\mathcal{P} = \begin{bmatrix} -fr_{11} & -fr_{12} & -fr_{13} & f\mathrm{R}_1^T\mathrm{T} \\ -fr_{21} & -fr_{22} & -fr_{23} & f\mathrm{R}_2^T\mathrm{T} \\ r_{31} & r_{32} & r_{33} & -\mathrm{R}_3^T\mathrm{T} \end{bmatrix}$$

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Camera Parameters and Projection Matrix

- If the two axes of the coordinate system (x_d, y_d) are not exactly perpendicular (non-rectangular pixels), the projection matrix takes the form:

$$\mathcal{P} = \begin{bmatrix} -\frac{f}{s_x} & s_\theta & o_x \\ 0 & -\frac{f}{s_y} & o_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{R} & -\mathbf{RT} \\ \mathbf{0} & 1 \end{bmatrix}$$

- s_θ: *skew factor,* proportional to $\frac{1}{\tan\theta}$,
- θ: the angle between the pixel coordinate system axes.
- Typically $\theta = 90^o$, and hence $s_\theta = 0$.

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Camera Parameters and Projection Matrix

- Assuming $s_\theta = 0$ and treating the ratios $a_x = -\frac{f}{s_x}$ and $a_y = -\frac{f}{s_y}$ as single quantities, by expressing the focal length in terms of pixel dimensions along the horizontal and vertical dimension, $\mathbf{P}_I$ can be rewritten as:

$$\mathbf{P}_I = \begin{bmatrix} a_x & 0 & o_x \\ 0 & a_y & o_y \\ 0 & 0 & 1 \end{bmatrix}$$

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Properties of the Projective Transformation

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- The projective transformation does not change the cross-ratio C_r of four collinear points.
- *Line at infinity*: the set of all points at infinity in $\mathbb{P}^2$.
- *Plane at infinity*: the set of all points at infinity in $\mathbb{P}^3$, formed by an infinite number of lines at infinity corresponding to different plane directions.
- *Vanishing points:* the points of intersection of the projected lines formed by parallel lines in the 3D Euclidean space.

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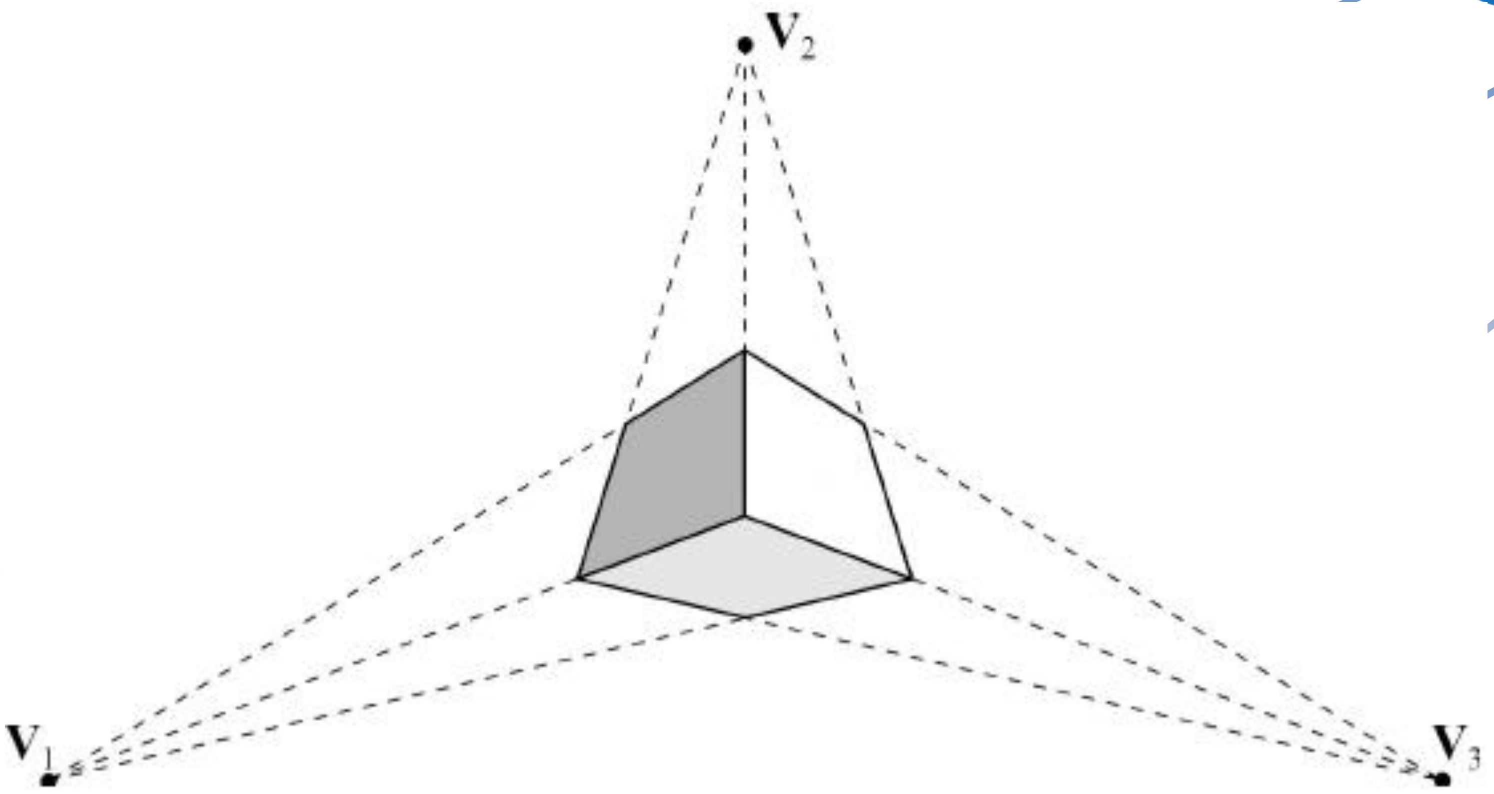
Properties of the Projective Transformation



Vanishing points

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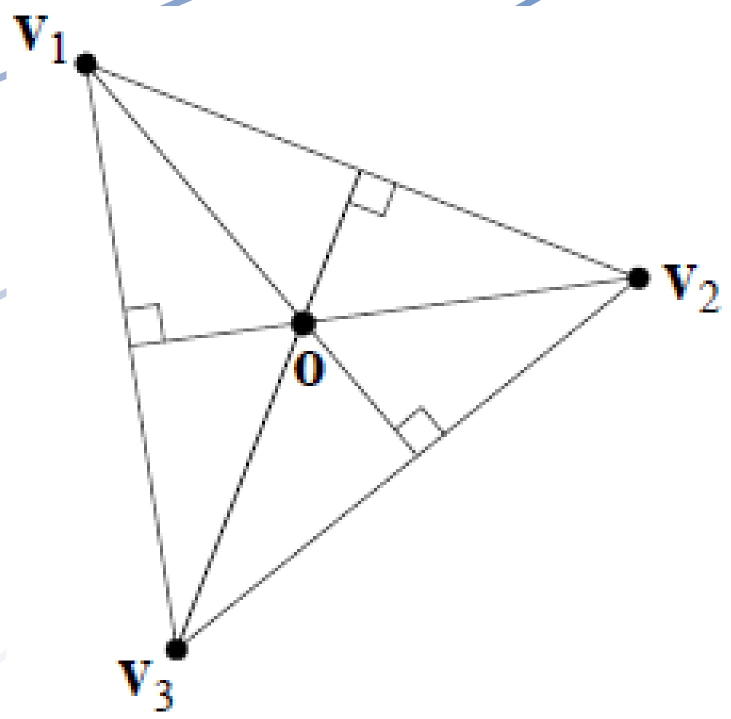
Properties of the Projective Transformation

- The orthocenter of the triangle formed by the vanishing points $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, corresponding to three perpendicular parallel line directions in the world reference system, is the principal point $\mathbf{o}$ on the image plane.



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Properties of the Projective Transformation

MultiDrone

- *Cross-ratio (or anharmonic ratio)* C_r: ratio of ratios of distances between collinear points.
  - It is a geometric property invariant under a projective transformation.

- For four points $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4$ in $\mathbb{P}^2$:

$$C_r(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4) = \frac{\Delta_{13}\Delta_{24}}{\Delta_{14}\Delta_{23}}$$

Δ_{ij}: the Euclidean distance between points $\mathbf{p}_i$ and $\mathbf{p}_j$.

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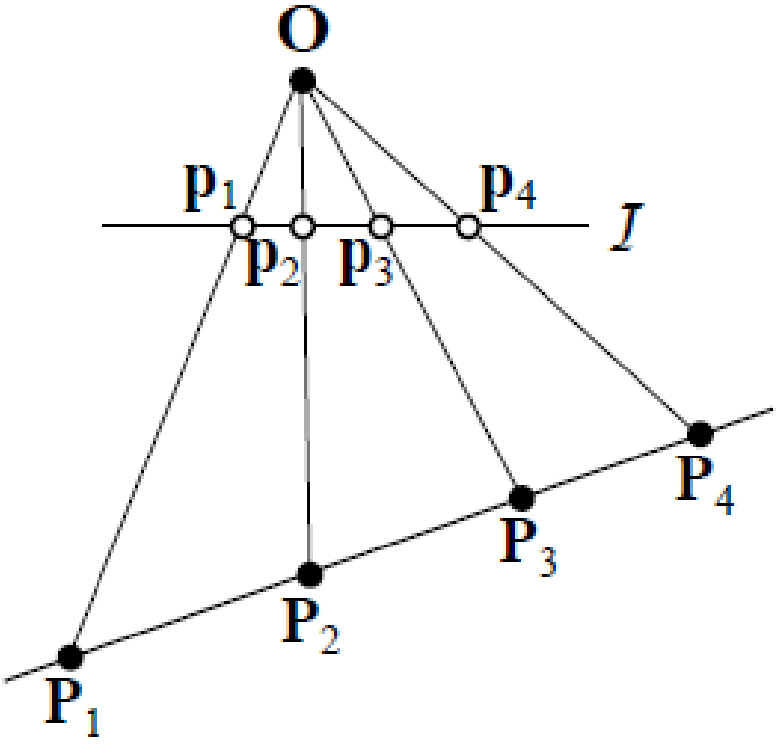
Properties of the Projective Transformation

- The cross-ratio of four collinear points remains invariant under a projective transformation: $C_r(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4) = C_r(\mathbf{P}_1, \mathbf{P}_2, \mathbf{P}_3, \mathbf{P}_4)$.



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Properties of the Projective Transformation

- *Vanishing lines*: the projections on the image of lines at infinity in $\mathbb{P}^3$, where parallel planes in the 3D Euclidean space intersect.
- *Horizon line*: the vanishing line of the ground plane and its parallel planes.
  - The projections of parallel lines lying on a plane that forms an angle θ with the ground, intersect either above or below the horizon line, depending on the sign of $\cos\theta$.

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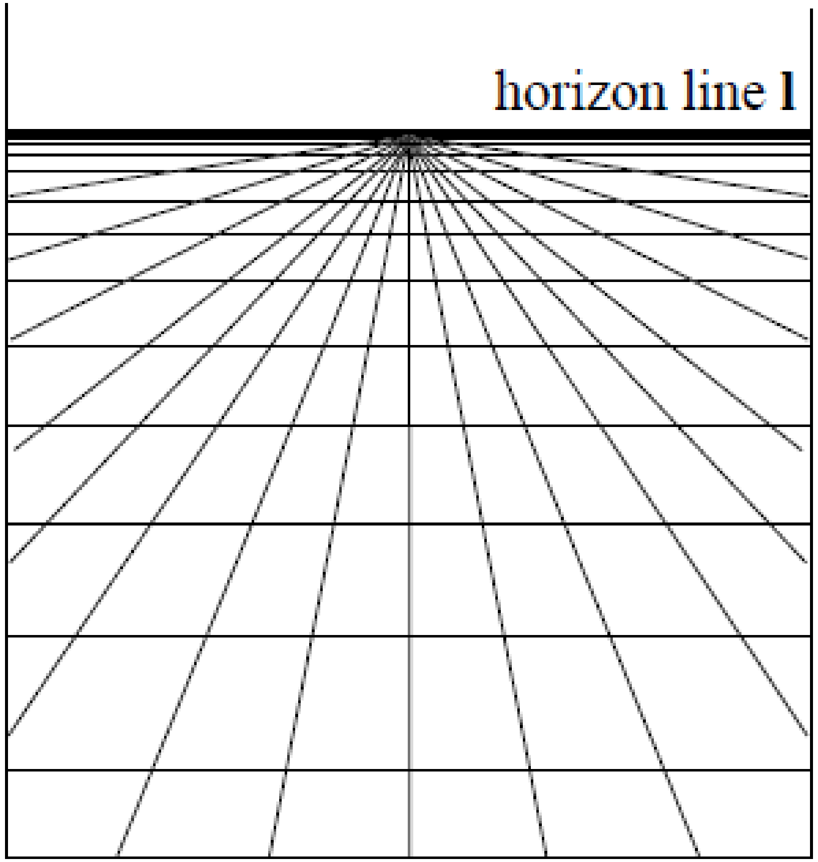
Properties of the Projective Transformation

- *Chirp effect*: the increase in local image spatial frequency proportionally to the distance of the projected scene area from the camera.
- It is evident in 2D image regions where distant and close-up scene parts are projected.



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Cameras at Infinity

- When the center of projection lies at infinity, instead of perspective projection, the orthographic camera and the weak-perspective camera model are used.
- Weak-perspective camera model projection matrix $\mathcal{P}_{wp}$:

$$\mathcal{P}_{wp} = \begin{bmatrix} -fr_{11} & -fr_{12} & -fr_{13} & f\mathrm{R}_1^T\mathrm{T} \\ -fr_{21} & -fr_{22} & -fr_{23} & f\mathrm{R}_2^T\mathrm{T} \\ 0 & 0 & 0 & \mathrm{R}_3^T(\overline{\mathrm{P}} - \mathrm{T}) \end{bmatrix}$$

$\overline{\mathbf{P}}$: centroid of the viewed object.

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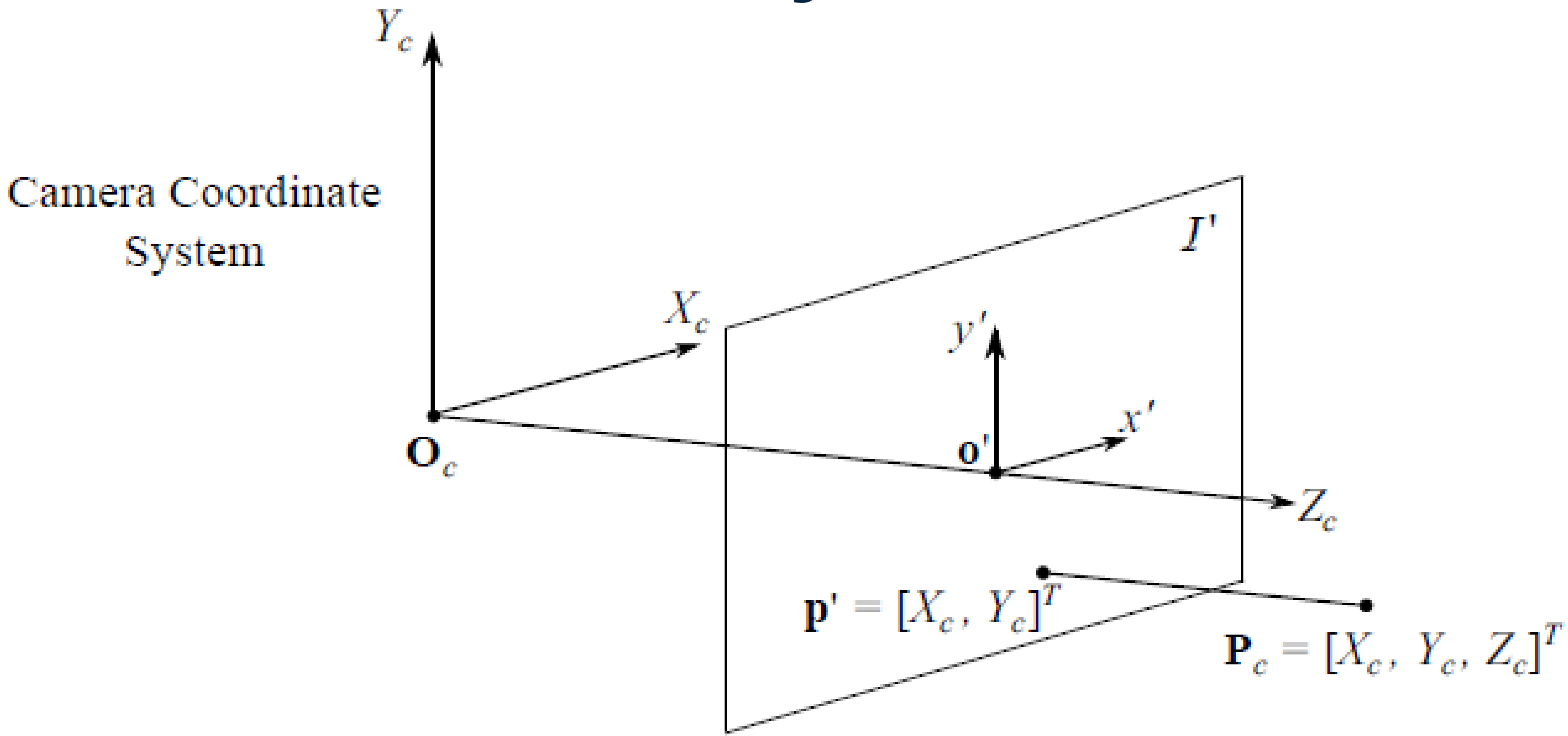
Cameras at infinity



Orthographic projection

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Cameras at Infinity

- The weak-perspective projection is valid, if the depth variations amongst the points of a viewed object are small, in comparison with its average distance from the camera (object depth), represented by $\mathbf{R}_3{}^T(\bar{\mathbf{P}} - \mathbf{T})$.
- Another camera-at-infinity model is the *affine camera model,* with projection matrix:

$$\mathcal{P}_{af} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ 0 & 0 & 0 & a_{34} \end{bmatrix}$$

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Cameras at Infinity

- The left 3×3 sub-matrix of $\mathcal{P}_{af}$ is singular.
- $\mathcal{P}_{af}$ has 9 independent entries.

- Main difference between the affine camera model and the projective framework:
  - In the projective framework, no distinction exists between points at infinity and the usual finite points of the affine space.

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Cameras at Infinity

- The affine camera model does **not** really describe any physical camera, but is used due to its simplicity.

- Main difference between the affine camera model and the weak-perspective model:
  - It preserves straight line parallelism, but **not** angles, since it may entail anisotropic scaling.

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Camera Calibration

- Camera calibration deals with determining the extrinsic and intrinsic camera parameters.

- To this end, a *calibration pattern*, also called *calibration grid* is employed, so that the projection matrix $\mathcal{P}$ of the camera can be computed from a single view by mapping known points $\mathbf{P}_w = [X_w, Y_w, Z_w]^T$ in world coordinates to their projections $\mathbf{p} = [x, y]^T$ on the image plane.

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Camera Calibration

- Taking as many such mappings as needed, a system of equations is formed.
- The solution of the system leads to the determination of the unknown extrinsic and intrinsic camera parameters.

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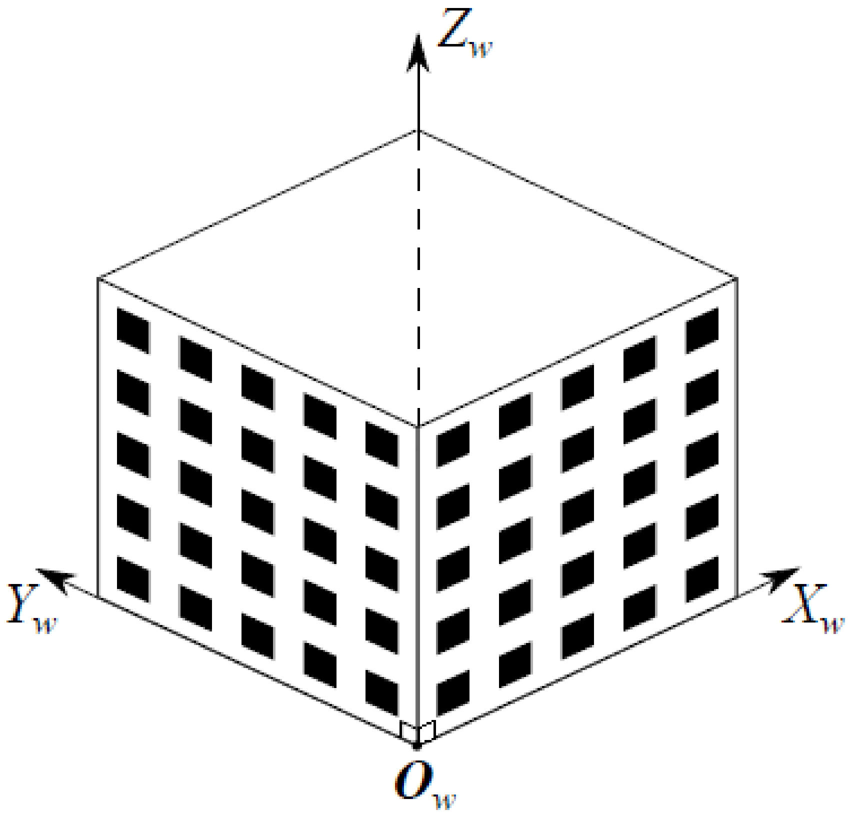
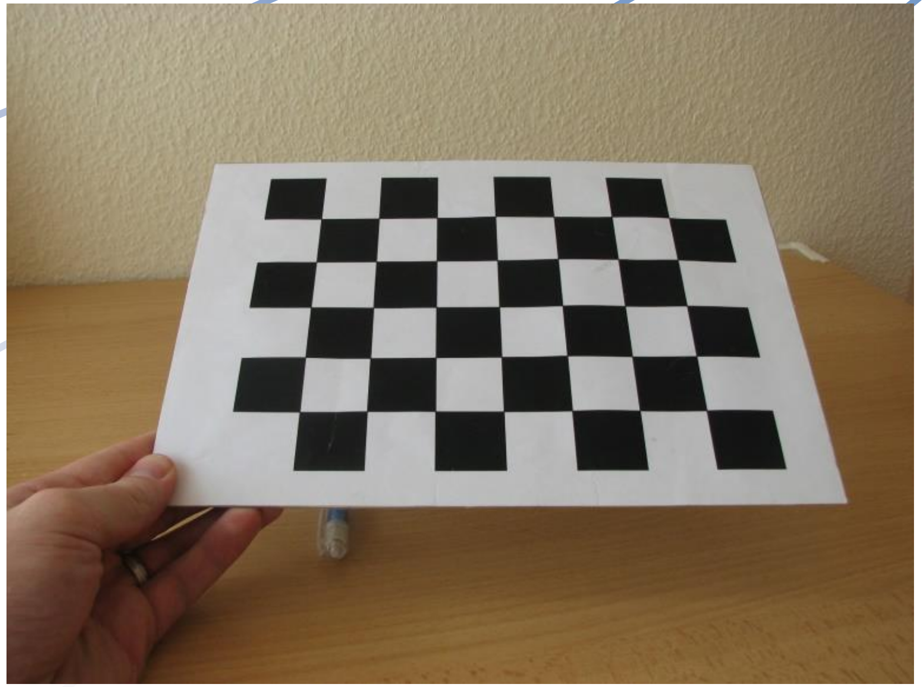
Camera Calibration





Calibration patterns.

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Camera Calibration

MultiDrone

- The calibration pattern is a 3D object of common, known dimensions and positioning, with a checkerboard pattern clearly visible on each side.
- Pattern dimensions must be known, in an accuracy much greater than the desired calibration accuracy.

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Camera Calibration

- The most popular calibration techniques utilizing solely a planar calibration pattern are:
  - *Direct camera parameter estimation* and
  - *Zhang's calibration method.*

- Other calibration methods do not require a calibration object and are jointly referenced by the term *self-calibration* or *autocalibration.*

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Direct camera parameter estimation

- $\mathbf{P}_w = [X_w, Y_w, Z_w]^T$: a known point in world coordinates.
- $\mathbf{P}_c = [X_c, Y_c, Z_c]^T$: the same point in camera coordinates.
- $\mathbf{p}_d = [x_d, y_d]^T$: its image point in pixel coordinates.

- The transformation between the world and camera coordinate systems involves an orthonormal 3×3 rotation matrix $\mathbf{R}$ and a 3×1 translation vector $\mathbf{T}$ (equivalent to determining the extrinsic camera parameters).

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Direct camera parameter estimation

$$\begin{bmatrix} X_c \\ Y_c \\ Z_c \end{bmatrix} = \mathrm{R} \begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} + \mathrm{T} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{33} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \end{bmatrix} + \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix}$$

- It can be decomposed into:

$$X_c = r_{11} X_w + r_{12} Y_w + r_{13} Z_w + T_x$$
$$Y_c = r_{21} X_w + r_{22} Y_w + r_{23} Z_w + T_y$$
$$Z_c = r_{31} X_w + r_{32} Y_w + r_{33} Z_w + T_z$$

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Direct camera parameter estimation

- Assuming $o_x = o_y = 0$, the point $\mathbf{P}_c$ in the camera coordinate system is related to the pixel coordinates of point $\mathbf{p}_d$ by:

$$x_d = -\frac{f}{s_x}\frac{X_c}{Z_c} \qquad y_d = -\frac{f}{s_y}\frac{Y_c}{Z_c}$$

and finally:

$$x_d = -\frac{f}{s_x}\frac{r_{11}X_w + r_{12}Y_w + r_{13}Z_w + T_x}{r_{31}X_w + r_{32}Y_w + r_{33}Z_w + T_z}$$

$$y_d = -\frac{f}{s_y}\frac{r_{21}X_w + r_{22}Y_w + r_{23}Z_w + T_y}{r_{31}X_w + r_{32}Y_w + r_{33}Z_w + T_z}$$

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Direct camera parameter estimation

MultiDrone

- Using a sufficient number of world coordinate and pixel coordinate point pairs, equations can be formulated and solved for the unknown camera parameters:
  - $r_{11}, r_{12}, \dots, r_{33}, T_x, T_y, T_z, s_x, s_y, f$.

- It should be noted that knowledge of the ratios f/s_x , f/s_y , rather than of all internal camera parameters suffices for camera calibration.

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Direct camera parameter estimation

- Internal camera parameters:
  - f: focal length in pixel length.
  - s_x, s_y: pixel size.
  - o_x, o_y: camera center coordinates.

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Direct camera parameter estimation

$$x_d = -\frac{f}{s_x}\frac{r_{11}X_w + r_{12}Y_w + r_{13}Z_w + T_x}{r_{31}X_w + r_{32}Y_w + r_{33}Z_w + T_z}$$

$$y_d = -\frac{f}{s_y}\frac{r_{21}X_w + r_{22}Y_w + r_{23}Z_w + T_y}{r_{31}X_w + r_{32}Y_w + r_{33}Z_w + T_z}$$

- Since the two equations have the same denominator, for each pair of 3D points $\mathbf{P}_{wi} = [X_{wi}, Y_{wi}, Z_{wi}]^T$ and their image points $\mathbf{p}_{di} = [x_{di}, y_{di}]^T$, $i = 1, \ldots, N$, we have:

$$x_{di}\frac{f}{s_y}(r_{21}X_{wi} + r_{22}Y_{wi} + r_{23}Z_{wi} + T_y) = y_{di}\frac{f}{s_x}(r_{11}X_{wi} + r_{12}Y_{wi} + r_{13}Z_{wi} + T_x)$$

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Direct camera parameter estimation

- By using the pixel aspect ration $a = {}^{s_x}/_{s_y}$, putting all the equation terms on the left side and employing N point pairs, we get:

$$
\begin{aligned}
x_{d1} r_{21} X_{w1} &+ \cdots + x_{d1} T_y - y_{d1} \alpha r_{11} X_{w1} - \cdots - y_{d1} \alpha T_x &= 0\\
x_{d2} r_{21} X_{w2} &+ \cdots + x_{d2} T_y - y_{d2} \alpha r_{11} X_{w2} - \cdots - y_{d2} \alpha T_x &= 0\\
\vdots \quad & \quad \vdots \quad \vdots \quad \quad \vdots \quad \quad \vdots \quad \vdots & \\
x_{dN} r_{21} X_{wN} &+ \cdots + x_{dN} T_y - y_{dN} \alpha r_{11} X_{wN} - \cdots - y_{dN} \alpha T_N &= 0.
\end{aligned}
$$

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Direct camera parameter estimation

- Each of the linear homogeneous equations has 8 unknown parameters:
  - $\mathbf{u} = \left[ar_{11}, ar_{12}, ar_{13}, r_{21}, r_{22}, r_{23}, T_x, T_y\right]^T \triangleq [u_1, u_2, \dots, u_8]^T$
- Expressing the homogeneous system of equation as a product of the matrix $\mathbf{X}$

$$\mathbf{X} \triangleq \begin{bmatrix} x_{d1}X_{w1} & x_{d1}Y_{w1} & x_{d1}Z_{w1} & x_{d1} & -y_{d1}X_{w1} & -y_{d1}Y_{w1} & -y_{d1}Z_{w1} & -y_{d1} \\ x_{d2}X_{w2} & x_{d2}Y_{w2} & x_{d2}Z_{w2} & x_{d2} & -y_{d2}X_{w2} & -y_{d2}Y_{w2} & -y_{d2}Z_{w2} & -y_{d2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{dN}X_{wN} & x_{dN}Y_{wN} & x_{dN}Z_{wN} & x_{dN} & -y_{dN}X_{wN} & -y_{dN}Y_{wN} & -y_{dN}Z_{wN} & -y_{dN} \end{bmatrix}$$

$$\mathbf{X}\mathbf{u} = 0$$

and the vector $\mathbf{u}$:

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Direct camera parameter estimation

- Thus, the desired solution is the *null space* of the matrix $\mathbf{X}$.
- Provided that $N \geq 7$ pairs are not coplanar:
  - Matrix $\mathbf{X}$ will have rank 7.
  - The system of equations will have one non-trivial solution $\mathbf{u} \neq \mathbf{0}$, obtained via the *singular value decomposition* (SVD) of matrix $\mathbf{X}$:
  $$\mathbf{X} = \mathbf{U\Sigma V}^T$$
  - $\mathbf{\Sigma}$ is a diagonal matrix containing the singular values. The solution $\mathbf{u}$ is the column of matrix $\mathbf{V}$ corresponding to the zero singular value of $\boldsymbol{\Sigma}$.

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Zhang's calibration method

- It uses a planar calibration pattern, posing in N different orientations $(N \geq 2)$, by moving either the pattern of the camera.
  - Precise knowledge of this motion is not required.
  - The calibration pattern can simply be printed on a paper and attached to any planar surface.
  - It has educed complexity.

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Zhang’s calibration method

- Let $\mathbf{P}_I$ be the intrinsic parameters matrix:

$$\mathbf{P}_I = \begin{bmatrix} \alpha_x & s_\theta & o_x \\ 0 & \alpha_y & o_y \\ 0 & 0 & 1 \end{bmatrix}$$

where $= [o_x, o_y]^T$ the principal point coordinates, s_θ the skew factor and $a_x = -\frac{f}{s_x}, a_y = -\frac{f}{s_y}$.

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Zhang's calibration method

- Assuming the calibration pattern lies on the scene plane $Z_w = 0$ (in world coordinates):

$$s \begin{bmatrix} x_d \\ y_d \\ 1 \end{bmatrix} = \mathbf{P}_I[\mathbf{R}_1|\mathbf{R}_2|\mathbf{R}_3|\mathbf{T}] \begin{bmatrix} X_w \\ Y_w \\ 0 \\ 1 \end{bmatrix} = \mathbf{P}_I[\mathbf{R}_1|\mathbf{R}_2|\mathbf{T}] \begin{bmatrix} X_w \\ Y_w \\ 1 \end{bmatrix}$$

where $\mathbf{P}_w = [X_w, Y_w, 0]^T$ a scene point on the calibration pattern, $\mathbf{p} = [x_d, y_d]^T$ its two-dimensional image and s is just a scale factor.

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Zhang's calibration method

- A 3×3 homography matrix $\mathbf{H}$ can be defined up to a scale factor, relating $\mathbf{P}$ and $\mathbf{p}$:

$$s\mathbf{p} = \mathbf{HP} = \mathbf{P}_I[\mathbf{R}_1|\mathbf{R}_2|\mathbf{T}]\mathbf{P}$$

Therefore:

$$\mathbf{H} = \lambda\mathbf{P}_I[\mathbf{R}_1|\mathbf{R}_2|\mathbf{T}]$$

where λ is a scale factor and $\mathbf{H} \triangleq [\mathbf{h}_1\mathbf{h}_2\mathbf{h}_3]$

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Zhang's calibration method

- From known 3D and 2D correspondences, such a homography can be estimated iteratively and by utilizing

$$s\mathbf{p} = \mathbf{HP} = \mathbf{P}_I[\mathbf{R}_1|\mathbf{R}_2|\mathbf{T}]\mathbf{P}$$

to form an objective function for optimization with the aid of a non-linear optimization algorithm, like Levenberg-Marquardt:

$$\min_{\mathbf{H}_i} E(\mathbf{H}_i) = \sum_{j=1}^{M} \|\mathbf{p}_{ij} - \hat{\mathbf{H}}_i \mathbf{P}_j\|^2, \ i = 1, \ldots, N.$$

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Zhang's calibration method

- Given a known $\mathbf{H}$, let ω be a symmetric matrix:

$$\boldsymbol{\omega} \triangleq \begin{bmatrix} \omega_{11} & \omega_{12} & \omega_{13} \\ \omega_{21} & \omega_{22} & \omega_{23} \\ \omega_{31} & \omega_{32} & \omega_{33} \end{bmatrix} = \mathbf{P}_I^{-T}\mathbf{P}_I^{-1} =$$

$$= \begin{bmatrix} \frac{1}{\alpha_x^2} & \frac{-s_\theta}{\alpha_x^2\alpha_y} & \frac{o_y s_\theta - o_x\alpha_y}{\alpha_x^2\alpha_y} \\ \frac{-s_\theta}{\alpha_x^2\alpha_y} & \frac{s_\theta^2}{\alpha_x^2\alpha_y^2} + \frac{1}{\alpha_y^2} & \frac{-s_\theta(o_y s_\theta - o_x\alpha_y)}{\alpha_x^2\alpha_y^2} - \frac{o_y}{\alpha_y^2} \\ \frac{o_y s_\theta - o_x\alpha_y}{\alpha_x^2\alpha_y} & \frac{-s_\theta(o_y s_\theta - o_x\alpha_y)}{\alpha_x^2\alpha_y^2} - \frac{o_y}{\alpha_y^2} & \frac{(o_y s_\theta - o_x\alpha_y)^2}{\alpha_x^2\alpha_y^2} + \frac{o_y^2}{\alpha_y^2} + 1 \end{bmatrix}$$

which can be represented by $\mathbf{b} = [\omega_{11}, \omega_{12}, \omega_{22}, \omega_{13}, \omega_{23}, \omega_{33}]^T$

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Zhang's calibration method

- Since $\mathbf{R}_1$ and $\mathbf{R}_2$ are orthogonal, $s\mathbf{p} = \mathbf{HP} = \mathbf{P}_I[\mathbf{R}_1|\mathbf{R}_2|\mathbf{T}]\mathbf{P}$ and the definition of ω entail that:

$$\mathbf{h}_1^T \omega \mathbf{h}_2 = 0 \qquad \mathbf{h}_1^T \omega \mathbf{h}_1 = \mathbf{h}_2^T \omega \mathbf{h}_2. \qquad \mathbf{h}_i^T \omega \mathbf{h}_j = \mathbf{v}_{ij}^T \mathbf{b}$$

where:

$$\mathbf{v}_{ij}^T \triangleq [h_{i1}h_{j1}, h_{i1}h_{j2}+h_{i2}h_{j1}, h_{i2}h_{j2}, h_{i3}h_{j1}+h_{i1}h_{j3}, h_{i3}h_{j2}+h_{i2}h_{j3}, h_{i3}h_{j3}]$$

- Based on the way $\mathbf{v}$ is defined,

$$\begin{bmatrix} \mathbf{v}_{12}^T \\ (\mathbf{v}_{11} - \mathbf{v}_{22})^T \end{bmatrix} \mathbf{b} = \mathbf{0}$$

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Zhang's calibration method

- By taking simultaneously into account N different images of the calibration pattern, and using N different homography matrices $\mathbf{H}_i, i = 1, \ldots, N$, the system of N corresponding equations can be compactly restated as:

$$\mathbf{Ab} = 0,$$

where $\mathbf{A}$ is a $2N \times 6$ matrix.

- This system can be solved for $\mathbf{b}$ applying SVD to $\mathbf{A} = \mathbf{UDV}^T$.

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Zhang's calibration method

- The intrinsic camera parameters can be estimated from ω:

$$o_y = \frac{(\omega_{12}\omega_{13} - \omega_{11}\omega_{23})}{\omega_{11}\omega_{22} - \omega_{12}^2}$$

$$\lambda = \omega_{33} - \frac{\omega_{13}^2 + o_y(\omega_{12}\omega_{13} - \omega_{11}\omega_{23})}{\omega_{11}}$$

$$\alpha_x = \sqrt{\frac{\lambda}{\omega_{11}}}$$

$$\alpha_y = \sqrt{\frac{\lambda\omega_{11}}{\omega_{11}\omega_{22} - \omega_{12}^2}}$$

$$s_\theta = -\frac{\omega_{12}\alpha_x^2\alpha_y}{\lambda}$$

$$o_x = \frac{s_\theta o_y}{\alpha_y} - \frac{\omega_{13}\alpha_x^2}{\lambda}.$$

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Zhang's calibration method

- Having computed the intrinsic camera parameters, the matrix $\mathbf{P}_I$ can also be estimated.
- Thus, by substituting $\mathbf{P}_I$ in the equations:

$$\mathbf{p}_{ij} = \lambda \mathbf{P}_I [\mathbf{R}_{1i} | \mathbf{R}_{2i} | \mathbf{T}_i] \mathbf{P}_j, \quad i = 1, \ldots, N, \quad j = 1, \ldots, M$$

we can solve for the columns of the rotation matrix and the translation vector, in order to obtain the extrinsic camera parameters (rotation matrix $\mathbf{R}_i$, translation vector $\mathbf{T}_i$).

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Zhang's calibration method

$$\mathbf{R}_{1i} = \lambda \mathbf{P}_I^{-1} \mathbf{h}_{1i}$$
$$\mathbf{R}_{2i} = \lambda \mathbf{P}_I^{-1} \mathbf{h}_{2i}$$
$$\mathbf{R}_{3i} = \mathbf{R}_{1i} \times \mathbf{R}_{2i}$$
$$\mathbf{T}_i = \lambda \mathbf{P}_I^{-1} \mathbf{h}_{3i}$$

$$\lambda = \frac{1}{\|\mathbf{P}_I^{-1}\mathbf{h}_{1i}\|} = \frac{1}{\|\mathbf{P}_I^{-1}\mathbf{h}_{2i}\|}.$$

- The above estimated results can be used as initializations for some repetitive optimization algorithm, so that refined results ones can be derived.

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Self-calibration

- These approaches:
  - do not require a calibration object and recover the camera parameters from image information alone;
  - are flexible but not very robust;
  - exploit properties of the absolute conic of the projective geometry;
  - typically require information equivalent to a partial 3D reconstruction of the scene.
- Self-calibration is strongly related to the geometry of multiple cameras.

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Q & A

Thank you very much for your attention!

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